HEAT AND MASS TRANSFER CORRELATIONS FOR THE DESIGN OF SMALL INDIRECT CONTACT COOLING TOWERS

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ABSTRACT

Cooling towers are increasingly used in buildings as a component of environmental cooling systems. When compared with industrial towers, cooling towers with lower capacities and smaller dimensions can be used for this application, with inlet water temperatures ranging from 22 to 25°C. Correlations for heat and mass transfer coefficients were obtained experimentally for a small-size indirect cooling tower, with a nominal cooling capacity of 10 kW. The new correlations are compared with existing ones and results of their application to a simplified cooling tower model are shown.

INTRODUCTION

Cooling towers are increasingly used in buildings as a component of environmental cooling systems. With chilled ceilings, because moderately high water temperatures can be used (18-20°C supply temperature), it is possible to cool water in an indirect contact cooling tower during most of the cooling period. The cooling tower could be combined with a supplementary refrigeration plant, or used alone if a short period of overheating is allowed or energy storage or night-cooling techniques are used.

Indirect contact cooling towers have been traditionally used to remove excess heat from various industrial processes with hot water temperatures between 32 and 46°C and typical cooling capacities above 40 kW. For chilled ceilings, cooling towers with lower capacities and smaller tower dimensions can be used with inlet water temperatures ranging from 22 to 25°C.

Existing simplified models allow the prediction of cooling tower performance but using as input heat and mass transfer correlations which were experimentally obtained for large-size (industrial) cooling towers. Those correlations do not apply to small-size cooling towers, as requested for most building applications.

The most important coefficients used in models are the mass transfer coefficient between spray water interface and air and the heat transfer coefficient between tubes and spray water. All correlation based models assume tube surface is completely wet, through a uniform distribution of spray water over the tube bundle. Therefore, mass and heat transfer occurs between an water film and air flow. For simultaneous heat and mass transfer, heat flux can be calculated through enthalpy potential, [1]:

\[ q = \alpha_m (h_i - h_{air}) \]  (1)

where \( \alpha_m \) is the mass transfer coefficient.

An enthalpy balance, for an elementary transfer surface \( dA \), can be expressed as

\[ m_{air} dh_{air} = \alpha_m (h_i - h_{air}) dA \]  (2)

which is known as the Merkel equation, [2], and which integration for the whole tower gives

\[ \frac{\alpha_m A}{m_{air}} = \frac{\int dh_{air}}{\int [h_i - h_{air}]} \]  (3)

The local energy balance, for an elementary tube surface \( dA \), can be written as

\[ K (T_w - T_i) dA = \alpha_m (h_i - h_{air}) dA - m_{spray} c_{p,spray} dT_{spray} \]  (4)

neglecting spray rate variation and with \( K \) being the heat transfer coefficient between water inside tube and film/air interface. It can be calculated by adding all thermal resistances:

\[ \frac{1}{K} = \frac{1}{\alpha_w} \frac{d_{ext}}{d_{int}} + \frac{d_{ext}}{2k_{tube}} \ln \frac{d_{ext}}{d_{int}} + \frac{1}{\alpha_{spray}} \]  (5)

using external tube surface as reference \( (d_{ext}) \).
When calculating tower performance, inlet conditions (water and air) are known and outlet conditions are the output. An iterative procedure is applied: water outlet temperature is guessed, which allows to calculate outlet air properties and the enthalpy integral in equation (3); since the integral value will not be equal to the first member of equation (3) - available number of transfer units, NTU – a new outlet temperature is used until equality is obtained.

Several models can be found in the literature, differing in simplifications assumed. The model to be considered, developed by Mizushima, [3], neglects spray temperature variation in equation (4) and considers interface conditions \( h_i, T_i \) constant throughout the cooling tower. Integration of equation (3) then gives

\[
\frac{a_m A}{m_{air}} = \ln \frac{h_i - h_{air, in}}{h_i - h_{air, out}}
\]  

The integration of equation (4) and substitution into (6) leads to

\[
\frac{T_{w, in} - T_i}{T_{w, out} - T_i} = \frac{h_i - h_{air, in}}{h_i - h_{air, out}} \exp \left( \frac{K}{a_m c_{pw} m_{w}} \right)
\]

which together with the function \( h_i = h_i (T_i) \) forms a non-linear set of equations for calculating \( T_i, h_i \).

**NOMENCLATURE**

- \( A \) area (m²)
- \( c_p \) specific heat (J/kg/K)
- \( d \) tube diameter (m)
- \( G \) mass velocity, at minimum tower section (kg/m²/s)
- \( h \) specific enthalpy (J/kg)
- \( K \) heat transfer coefficient between water inside tubes and spray/air interface (W/m²/K)
- \( k \) thermal conductivity (W/m/K)
- \( m \) mass flow rate (kg/s)
- \( q \) heat flux (W/m²)
- \( Re \) Reynolds number
- \( Sh \) Sherwood number
- \( T \) temperature (°C, K)
- \( x \) water vapour content in air or absolute humidity (kg water / kg dry air)

**Greek letters**

- \( \alpha_w \) mass transfer coefficient for water vapour, between spray water film and air (kg/m²/s)
- \( \alpha_{spray} \) heat transfer coefficient between tube external surface and spray water film (W/m²/K)
- \( \alpha_e \) heat transfer coefficient for water inside the tubes (W/m²/K)

\( \epsilon \) thermal efficiency

\( \Gamma \) spray mass rate per length of tube (kg/s/m)

**Subscripts**

- \( air \) air flow
- \( ext \) external
- \( i \) interface (spray water film/air)
- \( in \) at tower inlet
- \( int \) internal
- \( LM \) logarithmic mean difference
- \( out \) at tower outlet
- \( sat \) at saturation
- \( spray \) spray water
- \( tube \) tubes
- \( w \) water inside tubes
- \( wb \) wet bulb

**EXPERIMENTAL SETUP**

A new indirect contact cooling tower was designed in order to be used with chilled ceilings. Design conditions were a cooling capacity of 10 kW, for an inlet water temperature of 21°C, a water flow rate of 0.8 kg/s and an air wet bulb temperature of 16°C. The tower has a section of 0.6 m x 1.2 m and a height of 1.55 m. The tube bundle has 228 staggered tubes of 10 mm outside diameter, with a pitch of 25 mm and with a total transfer area of 8.6 m². This corresponds to a much smaller size than usual towers. The ratio between heat transfer surface and heat exchanger volume is 25 m²/m³.

The tower was manufactured by Sulzer Escher Wyss (Lindau, Germany). It is much smaller than cooling tower models usually built by this manufacturer. A forced draft configuration was chosen with a crossflow fan located at air entrance. This arrangement has a lower noise level, and also leads to a lower pressure drop. It was also chosen to facilitate air flow measurements. Fig.1 shows schematically the cooling tower and main variables involved.

A test facility was assembled at Porto to test this cooling tower. The thermal load was modeled with an electrical heater located in a water tank. Tower inlet water temperature was controlled by varying heating power. Fan speed was also controlled by means of a frequency controller, which allowed changing air flow rate. Spray water flow rate could be changed manually (valve with 7 fixed positions). Cooling water flow rate could also be changed, by varying pump speed and using regulating valves.

Tower water inlet and outlet temperatures were measured with PT100 probes. Air flow rate was measured with one vane anemometer at tower outlet section. In order to measure cooling water temperature evolution, several thermocouples were connected to the tubes. The data acquisition system used a data logger - HP 34970A – and HP VEE as software.
Figure 1: Indirect contact cooling tower and main variables.

**MASS TRANSFER COEFFICIENT**

The mass transfer coefficient can be obtained after experimental measurements and using a mass balance:

\[
m_{\text{air}} (x_{\text{air,out}} - x_{\text{air,in}}) = \alpha_m A \Delta x_{\text{LM}}
\]

where \( \Delta x_{\text{LM}} \) is the logarithmic mean humidity difference, defined as

\[
\Delta x_{\text{LM}} = \frac{x_{\text{air,out}} - x_{\text{air,in}}}{\ln \left( \frac{x_{\text{sat,i}} - x_{\text{air,in}}}{x_{\text{sat,i}} - x_{\text{air,out}}} \right)}
\]

Since it is difficult to know exactly the air humidity at tower outlet, due to changes along the section, an alternative method was used, using temperature (enthalpy) difference:

\[
m_{\text{air}} (h_{\text{air,out}} - h_{\text{air,in}}) = \alpha_m A \Delta h_{\text{LM}}
\]

with

\[
\Delta h_{\text{LM}} = \frac{h_{\text{air,out}} - h_{\text{air,in}}}{\ln \left( \frac{h_{\text{sat,Ti}} - h_{\text{air,in}}}{h_{\text{sat,Ti}} - h_{\text{air,out}}} \right)}
\]

Average spray temperature was used as interface temperature and outlet enthalpy was calculated after an enthalpy balance:

\[
h_{\text{air,out}} = h_{\text{air,in}} + \frac{m_{\text{w}} c_{\text{p}} (T_{w,\text{in}} - T_{w,\text{out}})}{m_{\text{air}}}
\]

Among others, Mizushina, [3], Parker, [4], and Niitsu, [5], correlated \( \alpha_m \) with air Reynolds number. Mizushina presented a more complete relationship involving also spray Reynolds number. Fig. 2 shows experimental results obtained with the small tower as a function of air flow rate ratio (flow rate divided by maximum flow rate). A correlation for the mass transfer coefficient as a function of air mass flow ratio is:

\[
\alpha_m = 0.17 \left( \frac{m}{m_{\text{max}}} \right)_{\text{air}}^{0.81}
\]

or

\[
\alpha_m = 0.064 G_{\text{air}}^{0.81}
\]

obtained for \( 0.4 < m_w < 0.8 \text{ kg/s},\ 10 < T_{w,\text{in}} < 20^\circ \text{C} \) and \( 15 < T_{w,\text{out}} < 28^\circ \text{C} \), with a correlation coefficient of 0.82.

Figure 2: Mass transfer coefficient as a function of air flow rate (maximum = 1.7 kg/s). Different air humidities are identified.

In order to compare this with existing correlations, [3,4,5], obtained for different geometries, a dimensionless representation was made – see Fig.3. The correlation for the small tower lies below existing ones (with the exception of Parker’s one), which were obtained for larger (industrial) towers.
**Figure 3:** Comparison of mass transfer coefficient correlations.

As Fig.2 shows, air humidity also seems to have an influence on mass transfer coefficient: higher humidities lead to higher coefficients. In order to express the effect of air humidity, a different correlation was attempted. The humidity was represented through the ratio dry/wet bulb air temperature (at tower inlet) – $T_{wb} / T_{wb}$ (in K). Fig.4 shows the relationship for 3 air flow rates. Note that correlation coefficients are much larger (>0.9) and that the effect of humidity is higher for higher flow rates. However, this type of correlation will increase the complexity of the numerical procedure for tower performance calculation. The effect of the error associated with the mass transfer coefficient on tower performance will be discussed in a later section.

**Figure 4:** Mass transfer coefficient as a function of $T_{air}/T_{wb}$ and air flow rate.

**SPRAY HEAT TRANSFER COEFFICIENT**

The heat transfer coefficient between tubes and spray water was calculated from experimental data and after calculating $K$. To calculate $K$ the following global (balance) equation was used:

$$m_w c_p \left( T_{w,in} - T_{w,out} \right) = K \Delta T_{LM}$$

(15)

with

$$\Delta T_{LM} = \frac{T_{w,out} - T_{w,in}}{\ln \left( \frac{T_{w,out} - T_{spray}}{T_{w,in} - T_{spray}} \right)}$$

(16)

$\alpha_{spray}$ was then obtained using a modified form of equation (5):

$$\alpha_{spray} = \left[ \frac{1}{K} - \frac{d_{ext}}{\alpha_n d_{int}} - \frac{d_{ext}}{2k_{turb}} \ln \left( \frac{d_{ext}}{d_{int}} \right) \right]^{-1}$$

(17)

Fig.5 shows experimental results for $\alpha_{spray}$ as a function of spray flow ratio (flow rate divided by maximum). A correlation for the heat transfer coefficient is:

$$\alpha_{spray} = 700 \left( \frac{m}{m_{max}} \right)_{spray}^{0.358}$$

(18)

or

$$\alpha_{spray} = 602 \left( \frac{\Gamma}{d_{ext}} \right)^{0.358}$$

(19)

obtained for $0.4 < m_a < 0.8 \text{ kg/s}$, $10 < T_{wb} < 20^\circ C$ and $15 < T_{wb} < 28^\circ C$, with a correlation coefficient of 0.74.

**Figure 5:** Heat transfer coefficient as a function of spray flow rate (maximum = 1.4 kg/s).

**Figure 6:** Comparison of mass transfer coefficient correlations.
Figure 6: Comparison of heat transfer coefficient correlations.

The effect of the heat transfer coefficient and its associated error on tower performance will be discussed in the following section.

SENSIBILITY ANALYSIS WITH SIMPLIFIED MODELS

The previous correlations can be applied to existing indirect contact cooling tower models. Mizushina’s model, [3], will be considered here, expressed through equations (6) and (7). Cooling tower performance can be expressed by its thermal efficiency, defined as

\[ \varepsilon = \frac{T_{w,in} - T_{w,out}}{T_{w,in} - T_{wb}} \]  

(20)

By applying the simplified model to the tested cooling tower, the results in Fig.7 were obtained. The influence of air and water flow rate on tower thermal efficiency can be analysed. Efficiency increases with the increase in air flow rate and decreases with the increase in water flow rate.

![Figure 7: Tower efficiency as a function of air and water flow rate (m_{spry}=1.4 kg/s, T_{wb}=16°C).](image)

For more details on the application of this and other models the interested reader is referred to [6].

The model can also be used to assess the influence of the error when determining mass and heat transfer coefficients on tower efficiency. Fig.8 shows the rate of change of efficiency with mass transfer coefficient (\( \partial \varepsilon / \partial m_w \)) for different values of air flow rate. Using \( \partial \varepsilon / \partial m_w \), the error on efficiency due to the error on mass coefficient can be assessed. For the maximum experimental air rate (1.7 kg/s) correlation (13) gives \( m_w = 0.17 \) kg/m²/s; if the maximum deviation in Fig.2 is assumed \( m_w \) will be equal to 0.32. The difference in thermal efficiency for the two values of \( m_w \) will be equal to 0.07 (\( \varepsilon = 0.46 \) compared to 0.53). This is not too large for an extreme situation. Therefore, \( f \)

![Figure 8: Rate of change of efficiency with mass transfer coefficient for different values of air flow rate (maximum spray and water rates).](image)

Fig.9 shows the rate of change of efficiency with heat transfer coefficient (\( \partial \varepsilon / \partial \alpha_{spry} \)) for different values of spray flow rate. The influence of the heat transfer coefficient on thermal efficiency is much lower than the influence of mass transfer coefficient. For the maximum experimental spray rate (1.4 kg/s) correlation (18) gives \( \alpha_{spry} = 697 \) W/m²/K; if the maximum deviation in Fig.5 is assumed \( \alpha_{spry} \) will be equal to 950. The difference in thermal efficiency for the two values of \( \alpha_{spry} \) will be equal to 0.007 (\( \varepsilon = 0.456 \) compared to 0.463). This is a negligible difference for this extreme situation. Therefore, correlation (19) can be used with a good degree of accuracy for all situations.

![Figure 9: Rate of change of efficiency with heat transfer coefficient for different values of spray flow rate (maximum air and water rates).](image)

CONCLUSIONS

A small-size indirect contact cooling tower was tested and mass and heat transfer coefficients were experimentally determined. Mass transfer coefficient was correlated with air...
flow rate. The effect of air humidity was also discussed. Heat transfer coefficient was correlated with spray water flow rate. It was found that the present correlations lead to mass and heat transfer coefficients that are lower than the ones for large (industrial) indirect contact cooling towers.

The correlations were applied to an existing simplified model and tower performance results were obtained. The model also showed that the correlations found have a good degree of accuracy when applied to all possible operating conditions.

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