Proceedings IRF2018: 6th International Conference Integrity-Reliability-Failure
Lisbon/Portugal 22-26 July 2018. Editors J.F. Silva Gomes and S.A. Meguid
Publ. INEGI/FEUP (2018); ISBN: 978-989-20-8313-1

PAPER REF: 7129

SOME QUASI-ISOTROPIC LAMINATES ARE MORE ISOTROPIC THAN OTHERS

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ABSTRACT

Because of their superior mechanical and environmental properties compared to traditional metals, fiber-reinforced composites have gained a widespread acceptance for various applications. However, laminated composites generally provide anisotropic stiffness properties depending on their stacking sequences. These anisotropic properties are not desired for certain applications that are subjected to multi-directional loadings requiring optimal stiffness properties in all directions. As a result, this paper aims to present a new lay-up strategy for multi-axial quasi-isotropic laminates to achieve the maximum degree of isotropy. The starting laminate configuration is a quasi-isotropic laminate in which extensional isotropy is inherently satisfied. As for the degree of bending isotropy, it can be quantified from a generalized eigenvalue problem of the bending stiffness tensor. A parametric design study was conducted on quasi-isotropic sub-laminates made from thin-ply non-crimp fabric **(NCF)**. Using both mid-plane symmetric and non-symmetric laminate configurations, the maximum degree of bending isotropy was achieved for non-symmetric laminates. A fully isotropic laminate **(FIL)** can then be obtained when both in-plane and bending stiffness tensors are equivalent to the isotropic stiffness counterpart with null coupling stiffness.

Keywords: composite laminate, non-crimp fabric **(NCF)**, quasi-isotropic, bending isotropy, fully isotropic laminate **(FIL)**.

INTRODUCTION

The tailoring potential of composite laminates to achieve high specific stiffness and strength has promoted them as promising candidates for constructing lightweight structures. The behavior of composite laminates is directly affected by the fiber orientation angles of their individual plies. As a result, the significant role of the designer is to tailor the stiffness properties of a composite laminate to encounter an applied load and yield a desirable laminate performance. However, certain applications are subjected to multi-directional loadings requiring optimal stiffness properties in all directions. In the case of in-plane multi-directional loading, quasi-isotropic laminates, which possess equivalent in-plane stiffness properties independent of any direction, show adequate performance if no coupling terms are present. These laminates are generally termed "Quasi" because they lack bending isotropy. On the contrary, when a laminate is also loaded normal to its plane in several directions, a need for isotropic performance in bending also arises. For example, large diameter mirrors that are used in reflector telescopes require an isotropic behavior, a low weight, and a negligible coefficient of thermal expansion (York, 2008). Another example includes that of an antenna structure which also necessitates equivalent isotropic properties (Fukunaga, 1990). These examples, along with several other automotive and aerospace applications that require lightweight isotropic properties, promote fully isotropic laminates **(FILs)** as ideal candidates. **FIL**s are laminates that possess both in-plane and bending isotropic properties with null coupling stiffnesses.

There has been some research efforts to find **FIL**s. Fukunaga presented a method to find fully isotropic laminates using lamination parameters, which are intermediate variables directly related to the stiffness matrices of the laminate. An **FIL** is characterized with isotropic stiffness properties where all the lamination parameters are set to zero. The inverse problem is then solved analytically to find the stacking sequences assuming only 4 different kinds of fiber orientation angles. **FIL**s with a minimum of 40 layers were obtained (Fukunaga, 1990). Wu et al. then proposed a mathematical model to obtain **FIL**s with a minimum of 36 plies by changing the stacking sequence order of quasi-isotropic laminates with mid-plane symmetry (Wu, 1992). Paradies also used lamination parameters to obtain laminates that are isotropic in bending only with a minimum of 16 plies of the same material. The concept of using a hybrid material laminate was also interestingly introduced to find a laminate satisfying bending isotropy with only 12 plies (Paradies, 1996). In addition, Grédiac formulated the design problem using lamination parameters and numerically obtained stacking sequences that are approximately isotropic with at least 12 plies using design optimization. The laminates were obtained by utilizing the steepest-descent method and a multiple point start algorithm (Grédiac, 1998). Vannucci and Verchery then obtained **FIL**s by utilizing the polar method and demonstrated that **FIL**s with the lowest number of 18 plies are obtained if the mid-plane symmetric constraint is relaxed for (0°, +60°, -60°) layers. Five solutions were found for laminates with 18 plies and 219 solutions were obtained for laminates with 27 plies (Vannucci, 2002). Furthermore, York demonstrated that the majority of **FIL**s are obtained using non-symmetric laminate configurations (York, 2008). Lee et al. also illustrated the use of mathematical modeling to obtain **FIL**s with up to 27 plies made of (0°, +60°, -60°) layers. Results agreed with Vannucci's findings except for laminates with 27 plies, where 340 solutions instead of 219 were claimed to be obtained (Lee, 2010).

FILs are desired from a design point of view because they eliminate the concern for weaknesses and counteract the "weak directions" of a laminate that are obtained due to unconsidered secondary loadings. **FIL** configurations ease the design problem for certain applications that require isotropic properties, where the designer benefits from the desirable mechanical and enviromental properties of composite materials as well as the isotropic behavior of conventional materials. As a result, a designer would be interested in obtaining an **FIL** with the smallest number of plies possible to produce efficient lightweight structures.

In the present study, a new lay-up strategy is proposed to attain laminates with the maximum degree of isotropy starting from an initial special *quasi-isotropic* material form made of [30/90/-30] and [-30/90/30] thin-ply *non-crimp fabric* **(QI-NCF)** that were supplied by CHOMARAT North America. Thin-ply **NCF** introduce attractive properties for lighweight applications, where they are constructed from a pre-assembly of multiple plies with different fiber orientation angles that result in a well-dispersed sub-laminate. Several desirable properties are obtained by using thin-ply **NCF** and are summarized in a recent study (Papila, 2016).

Starting from an initial "quasi-isotropic" construction, **QI-NCF**s are used to demonstrate a methodology to assess the degree of bending isotropy by formulating a generalized eigenvalue problem of the bending stiffness tensor. The classical lamination theory **(CLT)** provides a well-established methodology for evaluating elastic propoerties of a laminate with a known laminate configuration. No general rules are known today for designing fully isotropic laminates using a direct CLT framework. Most of the work done uses intermediate variables such as lamination parameters and polar invariants, which also possess beneficial properties to obtain design solutions for laminate classes other than **FIL**s (Vannucci,2002).

To demonstrate this new methodology, the generalized eigenvalue formulation of the degree of bending isotropy is introduced in the next section. The lay-up strategy for **QI-NCF** is then proposed, where successive **QI-NCFs** are rotated with respect to one reference **QI-NCF** to obtain the highest degree of bending isotropy. A parametric design study is carried on mid-plane symmetric and non-symmetric laminate configurations. An **FIL** is obtained when both, the in-plane and bending stiffness tensors are equivalent to the isotropic stiffness counterpart with null bending-extension coupling terms. The results of the parametric design study are then presented and discussed, followed by concluding remarks.

DEGREE OF BENDING ISOTROPY FORMULATION

The basis of laminate stiffness is formulated by using the classical lamination theory **(CLT)** satisfying the classical Kirchoff-Love assumptions for the laminate, with a through-the-thickness line perpendicular to the mid-plane that remains inextensible, straight, and perpendicular to the mid-plane after deformation. The laminate stiffness is represented by the extensional matrix **[A]**, the bending matrix **[D]**, and the bending-extension coupling matrix **[B]**, which can be defined as:

$$\begin{aligned} \boldsymbol{A_{ij}} &= \textstyle\sum_{k=1}^{N} \left(\overline{\boldsymbol{Q}}_{ij}\right)_k (z_k - z_{k-1}), \\ \boldsymbol{B_{ij}} &= \tfrac{1}{2}\textstyle\sum_{k=1}^{N} \left(\overline{\boldsymbol{Q}}_{ij}\right)_k (z_k^2 - z_{k-1}^2), \\ \boldsymbol{D_{ij}} &= \tfrac{1}{3}\textstyle\sum_{k=1}^{N} \left(\overline{\boldsymbol{Q}}_{ij}\right)_k (z_k^3 - z_{k-1}^3), \end{aligned} \tag{1}$$

where $\overline{\boldsymbol{Q}}_{ij}$'s are the laminate stiffness components in the laminate coordinate system of the k[th] layer and are given by:

$$\begin{aligned} \overline{\boldsymbol{Q}}_{11} &= \boldsymbol{U_1} + \boldsymbol{U_2}\cos 2\boldsymbol{\theta_k} + \boldsymbol{U_3}\cos 4\boldsymbol{\theta_k}, \\ \overline{\boldsymbol{Q}}_{12} &= \boldsymbol{U_4} - \boldsymbol{U_3}\cos 4\boldsymbol{\theta_k}, \\ \overline{\boldsymbol{Q}}_{22} &= \boldsymbol{U_1} - \boldsymbol{U_2}\cos 2\boldsymbol{\theta_k} + \boldsymbol{U_3}\cos 4\boldsymbol{\theta_k}, \\ \overline{\boldsymbol{Q}}_{66} &= \boldsymbol{U_5} - \boldsymbol{U_3}\cos 4\boldsymbol{\theta_k}, \\ \overline{\boldsymbol{Q}}_{16} &= (\boldsymbol{U_2}\sin 2\boldsymbol{\theta_k} + 2\boldsymbol{U_3}\sin 4\boldsymbol{\theta_k})/2, \\ \overline{\boldsymbol{Q}}_{26} &= (\boldsymbol{U_2}\sin 2\boldsymbol{\theta_k} - 2\boldsymbol{U_3}\sin 4\boldsymbol{\theta_k})/2, \end{aligned} \tag{2}$$

where $\boldsymbol{\theta_k}$ is the fiber orientation angle of the k[th] layer and $\boldsymbol{U_i}$ are the material invariants (Gürdal, 1999).

The **[A]**, **[B]**, and **[D]** matrices characterize the macro-mechanical behavior of composite laminates. As a result, the key role of the designer is to optimize the stacking sequence of the laminate to tailor the stiffness properties to meet the applied loads and obtain the optimal laminate performance. In most applications, designers have reached a tendency to use mid-plane symmetric laminates to avoid bending-extension coupling **[B]**=0 between in-plane forces and out-of-plane deformations and vice versa. By using this constraint, the designer omits the generality of the design problem and obtains only solutions restricted to the set of mid-plane symmetric laminates. Although this shortcut eases the design process to obtain null bending-extension coupling, it can lead to solutions that are optimal only in the class of considered mid-plane symmetric laminates and not globally. For example, laminates composed of an odd number of layers are automatically excluded, which are also qualified as candidates for being optimal solutions (Vannucci, 2006).

For a given laminate configuration, changing the stacking sequence order does not alter the **[A]** matrix, but it modifies the **[D]** matrix. Quasi-isotropic laminates, which have equally weighted layer composition, possess an **[A]** matrix that will always independent of the stacking sequence order and equivalent to the isotropic material counterpart. The **[D]** matrix, however, will not be isotropic because of the strong dependency on the composition and ordering of the stacking sequence of the laminate. To assess the degree of isotropy of the **[D]** matrix, an equivalent Voigt isotropic $[\bar{\boldsymbol{D}}]$ matrix, which is a representation of the average stiffness of the **D** matrix, can be defined as:

$$\bar{\boldsymbol{D}} = \frac{1}{2\pi}\int_0^{2\pi} \boldsymbol{D}_{\theta} d\theta, \tag{3}$$

which can be explicitly defined using matrix form as:

$$\bar{\boldsymbol{D}} = \begin{bmatrix} \bar{D}_{11} & \bar{D}_{12} & 0 \\ \bar{D}_{12} & \bar{D}_{11} & 0 \\ 0 & 0 & \bar{D}_{66} \end{bmatrix}, \tag{4}$$

and each of $\bar{D}_{11}, \bar{D}_{12}, \bar{D}_{66}$ are defined explicitly as:

$$\begin{aligned} \bar{D}_{11} &= (3D_{11} + 3D_{22} + 2D_{12} + 4D_{66})/8, \\ \bar{D}_{12} &= (D_{11} + D_{22} + 6D_{12} - 4D_{66})/8, \\ \bar{D}_{66} &= (D_{11} + D_{22} - 2D_{12} + 4D_{66})/8. \end{aligned} \tag{5}$$

To find the optimal stacking sequence that would maximize the degree of bending isotropy, the eigenvalues of [**D**] with respect to $[\bar{\boldsymbol{D}}]$ are calculated. A generalized eigenvalue problem of the bending stiffness matrix can be defined as:

$$(\boldsymbol{D} - \lambda\bar{\boldsymbol{D}})\boldsymbol{\kappa} = 0, \tag{6}$$

where the eigenvalues λ_i provide a measure for the relative bending stiffness between the **[D]** matrix and the isotropic counterpart $[\bar{\boldsymbol{D}}]$ (Peeters, 2017). The minimum eigenvalue λ_{min} quantifies the degree of bending isotropy as the distance in stiffness space between the **[D]** matrix of the laminate and the corresponding isotropic $[\bar{\boldsymbol{D}}]$. When the minimum eigenvalue $\lambda_{min}=1$, the laminate satisfies the conditions for bending isotropy.

LAY-UP STRATEGY

Two **NCF** thin-ply sub-laminates were used to conduct this parametric design study, namely **QI**$^{+}$ [30/90/-30] and **QI**$^{-}$ [-30/90/30]. An **NCF** may not simply provide its own mirror image alone by simple lamination practices, such as flipping over or rotating by a certain angle (Papila, 2016). As a result, the sole reason for using two sub-laminates **QI**$^{+}$ and **QI**$^{-}$ is having ability to achieve mid-plane symmetric laminates using **NCF**.

The lay-up strategy consists of studying the effect of orienting **QI-NCF**s with respect to a reference **QI-NCF** to achieve the maximum degree of bending isotropy. Starting from an initial quasi-isotropic construction, extensional isotropy is inherently satisfied. The degree of bending isotropy is then analyzed by solving the generalized eigenvalue problem for $\boldsymbol{\lambda_{min}}$. The goal is to find a laminate that is isotropic in extension and bending or a nearly isotropic one with the minimum number of plies required using thin-ply **NCF**. If the extension-bending coupling of the obtained laminate is null, then it qualifies to become a fully isotropic laminate. Starting from 2 **QI-NCF** stacks, the number of stacks is increased incrementally for both symmetric and non-symmetric laminate configurations to analyze the degree of bending isotropy. The lay-up strategy for analyzing the bending isotropy of [$\mathbf{QI^+}$ / $\mathbf{QI^-}$ + ϕ] starting from an initial symmetric laminate for ϕ=0 and increasing the angle ϕ can be visualized in Figure 1.

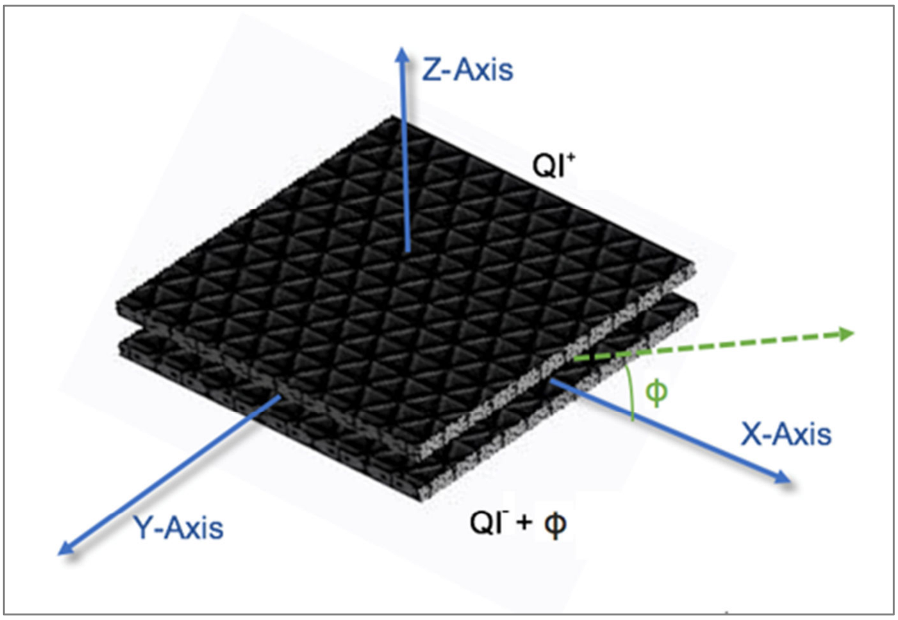

Fig. 1 - Lay-up Strategy for 2 **QI-NCF**s

To find the optimal laminate stacking sequence that would achieve an isotropic behavior in bending, the objective of this design problem would be the maximization of the minimal eigenvalue $\boldsymbol{\lambda_{min}}$ obtained using Eq. (6). Since the set of plies is determined a priori, either a genetic algorithm or a brute-force enumeration approach are qualified as efficient candidates for an optimization strategy. Due to the computationally inexpensive calculation of this parametric study, enumeration, where all possibilities are calculated, was chosen to be used. The minimum eigenvalue $\boldsymbol{\lambda_{min}}$ is evaluated for all the cases of $0 \leq \phi \leq 360$ with a unit step increment. As more **QI-NCF**s are stacked, the effect of rotating each sub-laminate on the degree of bending isotropy is studied by adding a variable ϕ_i for each additional sub-laminate.

As the number of **QI-NCF**s stacked increases, so does the degree of bending isotropy. As a reference, the minimal eigenvalue $\boldsymbol{\lambda_{min}}$= **0.266** that corresponds to 1 $\mathbf{QI^+}$ or $\mathbf{QI^-}$ sub-laminate is used. The degree of bending isotropy for 1 $\mathbf{QI^+}$ or $\mathbf{QI^-}$ sub-laminate is demonstrated in a polar plot in Figure 2 as a function of ϕ, where $0 \leq \phi \leq 360$.

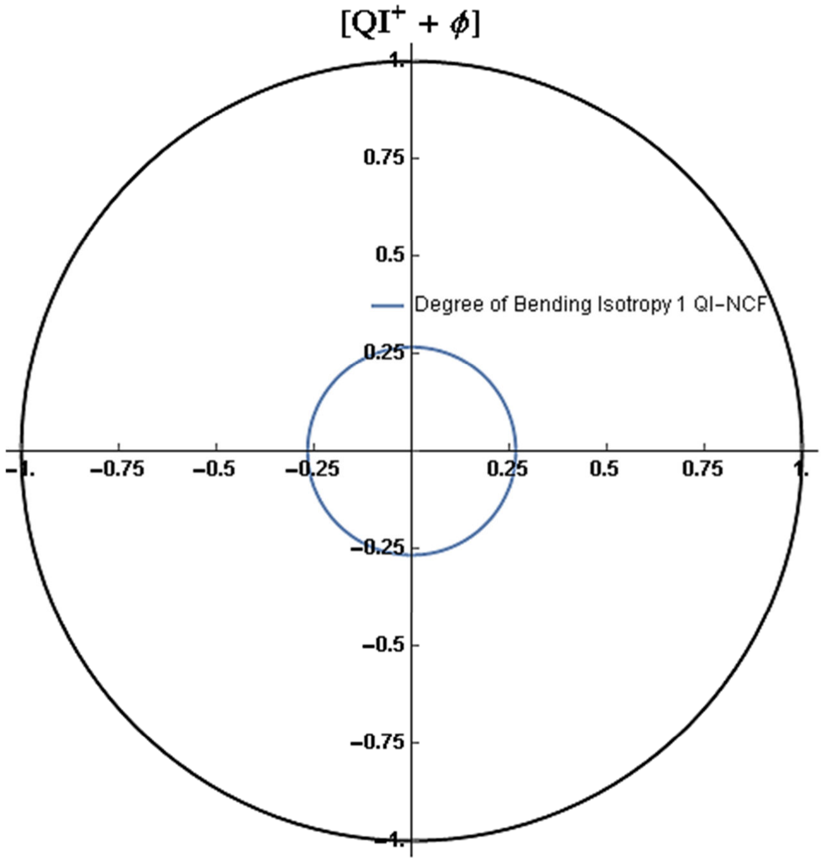

Fig. 2 - Polar Plot of Degree of Bending Isotropy for 1 **QI-NCF** as a function of ϕ

The inner circle corresponds to the value $\boldsymbol{\lambda_{min}}$= **0.266** and the outer circle denotes the value of 1 of a laminate satisfying bending isotropy. Orienting a whole laminate by a certain angle does not alter the degree of bending isotropy because it is analogous to changing the reference plane. For the parametric design study, we start by increasing the number of **QI-NCF**s used to achieve the maximum $\boldsymbol{\lambda_{min}}$ for symmetric and non-symmetric quasi-isotropic laminates.

RESULTS OF PARAMETRIC DESIGN STUDY

All the considered laminates are quasi-isotropic and satisfy conditions for extensional isotropy. This parametric design study aims to determine the optimal angles ϕ_i in which each **QI-NCF** is rotated with respect to the reference **QI**$^+$ sub-laminate to achieve the highest degree of bending isotropy.

The remaining key point left to address full isotropy is the null condition of the extension-bending coupling **[B]** matrix. For mid-plane symmetric laminates, the **[B]** matrix is zero. As for non-symmetric laminate configurations, the analysis of the degree of isotropy of extension-bending coupling would be the purpose of a future study. However, it will be demonstrated in the present study that non-symmetric configurations can provide a fully isotropic solution with a fewer number of plies than mid-plane symmetric laminates. The results of the parametric design study are presented below for both mid-plane symmetric and non-symmetric laminate configurations.

MID-PLANE SYMMETRIC QUASI-ISOTROPIC LAMINATES

The purpose in this section is to investigate the bending isotropy of mid-plane symmetric laminate configurations. As a result, the parametric design study should be carried on laminates that maintain the mid-plane symmetry constraint**.** However, for the first case of having 2 **QI-NCF** mid-plane symmetric laminate [$\mathbf{QI}^+$ / $\mathbf{QI}^-$ + ϕ], rotating one **QI-NCF** violates this condition, but it is presented in the following to demonstrate that non-symmetric laminate show a much higher degree of bending isotropy.

2 QI-NCF Mid-Plane Symmetric Laminate:

The results in Table 1 show that the maximum bending isotropy can be achieved by losing mid-plane symmetry. The minimum degree of bending isotropy is obtained when the laminate is mid-plane symmetric. The maximum degree of bending isotropy is achieved when ϕ= 60°.

Table 1 - Analysis of Degree of Bending Isotropy of **[$\mathbf{QI}^+$ / $\mathbf{QI}^-$ + $\boldsymbol{\phi}$]**

Initial Stacking Sequence	[QI^+ / QI^-] **OR [30/90/-30/-30/90/30]**	
	Stacking Sequence	$\boldsymbol{\lambda_{min}}$
Maximum Degree of Bending Isotropy	[$\mathbf{QI}^+$ / $\mathbf{QI}^-$ +60°] OR [30/90/-30/30/-30/90] **(Non-Symmetric)**	**0.54165**
Minimum Degree of Bending Isotropy	[$\mathbf{QI}^+$ / $\mathbf{QI}^-$] OR [30/90/-30/-30/90/30]	**0.26656**

A polar plot in Figure 3 shows the minimal eigenvalue $\boldsymbol{\lambda_{min}}$ for every angle of rotation ϕ.

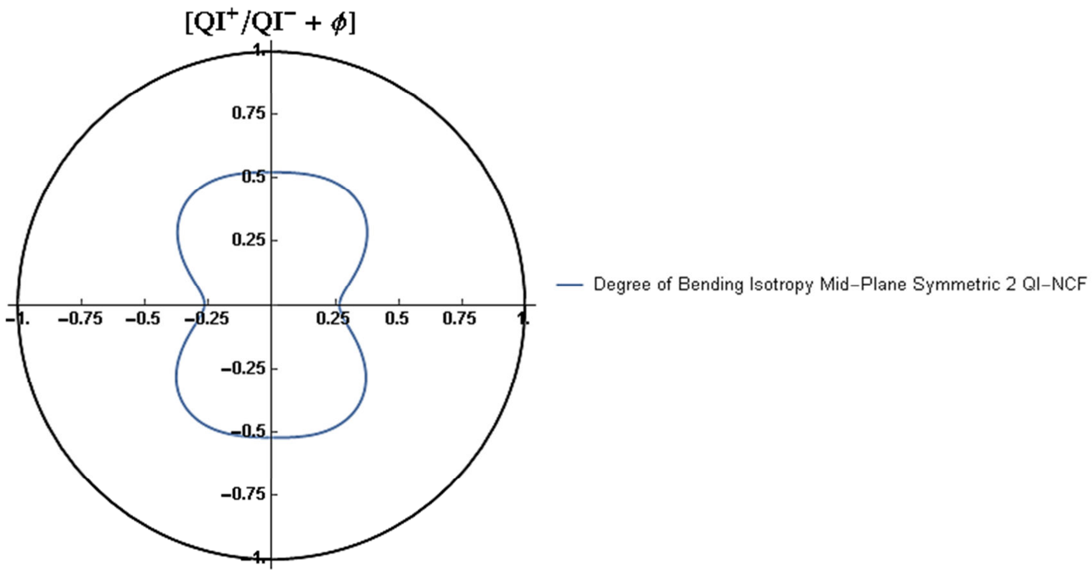

Fig. 3 - Polar Plot of Degree of Bending Isotropy for **[$\mathbf{QI}^+$ / $\mathbf{QI}^-$ + $\boldsymbol{\phi}$]**

4 QI-NCF Mid-Plane Symmetric Laminate:

Considering 4 **QI-NCF** mid-plane symmetric laminate [$\mathbf{QI}^+$/$\mathbf{QI}^+$/$\mathbf{QI}^-$/$\mathbf{QI}^-$], we can study either the effect of rotating the inner **QI** sub-laminates, the outer **QI** sub-laminates, or both. However, these cases would lead to the same results after rotating the reference frame. Thus, only the rotation of the inner **QI** sub-laminates is illustrated for [$\mathbf{QI}^+$/ $\mathbf{QI}^+$ + ϕ/ $\mathbf{QI}^-$ + ϕ/ $\mathbf{QI}^-$].

Table 2 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + ϕ/ QI$^-$ + ϕ/ QI$^-$]**

Initial Stacking Sequence	[QI$^+$/ QI$^+$/ QI$^-$/ QI$^-$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ - 60°/ **QI**$^-$ - 60° / **QI**$^-$]	**0.81658**
Minimum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ / **QI**$^-$ / **QI**$^-$]	**0.61038**

Since we are designing only one rotation angle ϕ, a polar plot shown in Figure 4 can demonstrate how the degree of bending isotropy changes as a function of angle of rotation. The results show that the using mid-plane symmetry without rotation of inner sub-laminates has the lowest degree of isotropy of only 61%. The maximum degree of bending isotropy (or full isotropy) is about 81.6% which is achieved for ϕ= -60°.

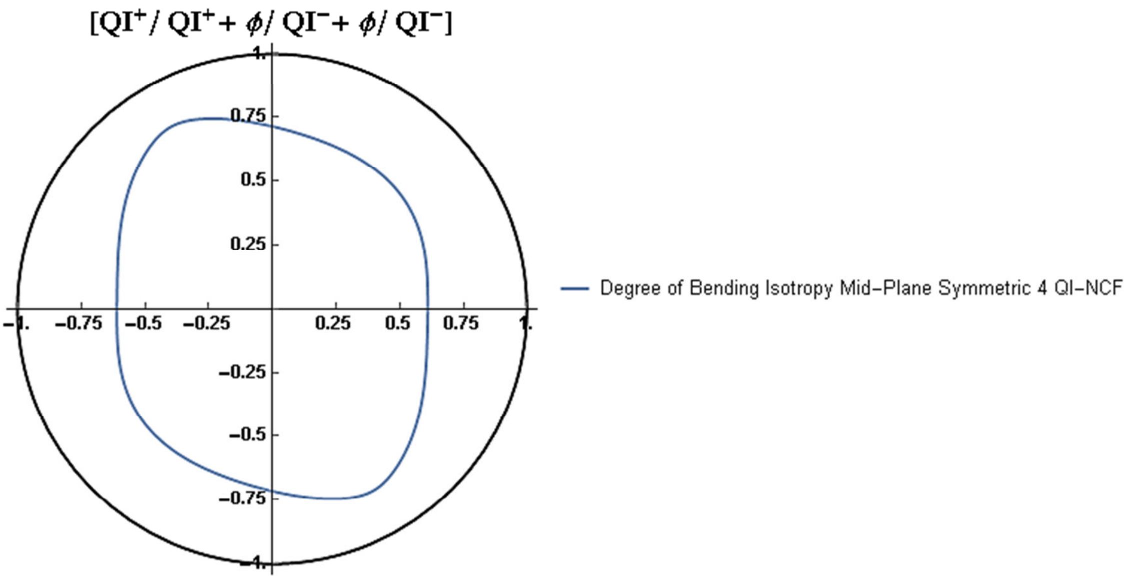

Fig. 4 - Polar Plot of Degree of Bending Isotropy for **[QI$^+$/ QI$^+$ + ϕ/ QI$^-$ + ϕ/ QI$^-$]**

6 QI-NCF Mid-Plane Symmetric Laminate:

Table 3: Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + ϕ_1/ QI$^+$ + ϕ_2 / QI$^-$ + ϕ_2/ QI$^-$ + ϕ_1 / QI$^-$]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$ / QI$^-$ / QI$^-$ / QI$^-$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ -60°/ **QI**$^+$+60° / **QI**$^-$+60° / **QI**$^-$ -60° / **QI**$^-$]	**0.93787**
Minimum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ / **QI**$^+$ / **QI**$^-$ / **QI**$^-$ / **QI**$^-$]	**0.73516**

The results show the same trend, where the maximum degree of bending isotropy is achieved for ϕ_1= -60° and ϕ_2= 60°. The maximum degree of isotropy achieved is around 93.8% isotropic. However, this is obtained when the inner sub-laminates are rotated at the optimum angles ϕ_1 and ϕ_2. Using regular mid-plane symmetry achieves only 73.5% of the isotropic behavior.

8 QI-NCF Mid-Plane Symmetric Laminate:

Table 4 - Analysis of Degree of Bending Isotropy of
[QI$^+$/ QI$^+$+ϕ_1/ QI$^+$+ϕ_2/ QI$^+$+ϕ_3/ QI$^-$+ϕ_3/ QI$^-$ + ϕ_2/ QI$^-$+ϕ_1/ QI$^-$]

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$ / QI$^-$ / QI$^-$ / QI$^-$ / QI$^-$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	**[QI$^+$** / **QI$^+$** -60°/ **QI$^+$** +60° / **QI$^+$** +90° / **QI$^-$** +90° / **QI$^-$** +60° / **QI$^-$** -60° / **QI$^-$**]	**0.96722**
Minimum Degree of Bending Isotropy	**[QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$ / QI$^-$ / QI$^-$ / QI$^-$ / QI$^-$]**	**0.79946**

Even with 8 **QI-NCF**, no laminate satisfying bending isotropy (or full isotropy) has been found for mid-plane symmetric configuration. However, the maximum degree of isotropy achieved is around 96.7% with ϕ_1= -60°, ϕ_2= 60°, and ϕ_3= 90°. The minimum degree of isotropy is only 79.9% which is obtained with regular mid-plane symmetry.

NON-SYMMETRIC QUASI-ISOTROPIC LAMINATES

The same design study is now carried out on non-symmetric quasi-isotropic laminates to confirm that they have an improved degree of bending isotropy. The results are shown below for non-symmetric quasi-isotropic laminates having 2 to 6 **QI-NCF**s.

2 QI-NCF Non-Symmetric Laminate:

Polar plot of the degree of isotropy is shown in Figure 5 to visualize the minimal eigenvalue $\boldsymbol{\lambda_{min}}$ as a function of angle of rotation ϕ. The results show the non-uniform distribution of $\boldsymbol{\lambda_{min}}$ for non-symmetric laminate configurations. It will be shown that as the number of **QI-NCF** increases, the polar plot of $\boldsymbol{\lambda_{min}}$ tends to a more uniform distribution.

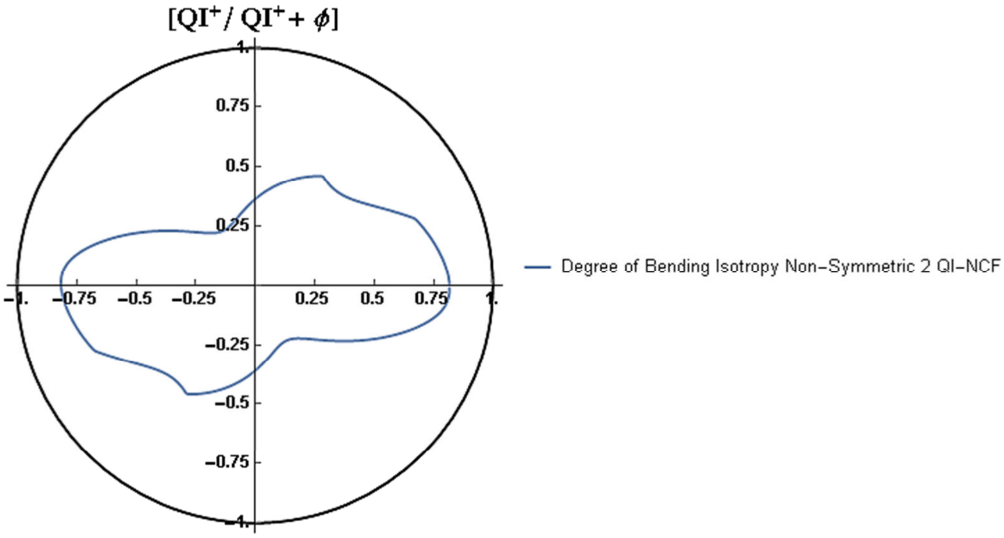

Fig. 5 - Polar Plot of Degree of Bending Isotropy for **[QI$^+$/ QI$^+$ + ϕ]**

The results tabulated in Table 5 show that the maximum bending isotropy is around 81.6% for 2 **QI-NCF** which can be achieved by using the same initial non-symmetric configuration [**QI**$^+$/ **QI**$^+$]. This is because the laminate is anti-symmetric having $D_{16} = D_{26} = 0$. The analysis shows that achieving bending isotropy using the generalized eigenvalue formulation enjoins the designer to avoid bending-twisting coupling since this behavior is not present in the isotropic material counterpart. However, the worst degree of isotropy also shows that improper design of the laminate can end up in a 26.6% bending isotropy.

Table 5 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + $\boldsymbol{\phi}$]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$]	
	Stacking Sequence	$\boldsymbol{\lambda_{min}}$
Maximum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$]	**0.81666**
Minimum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ -60°]	**0.26657**

3 QI-NCF Non-Symmetric Laminate:

As the number of **QI-NCF** increases, each **QI-NCF** has a design variable ϕ_i to study the effect of rotating the sub-laminate to increase the degree of bending isotropy. Maximum and minimum degree of isotropy and corresponding orientation angles are shown in Table 6.

Table 6 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + $\boldsymbol{\phi_1}$/ QI$^+$ + $\boldsymbol{\phi_2}$]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$]	
	Stacking Sequence	$\boldsymbol{\lambda_{min}}$
Maximum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ +60°/ **QI**$^+$]	**0.95926**
Minimum Degree of Bending Isotropy	[**QI**$^+$ / **QI**$^+$ +60°/ **QI**$^+$ -60°]	**0.51110**

The maximum degree of bending isotropy achieved is around 95.9% for ϕ_1= 60° and ϕ_2= -60°. Thus, a laminate with an odd number of plies having 3 **QI-NCFs** can also be qualified for having approximate optimal bending properties with a fewer number of plies than mid-plane symmetric configurations. As stated earlier, increasing the number of plies increases the uniformity of the distribution of $\boldsymbol{\lambda_{min}}$ as shown in Figure 6 and Figure 7. The polar plots are obtained after fixing each of ϕ_1 and ϕ_2 to be the optimal rotation angle and analyzing the degree of bending isotropy as a function of the other angle.

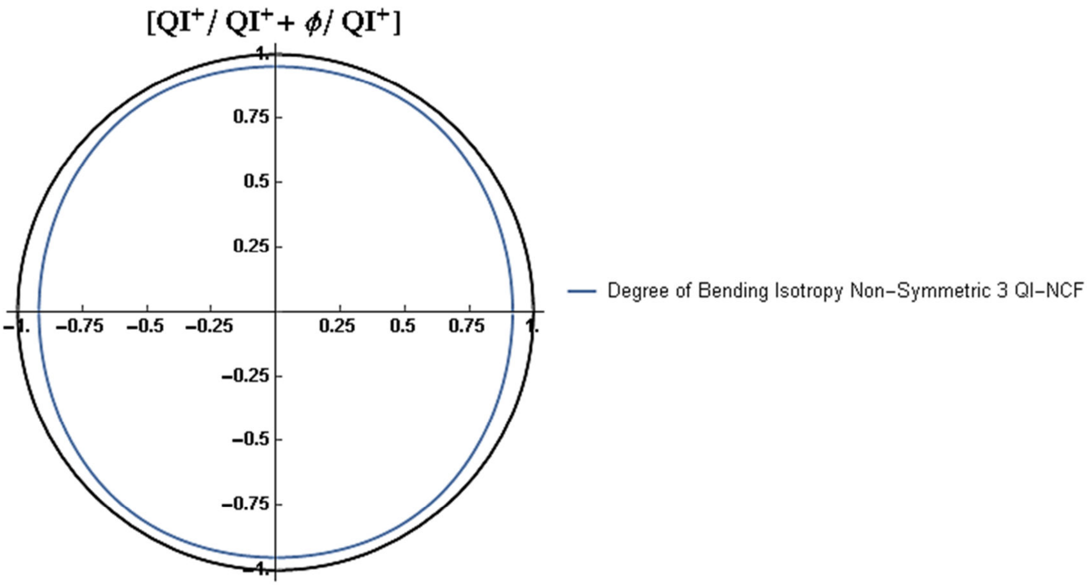

Fig. 6 - Polar Plot of Degree of Bending Isotropy for **[QI$^+$/ QI$^+$ + ϕ_1/ QI$^+$]** for **ϕ_2= 0°**

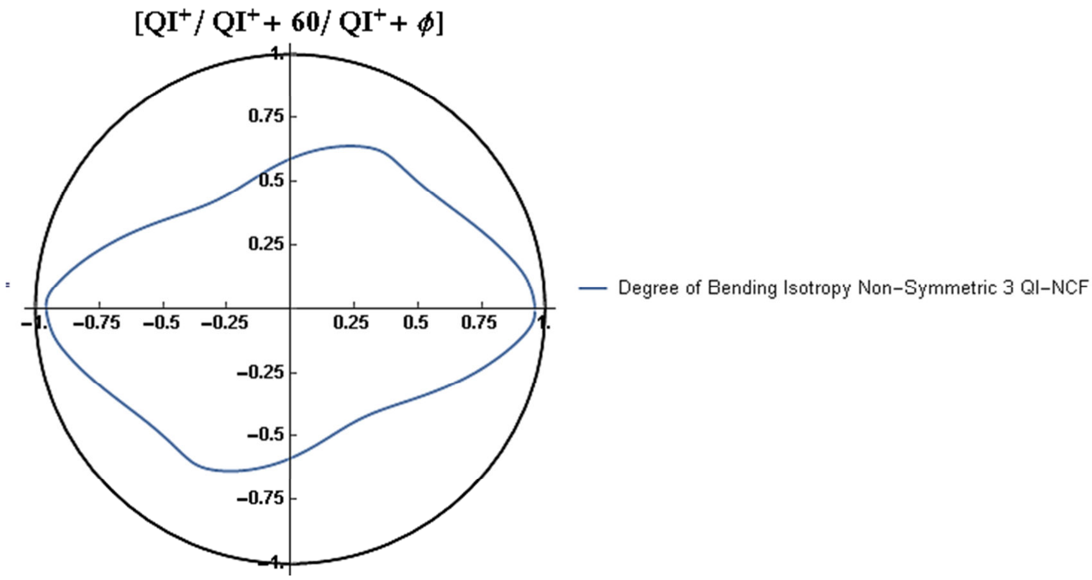

Figure 7 - Polar Plot of Degree of Bending Isotropy for **[QI$^+$/ QI$^+$ + 60°/ QI$^+$+ ϕ_2]** for **ϕ_1= 60°**

4 QI-NCF Non-Symmetric Laminate:

Table 7 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + ϕ_1/ QI$^+$ + ϕ_2 / QI$^+$ + ϕ_3]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** +33°/ **QI$^+$** -23° / **QI$^+$** +10°]	**0.99874**
Minimum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** / **QI$^+$** -60°/ **QI$^+$** -60°]	**0.61041**

Having 4 **QI-NCF** sub-laminates, the designer can approximately achieve an isotropic laminate in bending with a degree of isotropy of 99.87%. This is obtained when ϕ_1= 33°, ϕ_2= -23°, and ϕ_3= 10°. Comparing this to the optimal mid-plane symmetric laminate which has 81.65% bending isotropy, an increase of 22% is obtained in the degree of bending isotropy.

5 QI-NCF Non-Symmetric Laminate:

Table 8 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + ϕ_1/ QI$^+$ + ϕ_2 / QI$^+$ + ϕ_3/ QI$^+$ + ϕ_4]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** -60°/ **QI$^+$** +60° / **QI$^+$** -60° / **QI$^+$**]	**0.99702**
Minimum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** / **QI$^+$** +60° / **QI$^+$** -60° / **QI$^+$** -60°]	**0.68906**

The results also show that an approximate isotropic laminate in bending can be achieved for 5 **QI-NCF** sub-laminates. The maximum degree of bending isotropy is 99.7%, which is obtained when ϕ_1= -60°, ϕ_2= 60°, ϕ_3= -60°, and ϕ_4= 0°. Interestingly, this laminate has one additional **QI-NCF** stack, but it has a slightly lower degree of bending isotropy compared to the laminate with 4 **QI-NCF**.

6 QI-NCF Non-Symmetric Laminate:

Table 9 - Analysis of Degree of Bending Isotropy of **[QI$^+$/ QI$^+$ + ϕ_1/ QI$^+$ + ϕ_2 / QI$^+$ + ϕ_3/ QI$^+$ + ϕ_4/ QI$^+$ + ϕ_5]**

Initial Stacking Sequence	[QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$ / QI$^+$]	
	Stacking Sequence	λ_{min}
Maximum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** -60°/ **QI$^+$** +60° / **QI$^+$** +60° / **QI$^+$** -60°/ **QI$^+$**]	**1.0**
Minimum Degree of Bending Isotropy	[**QI$^+$** / **QI$^+$** / **QI$^+$** / **QI$^+$** -60°/ **QI$^+$** -60°/ **QI$^+$** -60°]	**0.73518**

For a laminate with 6 **QI-NCF**, the most interesting result is obtained. Not only the laminate shows a completely isotropic **[D]** matrix, but also the **[B]** matrix obtained is null. Thus, full isotropy is obtained for ϕ_1= -60°, ϕ_2= 60°, ϕ_3= 60°, ϕ_4= -60°, and ϕ_5= 0°. This result agrees with the fully isotropic laminate obtained by Vannucci for an 18-plied laminate (Vannucci, 2002). However, only one solution is obtained here since **NCF** have a predetermined stack of plies that cannot be interchanged. This demonstrates that non-symmetric laminates provide a much higher degree of bending isotropy than mid-plane symmetric configurations and may result in a fully isotropic laminate with a fewer number of plies.

CONCLUSIONS

This paper proposes a new lay-up strategy for thin-ply **NCF**, and it demonstrates a new methodology to assess the degree of bending isotropy. Starting from initial quasi-isotropic constructions in which extensional isotropy is inherently satified, the degree of bending isotropy is formulated as a generalized eigenvalue problem of the bending stiffness tensor, and variety of thin-ply NCF laminates are designed and analyzed. The results show that non-symmetric laminates have a much higher degree of bending isotropy than mid-plane symmetric laminates. The remaining key aspect to assess the degree of full isotropy of non-symmetric laminate configurations is the degree of extension-bending coupling, which is the purpose of future work.

For future work, the obtained laminates have been manufactured using VARTM and test coupons have been designed for three-point flexural testing to experimentally verify the results that were analyzed. Unfortunately, the results have not been finalized and would also be the purpose of future work. It is vital to understand this physical measure of bending isotropy. How is this measure affected by the presence extension-bending coupling in terms of the behavior of non-symmetric laminates? How do these stiffness properties relate to the response of the laminate in strength failure? Several interesting questions are left unanswered concerning the behavior of fully isotropic laminates.

ACKNOWLEDGMENTS

The authors at the *McNAIR* Center would like to thank CHOMARAT for their support in supplying the thin-ply non-crimp fabric materials. The authors are also grateful to Nicole Pascale and Tyler Dillard for their support and contributions to the experimental work currently in progress.

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