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THE ANALYSIS OF THE HELICOPTER TECHNICAL READINESS BY MEANS OF THE MARKOV PROCESSES

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ABSTRACT

The article presents a method of assessing the readiness of technical objects. The analysis focused on Mi-8 helicopters belonging to the Air Base. The Markov processes with discrete and continuous time were used to determine the readiness index. Based on the analysis of the operational process, nine operational states were distinguished, for which limit probabilities in discrete and continuous time were calculated. In addition, the dynamics of system changes in terms of striving for a stationary state were studied.

Keywords: helicopter operation and maintenance, Markov processes, technical readiness.

INTRODUCTION

Modern helicopters, thanks to their features, such as: the possibility of vertical take-off and landing, hovering, high manoeuvrability, mobility, range, velocity, resistance to ground fire, efficient own protection measures, as well as firepower and task execution accuracy, are the essential equipment of armed forces of many countries. From the point of view of operational features, the set of their advantages includes: ease of use and operation, and low encumbrance of crews with flight handling, which results facilitates the likelihood of an efficient task performance. Currently, helicopters are commonly used in national economy, for civilian, as well as military purposes. Their status in armed conflicts is of the basic combat asset in a modern battlefield (Tomaszek, 2013, 2016; Zieja, 2016; Żurek, 2009, 2014).

The helicopters operated in Poland are of obsolete design in terms of technical concept and manufacturing technology, and in many cases, they are multi-purpose objects of Soviet or Polish production. This article developed a method of assessing the readiness of Mi-8 helicopters with the application of the Markov processes. For this purpose, documentation data based on the Routine Maintenance Sheets were gathered, which are the basis for the register of Mi-8 helicopters over a period of two years (2014-2015). Next, the operational process was analysed, and an individual and cumulative database was developed. As a result, Markov models in discrete and continuous time were constructed (Jacyna-Gołda, 2017; Pham, 2006).

FORMULATION AND ANALYSIS OF HELICOPTER REGISTER STATES

The essence of mathematical modelling is the description of the studied phenomenon (process, system, set) using a mathematical language. The modelling process utilizes variables representing certain (and, therefore, significant from the point of view of the purpose) properties of the studied phenomenon (Brandimarte, 2014; Narayan, 2012; Tchórzewska-Cieślak, 2016). Studying an operational system requires the determination of all important factors, which define it. Secondary components, the ones, which unnecessarily complicate the model without significantly improving its quality should be ignored or omitted, and the ones similar in terms of the results should be grouped (Kierzkowski, 2017; Piegdoń 2018). Thorough examination of the process enables individualization of mutually disjoint operational states. It also imposes requirements for empirical data, used to construct the model. A minimum amount of its distinctive states, which enables calculating the basic operational indices, is enough to generally analyze an operational system. For a studied helicopter operational system, a set of the following states was distinguished:

S_1 - preparation for operation;

S_2 - test execution;

S_3 - refueling;

S_4 - readiness with a pilot;

S_5 - readiness without a pilot;

S_6 - protection;

S_7 - work on the ground;

S_8 - task performance;

S_9 - unfitness (maintenance and repair).

In order to ensure quality of an operation model, it is necessary to correctly select the permitted transitions of the object from the previous state to the next one (Knopik, 2016, 2018; Retsel, 2015). It was determined on the basis of the technical documentation and the current operational knowledge regarding the discussed operation process. The mathematical description of allowed transitions is a matrix of allowed transitions $S_i \rightarrow S_j$ from the previous state S_i(rows) to the next one S_j (column). The analyzed nine-state system has probable and prohibited transitions, according to Table 1 and the correlated graph shown in Figure 1, where:

0 - means a prohibited transition;
1 - means an allowed transition.

Table 1 - The matrix of allowed transitions of the Mi-8 helicopters operational process

$S_i \rightarrow S_j$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
S_1	0	1	0	0	0	1	1	0	1
S_2	1	0	1	1	1	0	1	0	0
S_3	1	0	0	1	1	0	1	0	0
S_4	0	1	1	0	0	0	1	0	0
S_5	1	0	1	0	0	1	0	0	0
S_6	1	0	1	0	0	0	0	0	0
S_7	0	1	1	1	1	0	0	1	0
S_8	0	0	0	0	0	0	1	0	0
S_9	1	1	0	0	0	1	0	0	0

As seen in Table 1, there are 72 theoretically possible interstate transitions, however, in the case of studying real operational processes, some transitions are prohibited. The studied sample narrowed this set to 30 allowed state changes, which is a natural consequence of the adopted process sequence. From the point of view of the process organization, narrowing the possible interstate transitions is understandable and is a consequence of the adopted (and not random) operational process organization.

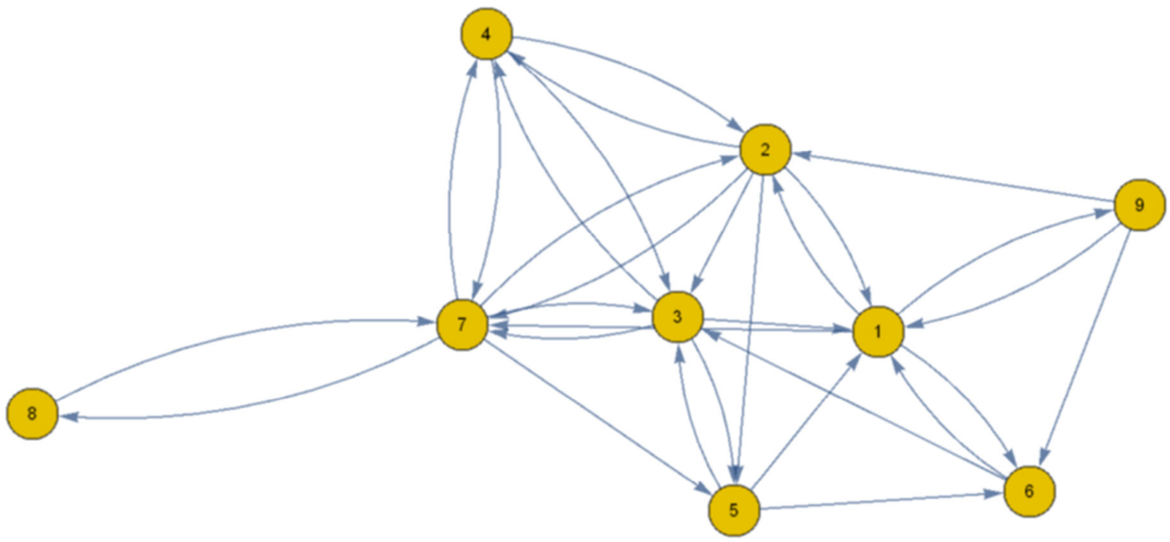

Fig. 1 - The nine-state operational model for Mi-8 helicopters

A graph showing the helicopter operation process including nine states. As can be seen, most of them are mutually communicating states ($S_1 \rightarrow S_2$, $S_1 \rightarrow S_6$, $S_1 \rightarrow S_9$, $S_2 \rightarrow S_4$, $S_3 \rightarrow S_4$, $S_3 \rightarrow S_5$, $S_3 \rightarrow S_7$, $S_4 \rightarrow S_7$, $S_7 \rightarrow S_8$). Operation is understood as a movement of the object over distinguished states, which form the phase space.

MARKOV PROCESS IN DISCRETE TIME

The first stage in constructing a Markov process in discrete time (Knopik, 2016; Werbińska-Wojciechowska, 2007, 2013) is the estimation of transition probabilities, as the values of estimators $\hat{p}_{ij}$ of elements $p_{ij}$and matrices P of probable transitions. The values of these estimators in a studied sample are frequencies w_{ij} of transitions from state S_i to state S_j, calculated according to the relationship (1):

$$\hat{\boldsymbol{p}}_{ij} = \boldsymbol{w}_{ij} = \boldsymbol{n}_{ij} / \boldsymbol{n}_i; \quad (1)$$

where:

n_{ij} - the number of transitions from state $S_i$to state S_j; n_i- the number of all transitions (exits) from state S_i;

w_{ij} - frequencies w_{ij} of transitions from state $S_i$to state S_j; n_i - number of observations of states S_i in the sample.

In the case of a nine-state operational process, the matrix of interstate transitions P adopts the following form:

$$P[p_{ij}] = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & p_{17} & 0 & p_{19} \\ p_{21} & 0 & p_{23} & p_{24} & p_{25} & 0 & p_{27} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & p_{35} & 0 & p_{37} & 0 & 0 \\ 0 & p_{42} & p_{43} & 0 & 0 & 0 & p_{47} & 0 & 0 \\ p_{51} & 0 & p_{53} & 0 & 0 & p_{56} & 0 & 0 & 0 \\ p_{61} & 0 & p_{63} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_{72} & p_{73} & p_{74} & p_{75} & 0 & 0 & p_{78} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{87} & 0 & 0 \\ p_{91} & p_{92} & 0 & 0 & 0 & p_{96} & 0 & 0 & 0 \end{bmatrix} \qquad (2)$$

Table 2 lists empirical likelihoods of interstate transitions for the tested group of Mi-8 helicopters.

Table 2 Probabilities of interstate transitions for Mi-8 helicopters

$p_i \rightarrow p_j$	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
p_1	0	0,447204	0	0	0	0,434782	0,018633	0	0,099378
p_2	0,012121	0	0,103030	0,721212	0,151515	0	0,012121	0	0
p_3	0,353535	0	0	0,242424	0,393939	0	0,010101	0	0
p_4	0	0,538922	0,005988	0	0	0	0,455089	0	0
p_5	0,057142	0	0,357142	0	0	0,585714	0	0	0
p_6	0,954545	0	0,045454	0	0	0	0	0	0
p_7	0	0,011299	0,282485	0,135593	0,033898	0	0	0,536723	0
p_8	0	0	0	0	0	0	1	0	0
p_9	0,058823	0,058823	0	0	0	0,882352	0	0	0

In relation to Markov processes with discrete time, it was proven (Pham, 2006) that if ergodic probabilities exist, then they can be calculated from the border of the transition matrix P in n steps, by solving a set of linear solutions or a simultaneous matrix equation, i.e., by passing from continuous t time to discrete time n, which is a number of the next experiment involving observation of a vehicle in time Δt, as per the relationship (3):

$$\bigwedge_j p_j = \lim_{n\to\infty} p_{ij}(n) = \sum_i p_i p_{ij} = p_j \Leftrightarrow P^T[p_j] = [p_j], \qquad \text{at } \sum_j p_j = 1 \qquad (3)$$

where:

P^T - transposed transition matrix P whereas $P = [p_j; i,j \in S]$; $[p_j]$ - vector of limit probabilities; p_{ij}- probabilities of transition from state i to state j.

The forms of a system of linear equations for ergodic probabilities $p_j(n)$ are shown by equations (4).

$$\begin{aligned} &0{,}012121p_2 + 0{,}353535p_3 + 0{,}057142p_5 + 0{,}954545p_6 + 0{,}058823p_9 - p_1 = 0 \\ &0{,}447204p_1 + 0{,}538922p_4 + 0{,}011299p_7 - 0{,}058823p_9 - p_2 = 0 \\ &0{,}103030p_2 + 0{,}005988p_4 + 0{,}357142p_5 + 0{,}045454p_6 + 0{,}282485p_7 - p_3 = 0 \\ &0{,}721212p_2 + 0{,}242424p_3 + 0{,}135593p_7 - p_4 = 0 \\ &0{,}151515p_2 + 0{,}393939p_3 + 0{,}033898p_7 - p_5 = 0 \\ &0{,}434782p_1 + 0{,}585714p_5 + 0{,}882352p_9 - p_6 = 0 \\ &0{,}018633p_1 + 0{,}012121p_2 + 0{,}010101p_3 + 0{,}455089p_4 + 1p_8 - p_7 = 0 \\ &0{,}536723p_7 - p_8 = 0 \\ &0{,}099378p_1 - p_9 = 0 \end{aligned} \qquad (4)$$

With simultaneous satisfaction of a requirement of system normalization (5).

$$\sum_{i=1}^{9} p_j = 1 \tag{5}$$

The solution of a system of equations (4) with a normalization requirement (5) was completed using the *Mathematica*, v.11 software. The solutions are abstruse functions in a set of complex numbers, which were not cited in this paper due to their complexity. Value of limit probabilities $p_j(n)$ are illustrated in Figure 2.

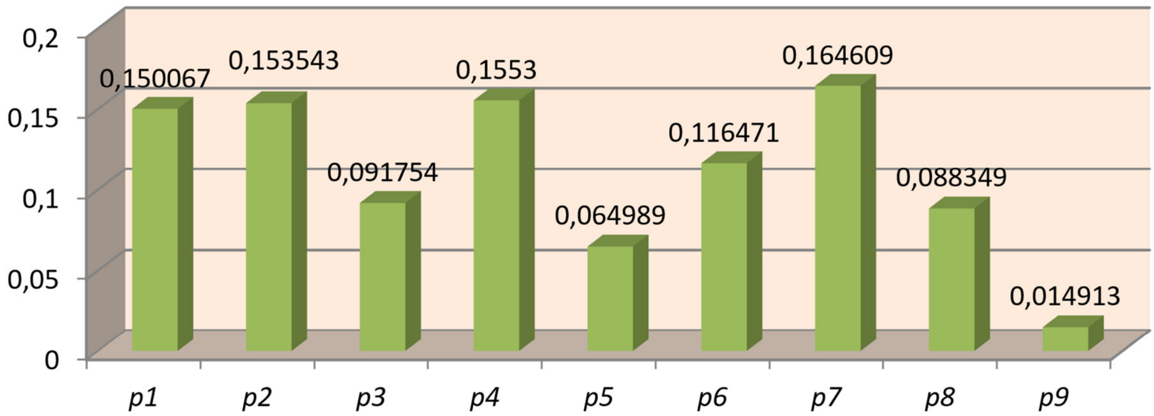

Fig. 2 - Limit probability $p_j(n)$ of the Markov process in discrete time for Mi-8 helicopters

Limit probabilities (Figure 2) relative to discrete time do not exceed values of 0.2. It proves the fact that there are no considerable disproportions regarding the priority for each of the nine operational states of a studied process. The biggest probability of entering is present for the states S_7 (work on the ground), S_4 (readiness with a pilot), S_2 (test execution) and S_1 (preparation for operation). The lowest probabilities were observed for the states S_9 (unfitness), S_5 (readiness without a pilot), S_8 (task performance) and S_3 (refuelling). However, we need to remember that the probabilities for the Markov chain are interpreted as a number of entries to a given state compared to all transitions from a distinguished set of states forming a phase space of the process. Therefore, they are interpreted in the quantitative and not qualitative sense (relative to the state duration).

MARKOV PROCESS IN CONTINUOUS TIME

The transition from discrete time to continuous time is through the intensity matrix Λ. Transition intensities $\lambda_{ij} \geq 0$for $i \neq j$ are defined as right-hand derivatives of transition probabilities relative to time, according to a relationship:

$$\boldsymbol{\lambda_{ij}\,(t_0) = d(p_{ij})/dt|t = t_{0+}} \tag{6}$$

Intensities $\lambda_{ii} \leq 0$ for $i = j$ are defined as complementation of the sum of intensity of transitions from state $S_i$for $i \neq j$ to 0 :

$$\boldsymbol{\lambda_{ii} + \Sigma_j \lambda_{ij} = 0} \tag{7}$$

hence:

$$\boldsymbol{\lambda_{ii} = -\,\Sigma_j \lambda_{ij}} \tag{8}$$

Modules $|\lambda_{ii}| = -\lambda_{ii}$are called intensities of exits from state S_i.They are not the intensities of returning from state S_i to S_i - as suggested by the notation. For the homogeneous Markov processes, the transition intensity is constant and equal to the inverse of average time ${}_{av}t_{ij}$ of an object staying in state S_i prior to state S_j:

$$\hat{\boldsymbol{\lambda}}_{ij} = \mathbf{1}/{}_{av}\boldsymbol{t}_{ij} \tag{9}$$

while:

$${}_{av}\boldsymbol{t}_{ij} = (\Sigma_j \boldsymbol{t}_{ij})/\boldsymbol{n}_i \tag{10}$$

where:

$t_{ij} = t_{k+1} - t_k$ only for ${}_{l}S_k = S_j$ - the time of an object staying in state S_i prior to state S_j, which is equal to the value of the discrete-continuous variable value for the observation number k.

${}_{av}t_{ij} = (\Sigma_j t_{ij})/{}_{tr}n_i$ means an average time spent in state $S_i$prior to state S_j.

Tab. 3 lists the transition intensities of matrices Λ for a nine-state operational process of Mi-8 helicopters.

Table 3 - Uniaxial tension test results

$\lambda_i \to \lambda_j$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
λ_1	-0,096710	0,010142	0	0	0	0,003961	0,016216	0	0,06639
λ_2	0,0799	-0,387914	0,058620	0,065348	0,050607	0	0,133333	0	0
λ_3	0,313948	0	-0,995498	0,352768	0,288782	0	0,04	0	0
λ_4	0	0,047199	0,013889	-0,078749	0	0	0,017661	0	0
λ_5	0,008681	0	0,012773	0	-0,026391	0,004936	0	0	0
λ_6	0,000319	0	0,000641	0	0	-0,000961	0	0	0
λ_7	0	0,086956	0,163934	0,172661	0,176471	0	-0,761588	0,161564	0
λ_8	0	0	0	0	0	0	0,015936	-0,015936	0
λ_9	0,002816	0,0025	0	0	0	0,00344	0	0	-0,008757

Transition intensities are expressed by the number of transitions per hour for a given object. After substituting the matrix Λ to the equation $[p_j]^T \cdot \Lambda = 0$, the following equation in matrix form was obtained for the studied operational process:

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix}^T \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & \lambda_{17} & 0 & \lambda_{19} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & \lambda_{37} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & \lambda_{46} & \lambda_{47} & \lambda_{48} & \lambda_{49} \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & 0 \\ \lambda_{61} & 0 & \lambda_{63} & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & 0 & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & \lambda_{84} & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & 0 & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

or, in the form of linear equation systems, as a relationship:

$$
\begin{aligned}
&-0{,}096710p_1 + 0{,}0799p_2 + 0{,}3139p_3 + 0{,}008681p_5 + 0{,}000319p_6 + 0{,}002816p_9 = 0\\
&0{,}010142p_1 - 0{,}387914p_2 + 0{,}047199p_4 + 0{,}086956p_7 + 0{,}0025p_9 = 0\\
&0{,}058620p_2 - 0{,}995498p_3 + 0{,}013889p_4 + 0{,}012773p_5 + 0{,}000641p_6 + 0{,}163934p_7 = 0\\
&0{,}065348p_2 + 0{,}352768p_3 - 0{,}078749p_4 + 0{,}172661p_7 = 0\\
&0{,}050607p_2 + 0{,}288782p_3 - 0{,}026391p_5 + 0{,}176471p_7 = 0\\
&0{,}003961p_1 + 0{,}004936p_5 - 0{,}000961p_6 + 0{,}00344p_9 = 0\\
&0{,}016216p_1 + 0{,}133333p_2 + 0{,}04p_3 + 0{,}017661p_4 - 0{,}761588p_7 + 0{,}015936p_8 = 0\\
&0{,}161564p_7 - 0{,}015936p_8 = 0\\
&0{,}06639p_1 - 0{,}008757p_9 = 0
\end{aligned} \qquad (12)
$$

This is a homogeneous system, which has an infinite number of solutions, among which there may be solutions satisfying the condition of normalization:

$$\sum_{i=1}^{9} p_j = 1 \qquad (13)$$

The solution to the above system (12) with a restriction (12) was obtained with the help of the *Mathematica* software (ver.11), and the result presented in Figure 3.

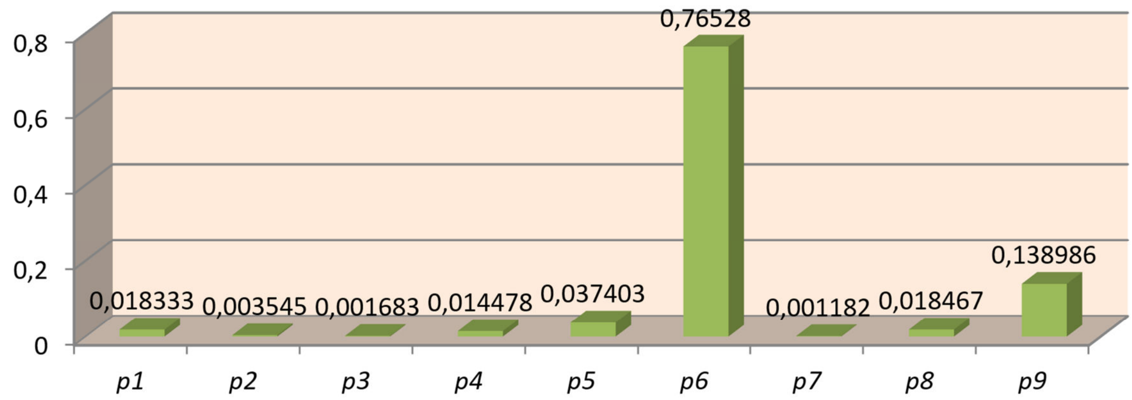

Fig. 3 - Limit probabilities $p_j(t)$ of the Markov process in continuous time for Mi-8 helicopters

Figure 3 shows that a Mi-8 type helicopter spends the most time in the states of S_6 (protection) and S_9 (unfitness). In relation to the other distinguished operational states, on average, it spends very little time, i.e., in the range of 0.037 in the stateS_5 (readiness without a pilot) to just over 0.001 for the states of S_7 (work on the ground) S_3 and (refuelling).

THE ANALYSIS OF CHANGES OF THE MI-8 HELICOPTER OPERATIONAL PROCESS IN TERMS OF THE DYNAMICS OF STRIVING FOR A STATIONARY STATE

The Smoluchowski - Chapman - Kolmogorov equation stems have the following matrix form:

$$\left(\prod_t' = \boldsymbol{d}\prod/\boldsymbol{dt} = \boldsymbol{\Lambda} * \prod\right) \wedge \left(\Sigma_j \boldsymbol{p}_j = \mathbf{1}\right) \qquad (14)$$

For the studied Markov process, they have the matrix form (15):

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \\ p_7(t) \\ p_8(t) \\ p_9(t) \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & \lambda_{17} & 0 & \lambda_{19} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & \lambda_{37} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & \lambda_{46} & \lambda_{47} & \lambda_{48} & \lambda_{49} \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & 0 \\ \lambda_{61} & 0 & \lambda_{63} & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & 0 & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & \lambda_{84} & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & 0 & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} = \begin{bmatrix} p_1{}'(t) \\ p_2{}'(t) \\ p_3{}'(t) \\ p_4{}'(t) \\ p_5{}'(t) \\ p_6{}'(t) \\ p_7{}'(t) \\ p_8{}'(t) \\ p_9{}'(t) \end{bmatrix} \quad (15)$$

or, it may be written in the form of a balanced differential equations system (16):

$$\begin{aligned}
p_1'(t) &= -\lambda_{12} \cdot p_2(t) - \lambda_{16} \cdot p_6(t) - \lambda_{17} \cdot p_7(t) - \lambda_{19} \cdot p_9(t) + \lambda_{21} \cdot p_2(t) + \lambda_{31} \cdot p_3(t) + \lambda_{51} \cdot p_5(t) + \lambda_{61} \\ &\quad \cdot p_6(t) + \lambda_{71} \cdot p_7(t) + \lambda_{91} \cdot p_9(t) \\
p_2'(t) &= -\lambda_{21} \cdot p_1(t) - \lambda_{23} \cdot p_3(t) - \lambda_{24} \cdot p_4(t) + \lambda_{12} \cdot p_1(t) + \lambda_{42} \cdot p_4(t) + \lambda_{92} \cdot p_9(t) \\
p_3'(t) &= -\lambda_{31} \cdot p_1(t) - \lambda_{34} \cdot p_4(t) - \lambda_{35} \cdot p_5(t) - \lambda_{37} \cdot p_7(t) + \lambda_{23} \cdot p_3(t) + \lambda_{43} \cdot p_4(t) + \lambda_{53} \cdot p_5(t) + \lambda_{63} \\ &\quad \cdot p_6(t) + \lambda_{73} \cdot p_7(t) + \lambda_{83} \cdot p_8(t) \\
p_4'(t) &= -\lambda_{42} \cdot p_2(t) - \lambda_{43} \cdot p_3(t) - \lambda_{46} \cdot p_6(t) - \lambda_{47} \cdot p_7(t) - \lambda_{48} \cdot p_8(t) - \lambda_{49} \cdot p_9(t) + \lambda_{24} \cdot p_2(t) + \lambda_{34} \\ &\quad \cdot p_3(t) + \lambda_{74} \cdot p_7(t) + \lambda_{84} \cdot p_8(t) \\
p_5'(t) &= -\lambda_{51} \cdot p_1(t) - \lambda_{53} \cdot p_3(t) - \lambda_{56} \cdot p_6(t) + \lambda_{35} \cdot p_3(t) + \lambda_{75} \cdot p_7(t) \\
p_6'(t) &= -\lambda_{61} \cdot p_1(t) - \lambda_{63} \cdot p_3(t) + \lambda_{16} \cdot p_1(t) + \lambda_{46} \cdot p_4(t) + \lambda_{56} \cdot p_5(t) + \lambda_{96} \cdot p_9(t) \\
p_7'(t) &= -\lambda_{71} \cdot p_1(t) - \lambda_{73} \cdot p_3(t) - \lambda_{74} \cdot p_4(t) - \lambda_{75} \cdot p_5(t) - \lambda_{78} \cdot p_8(t) + \lambda_{17} \cdot p_1(t) + \lambda_{37} \cdot p_3(t) + \lambda_{47} \\ &\quad \cdot p_4(t) + \lambda_{87} \cdot p_8(t) \\
p_8'(t) &= -\lambda_{83} \cdot p_3(t) - \lambda_{84} \cdot p_4(t) - \lambda_{87} \cdot p_7(t) + \lambda_{48} \cdot p_4(t) \cdot \lambda_{78} \\
p_9'(t) &= -\lambda_{91} \cdot p_1(t) - \lambda_{92} \cdot p_2(t) - \lambda_{96} \cdot p_6(t) + \lambda_{19} \cdot p_1(t) + \lambda_{49} \cdot p_4(t)
\end{aligned} \quad (16)$$

An analytically correct solution to a set of Ch-K-S system restricted with the normalization condition was determined with the use of the *Mathematica Markov Continuous* module. It was assumed that at the initial moment $t = 0$ the process $X(t)$ was in state S_1. The obtained observation probabilities of states $S_1 - S_9$ are, in practice, complex functions (these are not solutions according to the classic method). When analyzing the operational process dynamics of Mi-8 helicopters, it is essential to study characteristic times, after which the object reaches a state of equilibrium. Such tests are made available by the *Mathematica* ver.11 software. For the Mi-8 helicopters, the initial distribution vector of the following form $p_j = [1,0,0,0,0,0,0,0,0,]$ was adopted. During the initial period, the studied process was characterized by high change dynamics, which is shown in Figures 4 - 12.

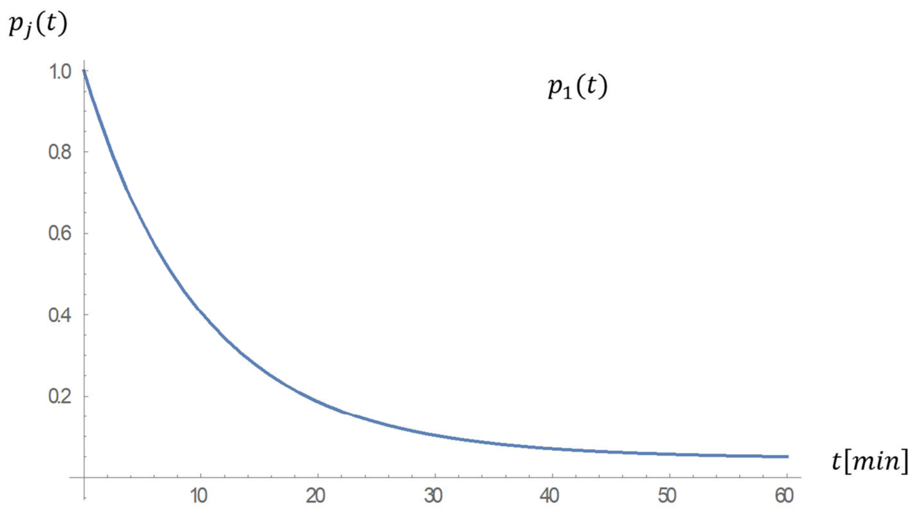

Fig. 4 - The probability change dynamics of a Mi-8 helicopter staying in state S_1 (preparation for operation) over a time of 60 minutes

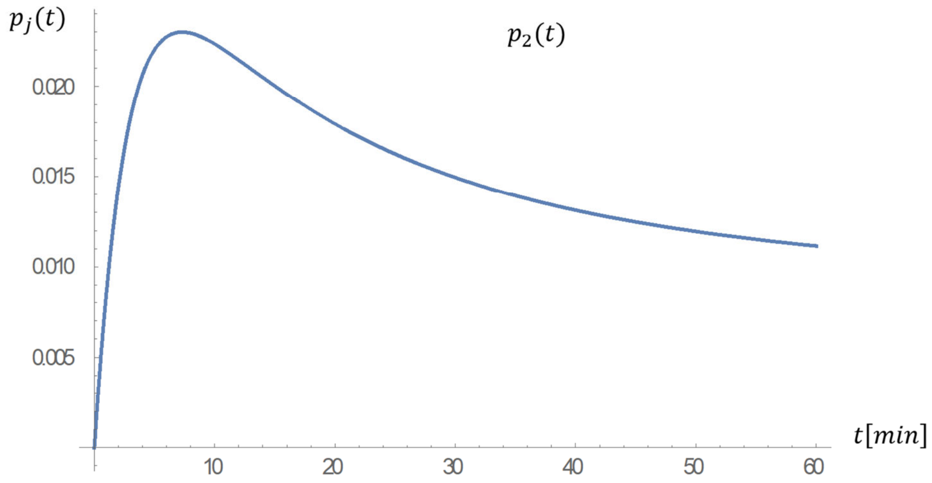

Fig. 5 - The probability change dynamics of a Mi-8 helicopter staying in state S_2 (test execution) over a time of 60 minutes

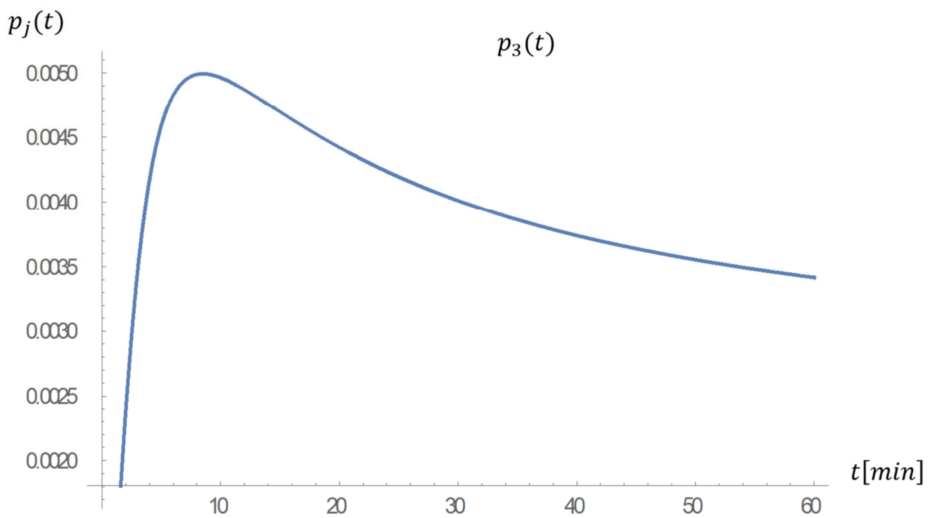

Fig. 6 - The probability change dynamics of a Mi-8 helicopter staying in state S_3 (refuelling) over a time of 60 minutes

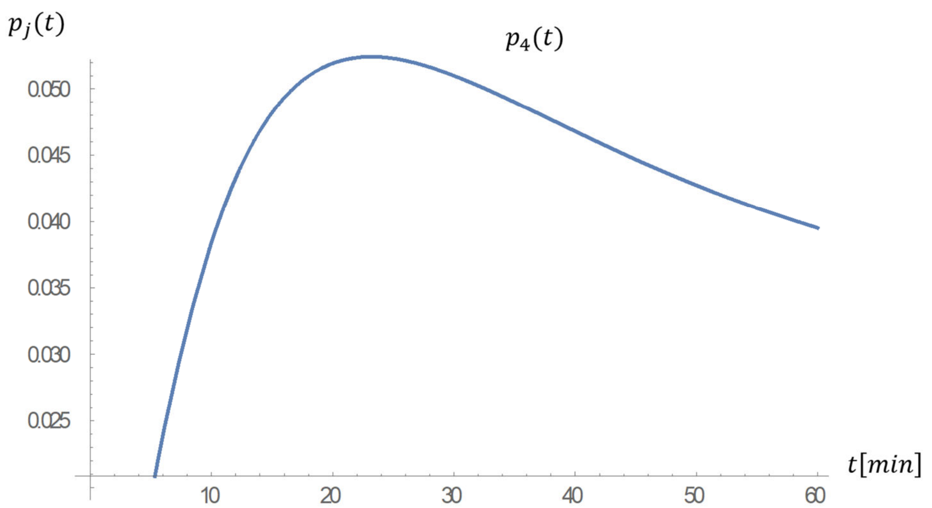

Fig. 7 - The probability change dynamics of a Mi-8 helicopter staying in state S_4 (readiness with a pilot) over a time of 60 minutes

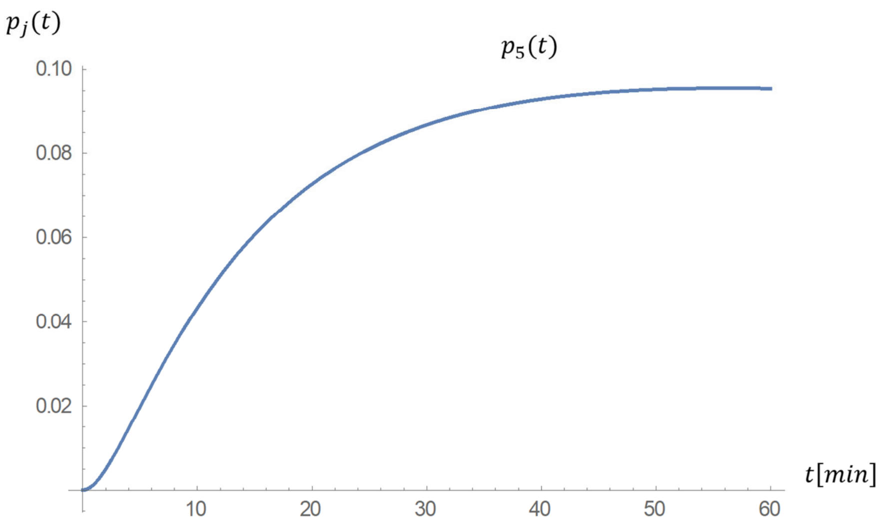

Fig. 8 - The probability change dynamics of a Mi-8 helicopter staying in state S_5 (readiness without a pilot) over a time of 60 minutes

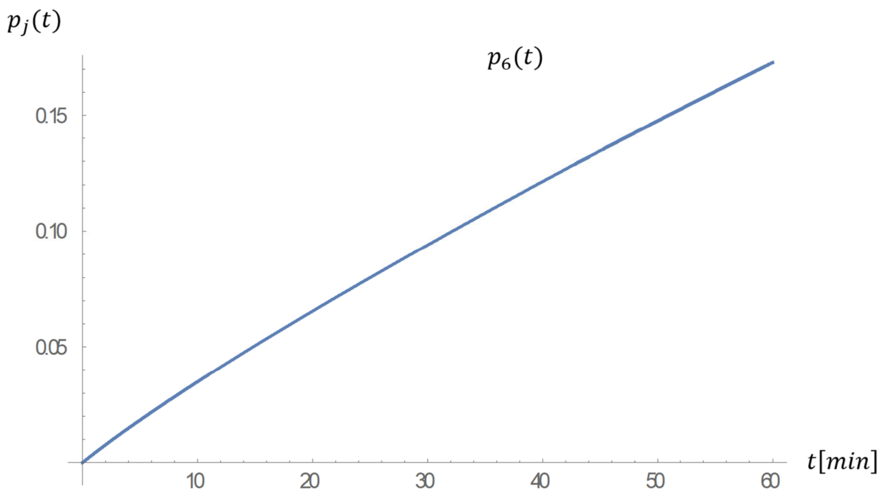

Fig. 9 - The probability change dynamics of a Mi-8 helicopter staying in state S_6 (protection) over a time of 60 minutes

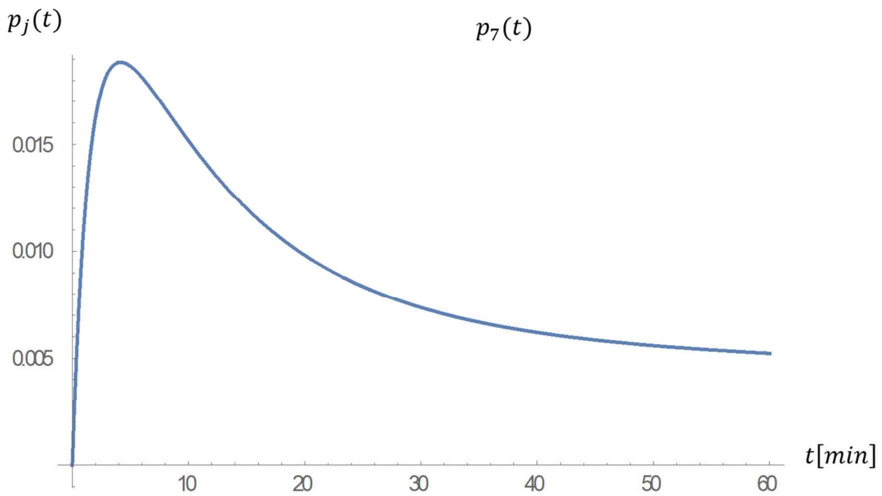

Fig. 10 - The probability change dynamics of a Mi-8 helicopter staying in state S_7 (work on the ground) over a time of 60 minutes

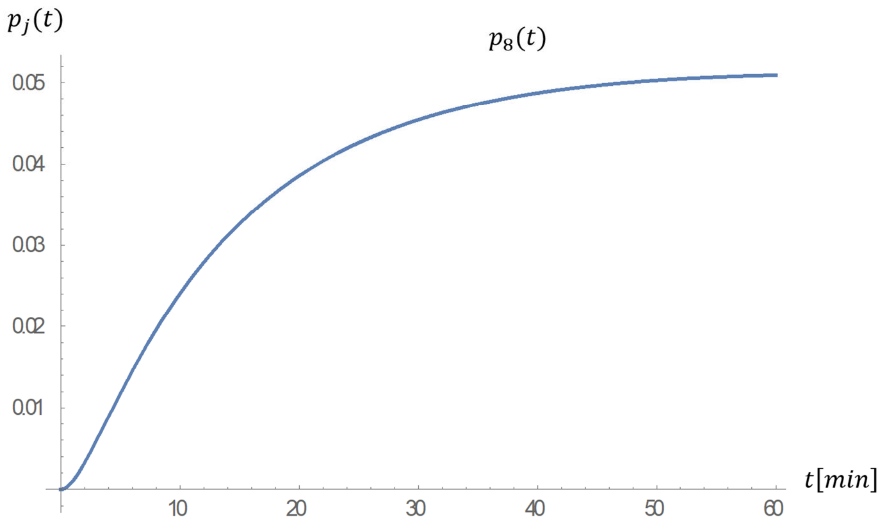

Fig. 11 - The probability change dynamics of a Mi-8 helicopter staying in state S_8 (task performance) over a time of 60 minutes

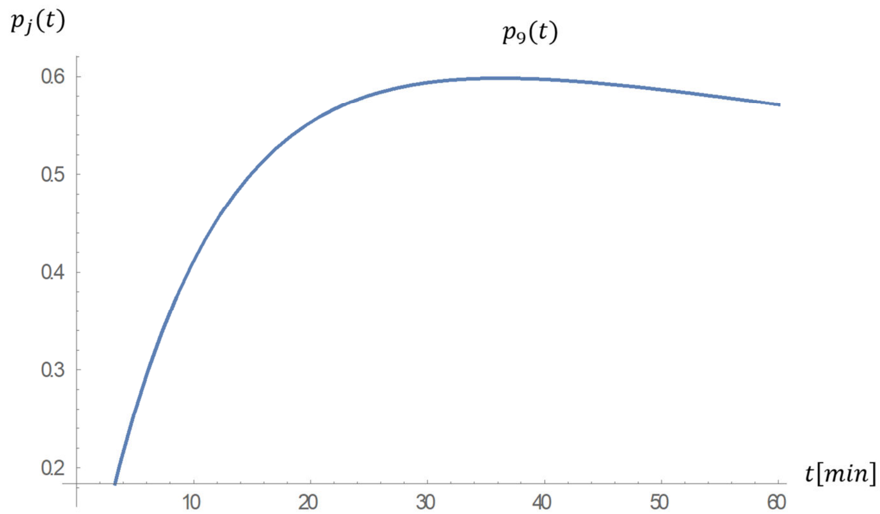

Fig. 12 - The probability change dynamics of a Mi-8 helicopter staying in state S_9 (unfitness) over a time of 60 minutes

As the curves presented in Figures 4 - 12 show, the studied process is characterized by significant dynamics of the changes in the initial phase for the distribution vector $p_j = [1,0,0,0,0,0,0,0,0]$. In practice, reaching a state of equilibrium is diversified for individual values of probabilities over time. After 4320 hours from the moment of forcing, all probabilities reach their limit values.

CONCLUSIONS

Table 4 lists the limit probability values for discrete $p_j(n)$ and continuous time $p_j(t)$ for Mi-8 helicopters.

Table 4 - Limit probabilities of a Mi-8 helicopter operational process in discrete $p_j(n)$ and continuous time $p_j(t)$ in a set of S_1-S_9 states

Set of Mi-8 helicopter states	$p_j(n)$	$p_j(t)$
S_1- preparation for operation	0.150067	0.018333
S_2 - test execution	0.153543	0.003545
S_3 - refuelling	0.091754	0.001683
S_4 - readiness with a pilot	0.153300	0.014478
S_5 - readiness without a pilot	0.064989	0.037403
S_6 - protection	0.116471	0.765280
S_7 - work on the ground	0.164609	0.001182
S_8 - task performance	0.088349	0.018467
S_9 - unfitness (maintenance and repair)	0.014913	0.138986

The limit probabilities of a 9-state Mi-8 helicopter model in the discrete and continuous time domain differ significantly (Table 4). The reasons stem from a different interpretation of the frequency relationships (the variable space of state change in discrete time) and the intensity of process transitions (the distribution of state durations variable in physical time). In the course of analyzing the limit probabilities for Mi-8 helicopters relating to the Markov process in discrete $p_j(n)$ and continuous time $p_j(t)$, the following final conclusions may be formulated:

a) for discrete time:

- the highest entry probabilities were observed for the states of:S_7 (work on the ground),
- almost identical for the states:S_2 (test execution) and S_4 (readiness with a pilot) due to the fact that these are concurrently positive correlated processes, since the readiness with a pilot is practically linked with test execution,
- slightly lower probabilities apply to the following states:S_1 (preparation for operation), S_6 (protection), S_3 (refuelling) and S_8 (task performance),
- the smallest entry probabilities apply to the states of:S_9 (unfitness) and S_5 (readiness without a pilot).

b) for continuous time:

- the calculated functional readiness index for Mi-8 helicopter is 0.820103 ($\mathrm{p}_1 + \mathrm{p}_2 + \mathrm{p}_4 + \mathrm{p}_6 + \mathrm{p}_8 = 0{,}820103$). Therefore, it is high, which proves a correctly executed operational process from the point of view of the supervision over technical objects in the inventory of Airlift Base,
- the highest probability of staying was observed for the states S_6 (protection) and S_9 (unfitness). The probabilities of staying in the other states are short-lived and do not have a significant impact on the calculated functional readiness index.

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