TESTING OF STEEL LOCAL MECHANICAL PROPERTIES AS RANDOM FUNCTIONS

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ABSTRACT

Local macroscopic mechanical properties of bar elements in steel building structures should be characterized by random functions of the same type. This idea has been inferred in present paper based on local hardness measurements. The possibility of trends occurring in the random sequences of hardness measured along the lengths of samples means that the test should be carried out on bars with a length comparable with the maximum lengths of technologically indivisible elements. Examination of this type of random sets on too short bars may lead to unreliable results even if a large statistical sample is used.

Keywords: structural steel, local mechanical properties, random functions, autocorrelation functions.

INTRODUCTION

Traditional determination of random bearing capacity for bar structures made of steel has been based on arbitrary subdivision of bars into sections described by mutually independent yield limits (Augusti, Baratta and Casciati, 1984). Such an approach turns out to be incorrect in terms of experimental investigations of random local mechanical properties functions. In the experimental measurements of steel hardness performed in random series, as opposed to the conventional measurements of tensile strength, the adverse phenomena of disregarding a substantial mass of the tested material located near the clamps and random location of the fracture within the sample are avoided.

In the analysis of experimental results presented in this paper, a model of sum of Gaussian stationary random noise and a random linear trend are proposed to be used for detailed description of local mechanical properties of structural steel.
LOCAL MACROSCOPIC MECHANICAL PROPERTIES OF STRUCTURAL STEEL AND CONCEPT OF THEIR STOCHASTIC MODELLING

Local macroscopic mechanical properties of thin-walled bars used in steel building structures vary in a random manner, both along the length of the bar as well as across the width and thickness of its cross section (Figure 1). This circumstance significantly complicates the tasks related to measurement and modelling of these properties, even in the case of steel bars rolled unidirectionally. A simplification of these tasks, appropriate especially in the case of length size effect research may consist in considering “integral” mechanical properties of the bar cross-section determined on flat tensile samples with a thickness equal to the thickness of the bar and width equal to the width of the selected cross section wall or a part thereof (Figure 2).

![Fig. 2 - Samples having the thickness of full thin-walled bar wall](image)

Fig. 2 - Samples having the thickness of full thin-walled bar wall

The yield limit $R_e$ and breaking strength $R_m$ exhibiting a clear departure from stationarity (Jastrzebski, 1961), as depicted in Figure 3, have been obtained during tensile test of samples taken from reinforcing bars having the diameter of 6 mm and coiled in 180 m long sections. This is related to the cooling conditions of the bars after drawing. Nevertheless this result indicates, that a detailed analysis of sufficiently long (in the order of 10 m) samples of this type
should be made with respect to the bars used in steel building structures. An evaluation of variability of local mechanical properties in structural steel may be obtained in an easiest way based on microhardness measurements. In fact, this property depends on local changes in material structure and may be measured at points specified a’priori, without losing access to the results at the location of clamps, as usually happens in the case of traditional tensile tests. The Brinell hardness measurement scheme used by the Authors, with application of a ball having the diameter \( D = 10 \text{ mm} \) pressed in with a force \( P = 30 \text{ kN} \) is depicted in Figure 4. The diameters of impressions have been measured by a workshop microscope with an accuracy of \( 10^{-3} \text{ mm} \) on a surface ground to a high degree of smoothness. The variation of random error measurements has been estimated at \( D(\Delta H) \approx 17.1 \text{ (MPa)}^2 \).

![Image](image.png)

Fig. 4 - Sampling scheme with step \( d \) as a function of random hardness \( H(x) \)

A SINGLE LONG SEQUENCE OF HARDNESS MEASUREMENTS

The three independent sets of Brinell hardness \( H \text{ [MPa]} \) measurements performed by the Authors at locations spaced every \( d = 2 \text{ cm} \) (Figure 4) on 20x8 mm flats originating in the same batch of mill products made of S235 steel and having the length of 6 m each, are depicted in detail in Figure 5. In the considerations that follow these flats are denoted by symbols “A”, “B” and “C”, respectively. At the initial stage of statistical analysis (Bendat and Piersol, 2010; Himmelblau, 1970; Lifshits, 1995; Lindgren, 2012) the occurrence of trends has been analysed (limiting the scope of analysis to linear trends) in sequences \( H_i^A \), \( H_i^B \) and \( H_i^C \), respectively.

For the linear trends expressed as \( H_i^t = ai + b \), with parameters \( a \) and \( b \) computed by the least squares method to fit the sequences “A”, “B” and “C”, the results presented in the Table 1 have been obtained.

<table>
<thead>
<tr>
<th>Flat</th>
<th>( a ) [MPa]</th>
<th>( b ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>“A”</td>
<td>-0.0094</td>
<td>138.1</td>
</tr>
<tr>
<td>“B”</td>
<td>-0.1490</td>
<td>141.2</td>
</tr>
<tr>
<td>“C”</td>
<td>+0.0107</td>
<td>139.0</td>
</tr>
</tbody>
</table>

The removal of trends corresponds to the following sequences:

\[
H_i^* = H_i - \Delta_i
\]

where:

\[
\Delta_i = H_i^t - \bar{H}
\]

is a local trend measure, while \( \bar{H} \) represents the average value of random sequence.
An analysis of the results presented in the Table 1 leads to the conclusion that the trend effect proved to be significant only in the case of the sequence “B”. In the case of sequences “A” and “C” one may assume, approximately, $H_i^* = H_i$.

Subsequently, an analysis of stationarity of the sequence $H_i^*$ with respect to the average value $\overline{H}$ and variances $D_H^*$ (Bendat and Piersol, 2010; Himmelblau, 1970) has been performed. Therefore statistical hypotheses on equality of averages $\overline{H}_j^*$ and equality of variances $D_{Hj}^*$ in $m$ subsequent sequences $H_j (j=1,2,\ldots,m)$ having equal lengths have been considered. In particular, “series numbers” (Wald – Wolfowitz) and “inversion numbers” (Mann –Kendall) tests have been applied, as these tests do not require compliance with the normality conditions of the tested sequences. The results obtained, which will not be discussed here in detail, allow for acceptance of these hypotheses. In the following course of this analysis the assumptions on stationarity and ergodicity (that the estimators may be computed based on single sufficiently long sequences) for averages $\overline{H}_j^*$ and variances $D_{Hj}^*$, and additionally on ergodicity with respect to the probability distribution have been adopted.

Based on the sequences of measurements $H_i^*$ corrected according to (1) (the influence of small random error in hardness measurements has been disregarded here) the histograms of frequency “w” have been prepared and are depicted in Figure 6. The width of histogram class, assumed as equal to $\Delta H = 10$ MPa uniformly for sequences “A”, “B” and “C”, roughly corresponds to...
optimum values (in the sense of the minimum least squares estimator error) estimated within the range of $\Delta_H = 9 \div 11$ MPa. According to the formula $\Delta_H \approx 3.5 \mu_H \sqrt{\frac{3}{n}}$ (Smirnoff and Dunin-Borkowski, 1969), that is obtained for the sets $H_i^*$ measured on the bars “A”, “B” and “C”.

![Fig. 6 - Hardness frequency histograms for flats “A”, “B” and “C”](image)

The values depicted in Figure 6, computed based on the consecutive histograms and corresponding frequencies $w_i(t=1,2,\ldots,p)$, include the average values for a sample $\overline{H}^*$, standard deviations $\mu_H^*$, as well as empirical coefficients of skewness $s^*$ and excess $e^*$, where:

$$s^* = \frac{\mu_H^*}{(\mu_H^*)^3} \quad \text{and} \quad e^* = \frac{\mu_H^*}{(\mu_H^*)^4} - 3$$

(3)
where for the central moments of the $k = 3$ and $4$ order the following holds:

$$
\mu_{Hk} = \frac{1}{p} \sum_{i=1}^{p} w_i \left( H_i^* - \overline{H}^* \right)^k
$$

(4)

The continuous line depicted in Figure 6 represents the density of normal probability distribution selected for empirical values of $\overline{H}^*$ and $\mu_H^*$. The standard deviations of random skewness $\mu_s$ and random excess $\mu_e$, determined on an $n$-element sample of the normal population are respectively equal to (Smirnoff and Dunin - Borkowski, 1969):

$$
\mu_s = \sqrt{\frac{6n - 1}{(n + 1)(n + 3)}} \quad \text{and} \quad \mu_e = \sqrt{\frac{24n(n - 2)(n - 3)}{(n - 1)^2(n + 3)(n + 5)}}
$$

(5)

In the calculations conducted by the Authors for $n = 285$, based on (5), the values of $\mu_e = 0.144$ and $\mu_e = 0.280$ have been obtained, respectively. These values are therefore mostly higher than the values computed for the sample, $\mu_s^*$ and $\mu_e^*$ (see Figure 6), and this speaks for the admissibility of treating the probability distribution of variable $H^*$ as normal distribution.

In Figure 7, the values of the normalized autocorrelation function are depicted, determined based on the sequences $H_i^*(i = 1, 2, ..., n = 285)$ of hardness measurements performed on bars “A”, “B” and “C”, treated hypothetically as implementations of ergodic random sequences.

Fig. 7 - Estimators of normalized hardness autocorrelation function for flats “A”, “B” and “C”
The autocorrelation function is defined as:

\[ \rho_H(r \cdot d) = \frac{K_H(r)}{K_H(0)} \quad (6) \]

where:

\[ K_H(r) = \frac{1}{n-r-1} \sum_{k=1}^{n-r} \left( H_k^* - \bar{H}^* \right) \left( H_{k+r}^* - \bar{H}^* \right) \quad (7) \]

for \( r = 0, 1, 2, \ldots, r_{\text{max}} \) and for \( r_{\text{max}} \leq \left( \frac{n}{5} \cdot \frac{n}{3} \right) \) is an estimator of the autocorrelation moment calculated in disregard of a small influence of random error present in hardness measurements. The fading away of empirical values of \( \rho_H(r \cdot d) \) for large values of \( r \) speaks in favor of the hypothesis on the ergodicity of the random functions considered, assumed a’priori in calculations of \( K_H(r) \) conducted according to (7).

Functions selected from a group of often used autocorrelation functions \( \rho(\Delta x) \) juxtaposed in the Table 2 in combination with corresponding spectral density functions \( s(\omega) \) are depicted in Figure 7 as well. These functions have been selected to fit the empirical values, under assumption of “fluctuation range” (correlation span) \( \theta \), which proved to be approximately equal for all three considered flats “A”, “B” and “C”. This correlation span has been determined as \( \theta = r_{\theta} d \approx 14 \text{ cm} \), meaning in turn, that \( r_{\theta} = 7 \), where:

\[ \theta = \sum_{r=0}^{\infty} |\rho_H(r \cdot d)|d \quad (8) \]

Table 2 - Examples of normalized autocorrelation functions \( \rho(\Delta x) \) and normalized spectral densities \( s(\omega) \) associated with them

<table>
<thead>
<tr>
<th>differentiable process</th>
<th>( \rho(\Delta x) )</th>
<th>( s(\omega) )</th>
<th>nondifferentiable process</th>
<th>( \rho(\Delta x) )</th>
<th>( s(\omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(\Delta x) )</td>
<td>( \rho(\Delta x) )</td>
<td>( \theta^2 e^{-\theta^2/4} )</td>
<td>( \rho(\Delta x) )</td>
<td>( \frac{4}{\theta^4} \exp -\frac{2 \Delta x}{\theta} )</td>
<td></td>
</tr>
<tr>
<td>( \pi(\Delta x) )</td>
<td>( \pi(\Delta x) )</td>
<td>( \pi(\Delta x) )</td>
<td>( \pi(\Delta x) )</td>
<td>( \theta \sin^2 \left( \frac{\omega \theta}{2} \right) )</td>
<td></td>
</tr>
<tr>
<td>( \theta )</td>
<td>( \theta )</td>
<td>( \theta )</td>
<td>( \theta )</td>
<td>( \theta - \theta )</td>
<td></td>
</tr>
<tr>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td></td>
</tr>
<tr>
<td>( \omega )</td>
<td>( \omega )</td>
<td>( \omega )</td>
<td>( \omega )</td>
<td>( \omega )</td>
<td></td>
</tr>
</tbody>
</table>

A SET OF SEQUENCES OF HARDNESS MEASUREMENTS

The hypothetical (abstract) general population of repetitive and randomly homogeneous bars with respect to hardness \( H \) corresponds to a single “long” ergodic implementation analysed in the previous section (considered after removal of possible trend). In the statistical analysis of random functions the most often a real population of repetitive elements randomly uniform with respect to the considered measurable property is considered. The sampling of such population consists of acquiring an ordered set of measurements. In the case considered by the Authors this would consist of acquiring an array of hardness measurements \( H = [H_i]_{n \times m} \), where the index \( i = 1, 2, \ldots, n \) enumerates consecutive measurements along the axis of the bar, while the
index \( j = 1, 2, \ldots, m \) enumerates bars in the considered sequence. Assuming initially the simplest model of the stationary random sequence \( H_j \), one may in particular perform calculations for a set of “cross sections” \( i = \text{const} \), and this in turn, with respect to the average values would yield:

\[
\overline{H_i} = \frac{1}{m} \sum_{j=1}^{m} H_{ij}
\]

(9)

and with respect to the autocorrelation functions:

\[
K_H^{(i)}(r) = \frac{1}{m-1} \sum_{j=1}^{m} (H_{ij} - \overline{H_i})(H_{i+r,j} - \overline{H_{i+r}})
\]

(10)

Thus, proceeding to the full set \( H_i \) treated as a set of \( n \) cross sections one obtains:

\[
\overline{H} = \frac{1}{n} \sum_{i=1}^{n} H_i \quad \text{and} \quad K_H(r) = \frac{1}{n} \sum_{i=1}^{n} K_H^{(i)}(r)
\]

(11)

The results of \( K_H(r) \) calculations for a population of 42 flats 40×8 mm having the length \( l = 2.5 \text{ m} \) each, made of S235 steel and originating in the same steel batch, subjected to Brinell hardness test every \( d = 4 \text{ cm} \) are depicted in Fig. 8. Due to the negative results of testing the hypothesis on the equality of mean values in longitudinal sequences, the figure also highlights the values of the autocorrelation function \( K_{H0}(r) \), such as:

\[
K_{H0}(r) = K_H(r) - D_{H0}
\]

(12)

where:

\[
D_{H0} = \frac{1}{m-1} \sum_{j=1}^{m} (\overline{H}_j - \overline{H})^2
\]

(13)

is a variance between groups (between averages \( \overline{H}_j \) for longitudinal sequences).

In Figure 9, a relatively complex analytical expression is depicted and selected to match the results of estimated normalized spectral density function \( s_H(\omega) \) and the corresponding normalized autocorrelation function \( \rho_H(\Delta t) \), with \( \Delta t = r \cdot d \). Important qualitative discrepancies between the results depicted in Figure 8 and Figure 9 and those presented earlier in Figure 7, especially the fact that the autocorrelation function does not vanish for \( i \to \infty \), indicate that the
approach based on application of the formulae (9) to (13) without satisfying the condition of sufficiently long implementations $H_i(i = 1, 2, \ldots, n)$, removal of the trend effect and equality of variances in particular implementations, should be considered incorrect.

**CONCLUDING REMARKS**

Determination of the characteristics of random mechanical properties of structural steel mentioned in the title of this paper requires measurements on bars with lengths close to the maximum lengths of elements originating from the same steel product (in the order of 10 m). Then, after removal of the trend, one may determine the characteristics of the random function based on single implementation. The appropriate general population of the bars is then treated as a hypothetical (abstract) one. In a real population of nominally identical bars the above mentioned linear trends will be expressed by coefficients $a$ and $b$ treated as random variables. In contrast to the random functions of time, in the case of bars considered here the orientation of trends with respect to the index “$i$” growth direction may be assumed for instance as always increasing. Only in the case of bars loaded by an unsymmetrical (relative to the stationarity center) distribution of axial forces one would have to introduce an alternating orientation of trend. The characteristics of the random functions determined in the simplest manner, as for stationary random function, for a real population of nominally identical bars may be burdened by substantial errors due to the influence of trends and variations in variances determined for individual implementations.

Determination of the characteristics of random local strength function on the basis of characteristics of random hardness function requires prior determination of the mutual correlation function for these quantities. This problem will be discussed by the Authors in a separate report.

**REFERENCES**


