THE ANALYSIS OF THE HELICOPTER TECHNICAL READINESS BY MEANS OF THE MARKOV PROCESSES

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ABSTRACT
The article presents a method of assessing the readiness of technical objects. The analysis focused on Mi-8 helicopters belonging to the Air Base. The Markov processes with discrete and continuous time were used to determine the readiness index. Based on the analysis of the operational process, nine operational states were distinguished, for which limit probabilities in discrete and continuous time were calculated. In addition, the dynamics of system changes in terms of striving for a stationary state were studied.

Keywords: helicopter operation and maintenance, Markov processes, technical readiness.

INTRODUCTION
Modern helicopters, thanks to their features, such as: the possibility of vertical take-off and landing, hovering, high manoeuvrability, mobility, range, velocity, resistance to ground fire, efficient own protection measures, as well as firepower and task execution accuracy, are the essential equipment of armed forces of many countries. From the point of view of operational features, the set of their advantages includes: ease of use and operation, and low encumbrance of crews with flight handling, which results facilitates the likelihood of an efficient task performance. Currently, helicopters are commonly used in national economy, for civilian, as well as military purposes. Their status in armed conflicts is of the basic combat asset in a modern battlefield (Tomaszek, 2013, 2016; Zieja, 2016; Żurek, 2009, 2014).

The helicopters operated in Poland are of obsolete design in terms of technical concept and manufacturing technology, and in many cases, they are multi-purpose objects of Soviet or Polish production. This article developed a method of assessing the readiness of Mi-8 helicopters with the application of the Markov processes. For this purpose, documentation data based on the Routine Maintenance Sheets were gathered, which are the basis for the register of Mi-8 helicopters over a period of two years (2014-2015). Next, the operational process was analysed, and an individual and cumulative database was developed. As a result, Markov models in discrete and continuous time were constructed (Jacyna-Gołda, 2017; Pham, 2006).

FORMULATION AND ANALYSIS OF HELICOPTER REGISTER STATES
The essence of mathematical modelling is the description of the studied phenomenon (process, system, set) using a mathematical language. The modelling process utilizes variables representing certain (and, therefore, significant from the point of view of the purpose)
properties of the studied phenomenon (Brandimarte, 2014; Narayan, 2012; Tchórzewska-Cieślak, 2016). Studying an operational system requires the determination of all important factors, which define it. Secondary components, the ones, which unnecessarily complicate the model without significantly improving its quality should be ignored or omitted, and the ones similar in terms of the results should be grouped (Kierzkowski, 2017; Piegdoń 2018). Thorough examination of the process enables individualization of mutually disjoint operational states. It also imposes requirements for empirical data, used to construct the model. A minimum amount of its distinctive states, which enables calculating the basic operational indices, is enough to generally analyze an operational system. For a studied helicopter operational system, a set of the following states was distinguished:

- $S_1$ - preparation for operation;
- $S_2$ - test execution;
- $S_3$ - refueling;
- $S_4$ - readiness with a pilot;
- $S_5$ - readiness without a pilot;
- $S_6$ - protection;
- $S_7$ - work on the ground;
- $S_8$ - task performance;
- $S_9$ - unfitness (maintenance and repair).

In order to ensure quality of an operation model, it is necessary to correctly select the permitted transitions of the object from the previous state to the next one (Knopik, 2016, 2018; Retsel, 2015). It was determined on the basis of the technical documentation and the current operational knowledge regarding the discussed operation process. The mathematical description of allowed transitions is a matrix of allowed transitions $S_i \rightarrow S_j$ from the previous state $S_i$ (rows) to the next one $S_j$ (column). The analyzed nine-state system has probable and prohibited transitions, according to Table 1 and the correlated graph shown in Figure 1, where:

0 - means a prohibited transition;
1 - means an allowed transition.

<table>
<thead>
<tr>
<th>$S_i \rightarrow S_j$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
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<th>$S_6$</th>
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<tbody>
<tr>
<td>$S_1$</td>
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<td>1</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$S_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>$S_4$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>$S_5$</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>$S_6$</td>
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<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>$S_7$</td>
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<td>$S_8$</td>
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<td>$S_9$</td>
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As seen in Table 1, there are 72 theoretically possible interstate transitions, however, in the case of studying real operational processes, some transitions are prohibited. The studied sample narrowed this set to 30 allowed state changes, which is a natural consequence of the adopted process sequence. From the point of view of the process organization, narrowing the possible interstate transitions is understandable and is a consequence of the adopted (and not random) operational process organization.

A graph showing the helicopter operation process including nine states. As can be seen, most of them are mutually communicating states ($S_1 \rightarrow S_2$, $S_1 \rightarrow S_6$, $S_1 \rightarrow S_9$, $S_2 \rightarrow S_4$, $S_3 \rightarrow S_4$, $S_3 \rightarrow S_5$, $S_3 \rightarrow S_7$, $S_4 \rightarrow S_7$, $S_7 \rightarrow S_8$). Operation is understood as a movement of the object over distinguished states, which form the phase space.

**MARKOV PROCESS IN DISCRETE TIME**

The first stage in constructing a Markov process in discrete time (Knopik, 2016; Werbińska-Wojciechowska, 2007, 2013) is the estimation of transition probabilities, as the values of estimators $\hat{p}_{ij}$ of elements $p_{ij}$ and matrices $P$ of probable transitions. The values of these estimators in a studied sample are frequencies $w_{ij}$ of transitions from state $S_i$ to state $S_j$, calculated according to the relationship (1):

$$\hat{p}_{ij} = w_{ij} = \frac{n_{ij}}{n_i};$$

where:

$n_{ij}$ - the number of transitions from state $S_i$ to state $S_j$; $n_i$ - the number of all transitions (exits) from state $S_i$;

$w_{ij}$ - frequencies $w_{ij}$ of transitions from state $S_i$ to state $S_j$; $n_i$ - number of observations of states $S_i$ in the sample.

In the case of a nine-state operational process, the matrix of interstate transitions $P$ adopts the following form:
Table 2 lists empirical likelihoods of interstate transitions for the tested group of Mi-8 helicopters.

Table 2 Probabilities of interstate transitions for Mi-8 helicopters

<table>
<thead>
<tr>
<th>$p_i \rightarrow p_j$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_5$</th>
<th>$p_6$</th>
<th>$p_7$</th>
<th>$p_8$</th>
<th>$p_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>0</td>
<td>0.447204</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.434782</td>
<td>0</td>
<td>0.018633</td>
<td>0.099378</td>
</tr>
<tr>
<td>$p_2$</td>
<td>0.012121</td>
<td>0</td>
<td>0.103030</td>
<td>0.721212</td>
<td>0.151515</td>
<td>0</td>
<td>0.012121</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_3$</td>
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<td>0</td>
<td>0</td>
<td>0.242424</td>
<td>0.393939</td>
<td>0</td>
<td>0</td>
<td>0.010101</td>
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</tr>
<tr>
<td>$p_4$</td>
<td>0</td>
<td>0.538922</td>
<td>0.005988</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.455089</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_5$</td>
<td>0.057142</td>
<td>0</td>
<td>0.357142</td>
<td>0</td>
<td>0</td>
<td>0.585714</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_6$</td>
<td>0.954545</td>
<td>0</td>
<td>0.045454</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_7$</td>
<td>0</td>
<td>0.011299</td>
<td>0.282485</td>
<td>0.135593</td>
<td>0.033898</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.536723</td>
</tr>
<tr>
<td>$p_8$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$p_9$</td>
<td>0.058823</td>
<td>0.058823</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.882352</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>

In relation to Markov processes with discrete time, it was proven (Pham, 2006) that if ergodic probabilities exist, then they can be calculated from the border of the transition matrix $P$ in $n$ steps, by solving a set of linear solutions or a simultaneous matrix equation, i.e., by passing from continuous $t$ time to discrete time $n$, which is a number of the next experiment involving observation of a vehicle in time $\Delta t$, as per the relationship (3):

$$
\hat{p}_{ij} = \lim_{n \to \infty} p_{ij}(n) = \sum_j p_{ij}p_{ij} = p_j \iff P^T[p_j] = [p_j], \quad \text{at } \sum_j p_j = 1
$$

where:

- $P^T$ - transposed transition matrix $P$ whereas $P = [p_{ij}; i,j \in S]$; $[p_j]$ - vector of limit probabilities; $p_{ij}$ - probabilities of transition from state $i$ to state $j$.

The forms of a system of linear equations for ergodic probabilities $p_j(n)$ are shown by equations (4).

$$
\begin{align*}
0.012121p_2 + 0.353535p_3 + 0.057142p_5 + 0.954545p_6 + 0.058823p_9 - p_1 = 0 \\
0.447204p_1 + 0.538922p_4 + 0.011299p_7 - 0.058823p_9 - p_2 = 0 \\
0.103030p_2 + 0.005988p_4 + 0.357142p_5 + 0.045454p_6 + 0.282485p_7 - p_3 = 0 \\
0.721212p_2 + 0.242424p_4 + 0.135593p_7 - p_4 = 0 \\
0.151515p_4 + 0.393939p_6 + 0.033898p_7 - p_5 = 0 \\
0.434782p_1 + 0.585715p_4 + 0.882352p_9 - p_6 = 0 \\
0.018633p_1 + 0.012121p_2 + 0.010101p_3 + 0.455089p_4 + 1p_6 - p_7 = 0 \\
0.536723p_3 - p_6 = 0 \\
0.099378p_1 - p_9 = 0
\end{align*}
$$
With simultaneous satisfaction of a requirement of system normalization (5).

\[ \sum_{i=1}^{9} p_j = 1 \]  \hspace{1cm} (5)

The solution of a system of equations (4) with a normalization requirement (5) was completed using the Mathematica, v.11 software. The solutions are abstruse functions in a set of complex numbers, which were not cited in this paper due to their complexity. Value of limit probabilities \( p_j(n) \) are illustrated in Figure 2.

![Fig. 2 - Limit probability \( p_j(n) \) of the Markov process in discrete time for Mi-8 helicopters](image)

Limit probabilities (Figure 2) relative to discrete time do not exceed values of 0.2. It proves the fact that there are no considerable disproportions regarding the priority for each of the nine operational states of a studied process. The biggest probability of entering is present for the states \( S_7 \) (work on the ground), \( S_4 \) (readiness with a pilot), \( S_2 \) (test execution) and \( S_1 \) (preparation for operation). The lowest probabilities were observed for the states \( S_9 \) (unfitness), \( S_5 \) (readiness without a pilot), \( S_8 \) (task performance) and \( S_3 \) (refuelling). However, we need to remember that the probabilities for the Markov chain are interpreted as a number of entries to a given state compared to all transitions from a distinguished set of states forming a phase space of the process. Therefore, they are interpreted in the quantitative and not qualitative sense (relative to the state duration).

**MARKOV PROCESS IN CONTINUOUS TIME**

The transition from discrete time to continuous time is through the intensity matrix \( \Lambda \). Transition intensities \( \lambda_{ij} \geq 0 \) for \( i \neq j \) are defined as right-hand derivatives of transition probabilities relative to time, according to a relationship:

\[ \lambda_{ij}(t_0) = \frac{d(p_{ij})}{dt}|_{t=t_0} \]  \hspace{1cm} (6)

Intensities \( \lambda_{ii} \leq 0 \) for \( i = j \) are defined as complementation of the sum of intensity of transitions from state \( S_i \) for \( i \neq j \) to 0:

\[ \lambda_{ii} + \sum_j \lambda_{ij} = 0 \]  \hspace{1cm} (7)

hence:

\[ \lambda_{ii} = - \sum_j \lambda_{ij} \]  \hspace{1cm} (8)
Modules $|\lambda_{li}| = -\lambda_{il}$ are called intensities of exits from state $S_i$. They are not the intensities of returning from state $S_i$ to $S_l$ - as suggested by the notation. For the homogeneous Markov processes, the transition intensity is constant and equal to the inverse of average time $\tau_{ij}$ of an object staying in state $S_i$ prior to state $S_j$:

$$\hat{\lambda}_{ij} = 1/\tau_{ij}$$

while:

$$\tau_{ij} = (\Sigma_j t_{ij})/n_i$$

where:

$$t_{ij} = t_{k+1} - t_k$$

only for $S_k = S_j$ - the time of an object staying in state $S_i$ prior to state $S_j$, which is equal to the value of the discrete-continuous variable value for the observation number $k$.

$\tau_{ij} = (\Sigma_j t_{ij})/n_i$ means an average time spent in state $S_i$ prior to state $S_j$.

Tab. 3 lists the transition intensities of matrices $\Lambda$ for a nine-state operational process of Mi-8 helicopters.

<table>
<thead>
<tr>
<th>$\lambda_i \rightarrow \lambda_j$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
<th>$\lambda_7$</th>
<th>$\lambda_8$</th>
<th>$\lambda_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>-0.996710</td>
<td>0.010142</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.003961</td>
<td>0.016216</td>
<td>0</td>
<td>0.00639</td>
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<tr>
<td>$\lambda_2$</td>
<td>0.0799</td>
<td>-0.387914</td>
<td>0.058620</td>
<td>0.065348</td>
<td>0.050607</td>
<td>0</td>
<td>0.13333</td>
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</tr>
<tr>
<td>$\lambda_3$</td>
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<td>0.288782</td>
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<td>0.04</td>
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<td>0</td>
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<tr>
<td>$\lambda_4$</td>
<td>0</td>
<td>0.047199</td>
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<td>-0.078749</td>
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<td>0</td>
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<td>$\lambda_5$</td>
<td>0.008681</td>
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<td>-0.026391</td>
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<tr>
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<td>0</td>
<td>-0.000961</td>
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<tr>
<td>$\lambda_7$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.015936</td>
<td>-0.015936</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda_9$</td>
<td>0.002816</td>
<td>0.0025</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00344</td>
<td>0</td>
<td>0</td>
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</table>

Transition intensities are expressed by the number of transitions per hour for a given object. After substituting the matrix $\Lambda$ to the equation $[p_j]^T \cdot \Lambda = 0$, the following equation in matrix form was obtained for the studied operational process:

$$[p_1]^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & \lambda_{17} & 0 & \lambda_{19} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & \lambda_{37} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & \lambda_{46} & \lambda_{47} & \lambda_{48} & \lambda_{49} \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & 0 \\ \lambda_{61} & 0 & \lambda_{63} & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & 0 & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & \lambda_{84} & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & 0 & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

or, in the form of linear equation systems, as a relationship:
This is a homogeneous system, which has an infinite number of solutions, among which there may be solutions satisfying the condition of normalization:

\[ \sum_{i=1}^{9} p_j = 1 \]  

(13)

The solution to the above system (12) with a restriction (12) was obtained with the help of the Mathematica software (ver.11), and the result presented in Figure 3.

Figure 3 shows that a Mi-8 type helicopter spends the most time in the states of \( S_6 \) (protection) and \( S_9 \) (unfitness). In relation to the other distinguished operational states, on average, it spends very little time, i.e., in the range of 0.037 in the state \( S_5 \) (readiness without a pilot) to just over 0.001 for the states of \( S_7 \) (work on the ground) \( S_3 \) and (refuelling).

**THE ANALYSIS OF CHANGES OF THE MI-8 HELICOPTER OPERATIONAL PROCESS IN TERMS OF THE DYNAMICS OF STRIVING FOR A STATIONARY STATE**

The Smoluchowski - Chapman - Kolmogorov equation stems have the following matrix form:

\[ \Pi_t = d\Pi_t/\Pi_t = A \cdot \Pi_t \wedge (\sum_{j} p_j = 1) \]  

(14)

For the studied Markov process, they have the matrix form (15):
or, it may be written in the form of a balanced differential equations system (16):

\[
\begin{align*}
p'_1(t) &= -\lambda_{12} \cdot p_2(t) - \lambda_{16} \cdot p_4(t) - \lambda_{17} \cdot p_7(t) - \lambda_{19} \cdot p_9(t) + \lambda_{21} \cdot p_2(t) + \lambda_{31} \cdot p_3(t) + \lambda_{51} \cdot p_5(t) + \lambda_{61} \cdot p_6(t) + \lambda_{71} \cdot p_7(t) + \lambda_{91} \cdot p_9(t) \\
p'_2(t) &= -\lambda_{21} \cdot p_1(t) - \lambda_{23} \cdot p_3(t) - \lambda_{24} \cdot p_4(t) + \lambda_{12} \cdot p_1(t) + \lambda_{42} \cdot p_4(t) + \lambda_{92} \cdot p_9(t) \\
p'_3(t) &= -\lambda_{31} \cdot p_1(t) - \lambda_{34} \cdot p_4(t) - \lambda_{35} \cdot p_5(t) - \lambda_{37} \cdot p_7(t) + \lambda_{23} \cdot p_3(t) + \lambda_{43} \cdot p_4(t) + \lambda_{53} \cdot p_5(t) + \lambda_{63} \cdot p_6(t) + \lambda_{73} \cdot p_7(t) + \lambda_{83} \cdot p_8(t) \\
p'_4(t) &= -\lambda_{42} \cdot p_2(t) - \lambda_{43} \cdot p_3(t) - \lambda_{46} \cdot p_6(t) - \lambda_{47} \cdot p_7(t) - \lambda_{48} \cdot p_8(t) - \lambda_{49} \cdot p_9(t) + \lambda_{24} \cdot p_2(t) + \lambda_{34} \cdot p_3(t) + \lambda_{74} \cdot p_7(t) + \lambda_{84} \cdot p_8(t) \\
p'_5(t) &= -\lambda_{51} \cdot p_1(t) - \lambda_{53} \cdot p_3(t) - \lambda_{56} \cdot p_6(t) + \lambda_{35} \cdot p_3(t) + \lambda_{75} \cdot p_7(t) \\
p'_6(t) &= -\lambda_{61} \cdot p_1(t) - \lambda_{63} \cdot p_3(t) + \lambda_{16} \cdot p_1(t) + \lambda_{46} \cdot p_4(t) + \lambda_{56} \cdot p_5(t) + \lambda_{96} \cdot p_9(t) \\
p'_7(t) &= -\lambda_{71} \cdot p_1(t) - \lambda_{73} \cdot p_3(t) - \lambda_{74} \cdot p_4(t) - \lambda_{75} \cdot p_5(t) - \lambda_{78} \cdot p_8(t) + \lambda_{17} \cdot p_1(t) + \lambda_{37} \cdot p_3(t) + \lambda_{47} \cdot p_4(t) + \lambda_{87} \cdot p_8(t) \\
p'_8(t) &= -\lambda_{81} \cdot p_1(t) - \lambda_{83} \cdot p_3(t) - \lambda_{84} \cdot p_4(t) - \lambda_{97} \cdot p_7(t) + \lambda_{18} \cdot p_1(t) + \lambda_{49} \cdot p_4(t) \\
p'_9(t) &= -\lambda_{91} \cdot p_1(t) - \lambda_{92} \cdot p_2(t) - \lambda_{96} \cdot p_6(t) + \lambda_{19} \cdot p_1(t) + \lambda_{49} \cdot p_4(t)
\end{align*}
\]

An analytically correct solution to a set of Ch-K-S system restricted with the normalization condition was determined with the use of the Mathematica Markov Continuous module. It was assumed that at the initial moment \( t = 0 \) the process \( X(t) \) was in state \( S_1 \). The obtained observation probabilities of states \( S_1 - S_9 \) are, in practice, complex functions (these are not solutions according to the classic method). When analyzing the operational process dynamics of Mi-8 helicopters, it is essential to study characteristic times, after which the object reaches a state of equilibrium. Such tests are made available by the Mathematica ver.11 software. For the Mi-8 helicopters, the initial distribution vector of the following form \( p_1 = [1,0,0,0,0,0,0,0] \) was adopted. During the initial period, the studied process was characterized by high change dynamics, which is shown in Figures 4 - 12.

\( p_1(t) \)

![Fig. 4 - The probability change dynamics of a Mi-8 helicopter staying in state \( S_1 \) (preparation for operation) over a time of 60 minutes](image-url)
Fig. 5 - The probability change dynamics of a Mi-8 helicopter staying in state $S_2$ (test execution) over a time of 60 minutes.

Fig. 6 - The probability change dynamics of a Mi-8 helicopter staying in state $S_3$ (refuelling) over a time of 60 minutes.

Fig. 7 - The probability change dynamics of a Mi-8 helicopter staying in state $S_4$ (readiness with a pilot) over a time of 60 minutes.
Fig. 8 - The probability change dynamics of a Mi-8 helicopter staying in state $S_5$ (readiness without a pilot) over a time of 60 minutes.

Fig. 9 - The probability change dynamics of a Mi-8 helicopter staying in state $S_6$ (protection) over a time of 60 minutes.

Fig. 10 - The probability change dynamics of a Mi-8 helicopter staying in state $S_7$ (work on the ground) over a time of 60 minutes.
As the curves presented in Figures 4 - 12 show, the studied process is characterized by significant dynamics of the changes in the initial phase for the distribution vector $p_j = [1,0,0,0,0,0,0,0,0]$. In practice, reaching a state of equilibrium is diversified for individual values of probabilities over time. After 4320 hours from the moment of forcing, all probabilities reach their limit values.
CONCLUSIONS

Table 4 lists the limit probability values for discrete \( p_j(n) \) and continuous time \( p_j(t) \) for Mi-8 helicopters.

<table>
<thead>
<tr>
<th>Set of Mi-8 helicopter states</th>
<th>( p_j(n) )</th>
<th>( p_j(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 ) - preparation for operation</td>
<td>0.150067</td>
<td>0.018333</td>
</tr>
<tr>
<td>( S_2 ) - test execution</td>
<td>0.153543</td>
<td>0.003545</td>
</tr>
<tr>
<td>( S_3 ) - refuelling</td>
<td>0.091754</td>
<td>0.001683</td>
</tr>
<tr>
<td>( S_4 ) - readiness with a pilot</td>
<td>0.153300</td>
<td>0.014478</td>
</tr>
<tr>
<td>( S_5 ) - readiness without a pilot</td>
<td>0.064989</td>
<td>0.037403</td>
</tr>
<tr>
<td>( S_6 ) - protection</td>
<td>0.116471</td>
<td>0.765280</td>
</tr>
<tr>
<td>( S_7 ) - work on the ground</td>
<td>0.164609</td>
<td>0.001182</td>
</tr>
<tr>
<td>( S_8 ) - task performance</td>
<td>0.088349</td>
<td>0.018467</td>
</tr>
<tr>
<td>( S_9 ) - unfitness (maintenance and repair)</td>
<td>0.014913</td>
<td>0.138986</td>
</tr>
</tbody>
</table>

The limit probabilities of a 9-state Mi-8 helicopter model in the discrete and continuous time domain differ significantly (Table 4). The reasons stem from a different interpretation of the frequency relationships (the variable space of state change in discrete time) and the intensity of process transitions (the distribution of state durations variable in physical time). In the course of analyzing the limit probabilities for Mi-8 helicopters relating to the Markov process in discrete \( p_j(n) \) and continuous time \( p_j(t) \), the following final conclusions may be formulated:

a) for discrete time:
   - the highest entry probabilities were observed for the states of \( S_7 \) (work on the ground),
   - almost identical for the states \( S_2 \) (test execution) and \( S_4 \) (readiness with a pilot) due to the fact that these are concurrently positive correlated processes, since the readiness with a pilot is practically linked with test execution,
   - slightly lower probabilities apply to the following states: \( S_1 \) (preparation for operation), \( S_6 \) (protection), \( S_3 \) (refuelling) and \( S_8 \) (task performance),
   - the smallest entry probabilities apply to the states of \( S_9 \) (unfitness) and \( S_5 \) (readiness without a pilot).

b) for continuous time:
   - the calculated functional readiness index for Mi-8 helicopter is 0.820103 (\( p_1 + p_2 + p_4 + p_6 + p_8 = 0.820103 \)). Therefore, it is high, which proves a correctly executed operational process from the point of view of the supervision over technical objects in the inventory of Airlift Base,
   - the highest probability of staying was observed for the states \( S_6 \) (protection) and \( S_9 \) (unfitness). The probabilities of staying in the other states are short-lived and do not have a significant impact on the calculated functional readiness index.
REFERENCES


