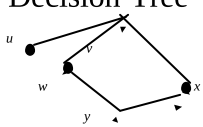
Trees & Game Theory

Bidding

Auctions

Negotiations

Decision Tree



A directed graph *T* is said to be a *tree* if

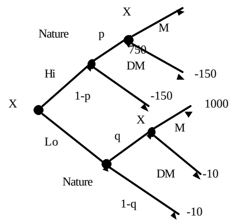
1. There exists a distinguished node *R* (called the root of the tree) that has no edges going into it, and
2. For every other *u* of the graph there exists exactly one path from the root *R* to *u*.

Single-person Decisions

- Generally *decision graph* is employed to describe the sequential decision process of single person.
- A *decision graph* is any directed graph having a unique root *R*, in the sense that

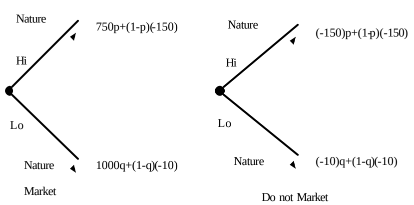
1. *R* is the only node with no edge ending into it;
2. For every node *N* other than *R*, there is at least one path from *R* to *N*;
3. There is at least one terminal node; and
4. From every non-terminal node *N*, there is at least one path from *N* to a terminal node.

Uncertainty and Single-person Decisions

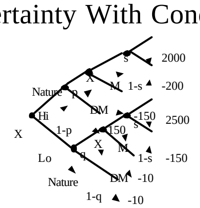


X – Decision Node
Nature - Consequence
Hi – High Investment
Lo – Low Investment
M – Market
DM – Do not Market
P – Probability of success under high investment
q - Probability of success under low investment

Uncertainty and Single-person Decisions (Cont'd)



Uncertainty With Conditional Probability



s – Marketing is Successful
1-s – Marketing isn't successful

- Bayes' Formula – if an event B is known to have occurred what is the probability that another event A will happen?

$$P(A/B)=\frac{P(B/A)P(A)}{P(B/A)P(A)+P(B/A^c)P(A^c)}$$

Conditional Probability

- "I have two canvas book bags filled with poker chips. The first bag contains 70 green chips and 30 white chips, and I shall refer to this as the predominantly green bag. The second bag contains 70 white chips and 30 green chips, and I shall refer to this as the predominantly white bag. The chips are all identical except for color. I now mix up the two bags so that you don't know which is which and put one of them aside. I shall be concerned with your judgments about whether the remaining bag is predominantly green or not. Now suppose that you choose 12 chips at random with replacement from this remaining bag and it turns out that you draw eight green chips and four white chips, in some particular order. What do you think the odds are that the bag you have sampled from is predominantly green?"
- Professor ward Edwards

Which Bag?

Assume P (GB) = .5 & P (WB) = .5

If A is event "g g w g w g g w g w g g"

then

$P(A|GB) = .7 \times .7 \times .3 \times \ldots \times .7 = (.7)^8(.3)^4$

$=0.000467$

$P(A|WB) = .3 \times .3 \times .7 \times \ldots \times .3 = (.7)^4(.3)^8$

$=0.0000158$

Bayes' Solution

$$P(GB \mid A) = \frac{P(A \mid GB)P(GB)}{P(A \mid GB)P(GB) + P(A \mid WB)P(WB)} = 0.967!$$

- Is this a good "bet" given such probabilities?
- What happens if the event list is shorter or longer? (Basis for sampling theory.)

n-player Game Tree

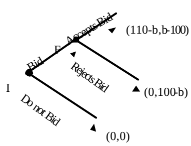
A tree *T* is said to be an *n-player game tree* if

a. Each non-terminal node of the tree is 'owned' by exactly one of the players
b. At each terminal node *v* of the tree an *n*-dimensional ''payoff' vector $p(v)=(p_1(v),\ p_2(v),\ \ldots,\ p_n(v))$ is assigned

A *sequential game* is an n-player game tree such that the decision nodes have been partitioned into information sets that belong to the players.

Sequential Games - Perfect Information

- A *sequential game* is a game of perfect information if every information set is a singleton. Otherwise it is a game with imperfect information.
- A *sequential game* of perfect information is a sequential game in which a player knows exactly what choices have been made in the game at the time she has to make a choice.

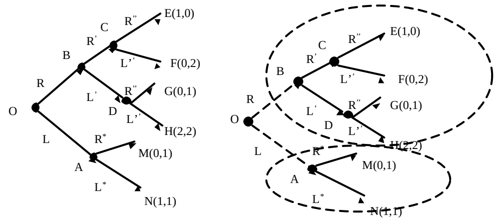


Sequential Games - Subgames

A *subgame* of an *n*-player extensive form game is another extensive form *n*-player game such that:

1. Its game tree is a branch of the original game tree
2. The information sets in the branch coincide with the information sets of the original game and cannot include nodes that are outside the branch
3. The payoff vectors of the terminal nodes of the branch are precisely the same as the payoff vectors of the original game at these terminal nodes

Sequential Games With Imperfect Information



3/19/2003

Contract Bidding

- Basic elements
- direct job costs
- mark up or return
  - overhead
  - profit

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Multiple Objectives

- Maximum profit
- Win award
- Want many awards to fill capacity
  - High volume vs. High profit
  - Optimum markup

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Average Return

$$\overline{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$$

$$E[\overline{R}] = \frac{1}{N}\sum_{i=1}^{N} E[R_i]$$

$$V[\overline{R}] = \overline{s}^2 / N$$

Decision

- If σ does not grow as a function of N, the variance approaches zero.
- Then the sample mean value will equal expected value.
- Strategy should maximize expected return for each contract to result in the largest long-run average return?

Competitor's Markup

- C - estimate of direct job cost
- X_0 - competitor's bid
- X - competitor's percentage markup random variable

$$X = \frac{(X_0 - C)}{C} 100 \quad \%$$

“Exceedance” Probability

- Analysis of other bids

$$p(x) = P(X > x) = 1 - F(x)$$
$$p(x_0) = P(X > 0) = P(X_0 > C)$$

- $P(X_0 > C)$ is probability that competitor’s bid X_0 exceeds the direct job cost C.

Consequence

- Bidder’s own percentage markup is k
- Bidder wins when $x>k$
- Percentage return is a random variable with two discrete values $(k, 0)$

Win Probabilities

$$P(R = k) = P(X > k)$$
$$P(R = 0) = P(X \le k)$$
$$E[R] = kP(R = k) + (0)P(R = 0) = kP(X > k)$$

- E[R] - expected percentage return

Single Competitor

$$p(x) = p_0 e^{-x/q}, x \geq 0, q > 0$$
$$E[R] = kp_0 e^{-k/q}$$

- Only positive markups are considered!
- Only positive values for x.

Optimum Markup

$$k^* = q$$
$$E^*[R] = \frac{qp_0}{e}$$

- Find maximum by taking first two derivatives (Necessary and sufficient)

Multiple Bidders

- General Relationship:

$$p_i(x_i) = p_{0i} e^{-x_i/q_i}, x_i \geq 0, q_i > 0, \; for\, i = 1,\ldots,n$$
$$\prod_{i=1}^{N} P(X_i > k) = \prod_{i=1}^{N} p_i(k)$$

Multiple Bidder Solution

$$E[R] = k\prod_{i=1}^{N} P(X_i > k) = k\prod_{i=1}^{N} p_i(k)$$

$$E[R] = k\tilde{p}_0 e^{-k/\tilde{q}}, \tilde{p}_0 = p_{01} p_{02} \ldots p_{0N}$$

$$and\ \frac{1}{\tilde{q}} = \frac{1}{q_1} + \frac{1}{q_2} + \ldots + \frac{1}{q_N}$$

Multiple Bidders Solution

- Optimal Percentage Markup

$$k^* = \tilde{q}$$

$$E^*[R] = \frac{\tilde{q}\tilde{p}_0}{e}$$

Multiple Bidders Solution

- Optimal Percentage Markup
  - Same distribution for all bidders

$$k^* = \frac{q}{N}$$

$$E^*[R] = \frac{q p_0^N}{Ne}$$

Multiple Bidder Implications

- Both optimal percentage markup and maximum expected percentage return are reduced as number of bidders increases
- Job return is reduced considerably when competition for the contract is increased

Bid Error

- Skewness - curve falls off less rapidly from the peak on the high side of k* than on the low side
  - overestimate of k* is less of a loss than an underestimate by the same amount

Cost of Uncertainty

- Opportunity loss or regret
- Better decision could be made after the future starts to unfold and decision is reconsidered in retrospect.
- Loss incurred due to inability to predict, exactly, change factor outcomes.

Opportunity Loss

- Difference between value indicated in consequence node and best that could have been achieved by considering all possible decisions and same outcome.
- Random variable.
- If alternative chosen produces smallest loss, then, of course, the opportunity loss is zero.

Expected Opportunity Loss

- Represents the long-term average cost that results from having less than perfect information.
- Computation of E[L] provides information to evaluate risks of each alternative but also for value of information collected and technology developed to reduce uncertainty.

Cost of Uncertainty

- Cost of uncertainty “E[L]”
- Can be described by either
  - table
  - loss function

Bidding Problem

- Opportunity loss represented by difference between X and R
  - X percentage markup by competitor
  - X continuous variable
  - R percentage return corresponding to percentage markup k by bidder
  - R discrete variable, values k and 0

Bidding E[L]

Opportunity loss:

$$l(k,x) = \{ \begin{matrix} x-k, & for\ 0 \le k < x \\ x, & for\ x \le k < \infty \end{matrix}$$

$$L = \{ \begin{matrix} X, & for\ 0 \le X \le q \\ X-q, & for\ X > q \end{matrix}$$

X is random variable with known density of X

Value for Perfect Information

$$E[L] = \int_0^q x\ f(x)\,dx + \int_q^\infty (x-q)\ f(x)\,dx$$

$$F(x) = 1 - p(x)$$

$$p(x) = p_0 e^{-k/q}$$

$$f(x) = -\frac{dp(x)}{dx} = \frac{p_0 e^{-x/q}}{q}$$

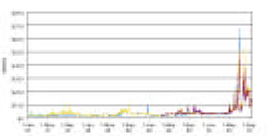
$$E[L] = 0.632\ p_0 q$$

Questions?

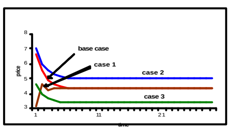
- Interesting Case is Dynamic Simulation of Strategies as information is found

Introduction

- Electric power industry restructuring
- Electric market dynamics



Price dynamics in California market



Price dynamics in [Yang,2002]

Introduction II

- Changes in Generation company (GENCO) decision making

	Past market	**Restructured market**
Demand	Assured	Unsecured
Operation	Cost-based	Profit –based
Competition	Indirect	Direct
Risk	Low	High

Short-term Electric Market

- Modeling of GENCOs decision-making with
  - different market participants expectations;
  - different competitions
  - different market organizations;
  - different gain maximizing horizon
  - with and without uncertainty.
- Modeling of ESCOs decision making
- Modeling of other ancillary service providers
- Modeling of other markets (fuel, emissions, etc.)

Dynamics Modeling And Simulation

- Electric market dynamics properties
  - stability criteria and equilibrium calculation;
  - equilibrium properties
  - different transition processes
  - market properties
- Replication of actual markets

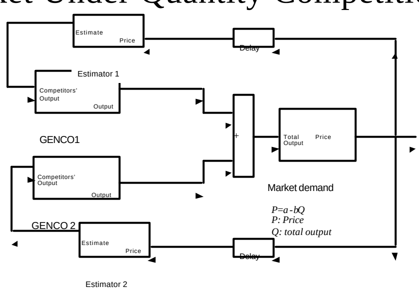
Decisions In Short- Term

- A short-term optimal decision-making model for GENCOs to maximize profit with effects of
  - market organizations;
  - short-term technical constrain
  - short-term market constraints;
  - bidding strategy and
  - uncertainty.

Method utilized

- Control theory
  - Model decision-making as control processes
  - Model electric market as a discrete time control system
- Decision analysis
  - A probabilistic framework to assist in discussions, compare alternatives, and find optimal actions by decision-makers
- Decision theory
  - How to make a series of decisions and take a series of actions in a state space, where the changes of state are controlled by those decisions and actions.

Market Under Quantity Competition



Market Dynamic Simulations - Quantity Competition

- Traditional Cournot model
  - Firms assume competitor will not change the output decisions no matter how much they produces
- Extension of Cournot model
  - GENCOs estimate competitors output and make output decision based on the estimate.

$$q_i(t) = \frac{a - c_{i1}}{2b} - \frac{1}{2}\hat{q}_j(t)$$

GENCOs - Naive Expectation

- Naïve expectation
  - GENCO i believes that GENCO j will not change its output

$$q_j^{\wedge}(t) = q_j(t-1)$$

- Market model

$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{a-c_{11}}{2b} \\ \frac{a-c_{12}}{2b} \end{bmatrix} \qquad p(t) = \begin{bmatrix} -b & -b \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$

GENCOs - Naive Expectation II

- Different estimate of the demand $p = a_i - b_i Q$
- Market system

$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{a_1-c_{11}}{2b_1} \\ \frac{a_2-c_{12}}{2b_2} \end{bmatrix} \qquad p(t) = \begin{bmatrix} -b & -b \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$

$$c_i(q_i) = c_{i2} q_i^2 + c_{i1} q_i + c_{i0}$$

- With quadratic cost function
- Market model

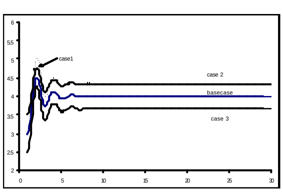
$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{b_1}{2(b_1+c_{21})} \\ -\frac{b_2}{2(b_2+c_{22})} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{a_1-c_{11}}{2(b_1+c_{21})} \\ \frac{a_2-c_{12}}{2(b_2+c_{22})} \end{bmatrix} \qquad p(t) = \begin{bmatrix} -b & -b \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$

GENCOs - Naive Expectation III

- Case study

		Parameters		
Case	base	1	2	3
a	6	7	6	6
b	0.1	0.2	0.1	0.1
c_{11}	3	3	4	2
c_{21}	3	3	3	3
Stable?	Yes	yes	yes	Yes
		Equilibrium properties		
Q_1	10	6.667	3.333	16.667
q_2	10	6.667	13.333	6.667
Q	20	13.333	16.667	23.333
Price	4	4.333	4.333	3.667
		Marketproperties		
MP_1,MP_2	0.333,0.333	0.444,0.44	[illegible]	[illegible]
MS_1,MS_2	0.5,0.5	0.5,0.5	0.2,0.8	0.714,0.286
	Market system properties			
Controllability	Not controllable			
Observability	Notobservable			

Parameters and market properties

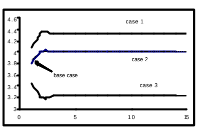


price dynamics

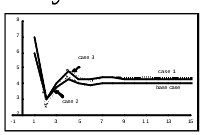
GENCOs - Other Expectations

- Forward expectation
  - adjust expectation according to the other's output history and possible final equilibrium
- Adaptive expectation,
  - Adjust expectation according to the other's true output and the forecasting error in the last period
- Moving average expectation
  - Assign data in the past with weights to reflect ability in forecasting.
- Mixed expectations

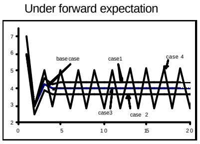
Electric Market Dynamics



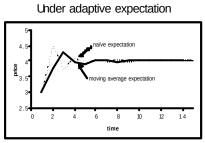
Under forward expectation



Under adaptive expectation



Under moving average expectation

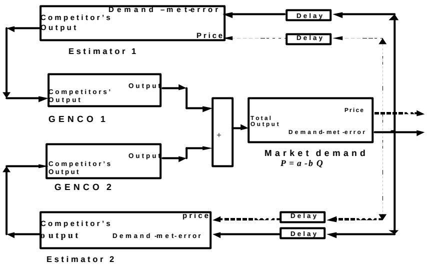


naïve expectation Vs. moving average expectation

Market Simulations Conclusions

- Under all expectations, market price/quantity equilibrium, together with market share and market power, depend on all parameters except c_{i0} ;
- c_{i0} does not influence market stability and equilibrium
- System demonstrates uncontrollability
- Under different expectations
  - GENCOs make different decisions
  - Market has different dynamics

Demand-met-error Feedback

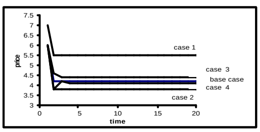


Introduction of Demand-met-error Feedback

- Market model

$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} -\frac{a_2}{2} \\ -\frac{a_1}{2} \end{bmatrix} u(t) + \begin{bmatrix} \frac{a - c_{11}}{2b} \\ \frac{a - c_{21}}{2b} \end{bmatrix} \qquad p(t) = [-b \quad -b] \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$

- Market property major change:
  - Controllable
  - Less dynamic



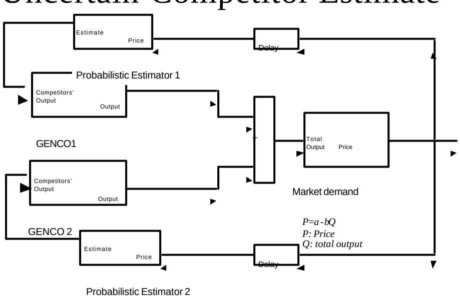
Discussions

- Examples of other control schemes
  - Introduction of price cap or price floor (saturation processes);
  - Limitation on maximum market share of market participants;
  - Information flow control
  - Adjustment of transaction cost;
  - Load management
- Models used to study other market
  - With proper modifications: time settings, constraints, information available…

Quantity Competition - Uncertainty

- Uncertainty source
- information delay
  - One –period
  - More than one period
- Uncertainty associated with GENCO forecast
  - Forecast of demand
  - Forecast of competitor

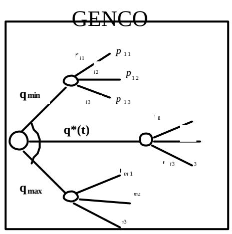
Uncertain Competitor Estimate



Uncertain Competitor Estimate II

Estimate of competitor decision

Case	Output of GENCO j at time t	Probability
1	$q_j(t-1)g_{i1}$	r_{i1}
2	$q_j(t-1)g_{i2}$	r_{i2}
3	$q(t-1)g_{i3}$	r_{i3}

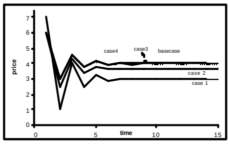


Uncertain Competitor Estimate III

- Market model

$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\sum_k r_{1k} g_{1k}}{2} \\ -\frac{\sum_k r_{2k} g_{2k}}{2} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{a_1 - c_{11}}{2b_1} \\ \frac{a_2 - c_{21}}{2b_2} \end{bmatrix}$$

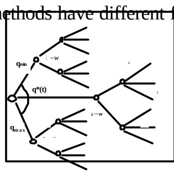
$$p(t) = \begin{bmatrix} -b & -b \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$



- Conclusions
  - equilibrium, if any, depends on all system parameters except c_{i0};
  - c_{i0}, does not influence market stability or equilibrium

Uncertain Forecasting Techniques

- GENCO1
  - adaptive expectation and forward expectation
  - Different methods have different forecasting accuracies



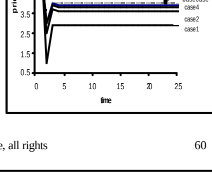
- GENCO2 uses adaptive expectation only.

Uncertain Forecasting Techniques II

- Market model　　　Market dynamics

$$\begin{bmatrix} q_1(t+1) \\ \hat{q}_1(t+1) \\ q_2(t+1) \\ \hat{q}_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{vb_1 + (1-w)(1-l)}{2} & -\frac{v(1-b_1)}{2} \\ b_1 & 1-b_1 & 0 & 0 \\ -\frac{b_1}{2} & -\frac{1-b_1}{2} & 0 & 0 \\ 0 & 0 & vb_1 + (1-w)(1-l) & v(1-b_1) \end{bmatrix} \begin{bmatrix} q_1(t) \\ \hat{q}_1(t) \\ q_2(t) \\ \hat{q}_2(t) \end{bmatrix} + \begin{bmatrix} \frac{\sum_k r_{1k} a_{1k} - c_{11}}{2\sum_k r_{1k} b_{1k}} - \frac{(1-w)l}{2} q_2^* \\ 0 \\ \frac{a_2 - c_{21}}{2b_2} \\ (1-w)l q_2^* \end{bmatrix}$$

$$p(t) = \begin{bmatrix} -b & 0 & -b & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ \hat{q}_1(t) \\ q_2(t) \\ \hat{q}_2(t) \end{bmatrix} + a$$

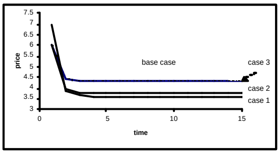


Uncertain Response To Demand-met-error Feedback

- Market model

$$\begin{bmatrix} q_1(t+1) \\ q_2(t+1) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} -\frac{\sum P_{2k} a_{2k}}{2} \\ -\frac{\sum P_{1k} a_{1k}}{2} \end{bmatrix} u(t) + \begin{bmatrix} \frac{a_1 - c_1}{2b_1} \\ \frac{a_2 - c_2}{2b_2} \end{bmatrix} \qquad p(t) = \begin{bmatrix} -b & -b \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + a$$

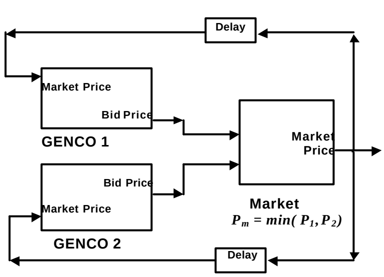
- Market price dynamics



Multi-period Profit Maximization - Quantity Competition

- GENCO1 decision with estimate of GENCO2 in next two periods
  - When output in different time periods are independent or interdependent
- GENCO1 decision with estimate of GENCO2's output strategy
  - When output in different time periods are independent or interdependent
- GENCO1 decisions with probabilistic estimate of GENCO2's output
- GENCO1 decisions with probabilistic estimate Of GENCO2's strategy

Under Price Competition

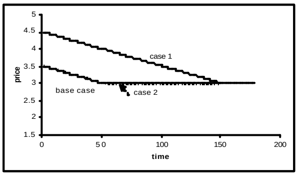


Under Price Competition II

- Market model

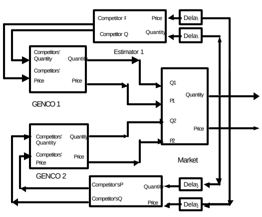
$$p_1(t) = f(VC_1, p_1(t-1), p_m(t-1))$$
$$p_2(t) = f(VC_2, p_2(t-1), p_m(t-1))$$
$$[q_1(t), q_2(t), p_m(t)] = auction(p_1(t), p_2(t), q_d(t))$$

- Assume both GENCOs can meet demand
- Market dynamics--testing other's production cost (marginal const)

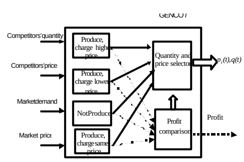


Under Price And Quantity Competition

- Market organization

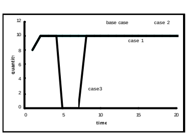


- GENCO decision-making

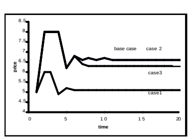


Under Price And Quantity Competition II

- Market dynamics



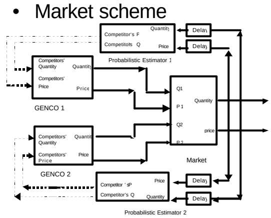
Quantity



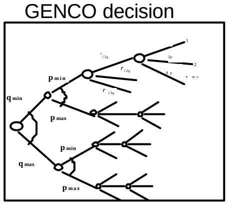
Price

Uncertain Price And Quantity Competition III

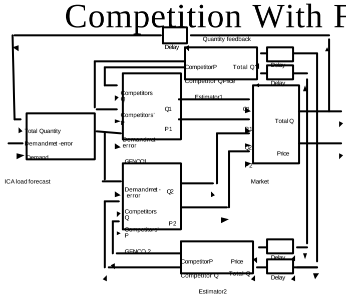
- Market scheme

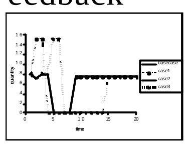


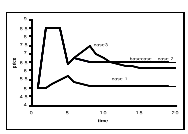
GENCO decision



Under Price And Quantity Competition With Feedback

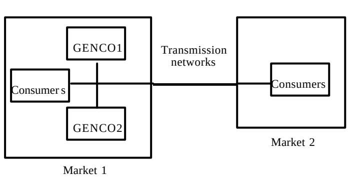






Profit Maximizing - Two Markets

- Two electric markets connected by transmission networks



Profit Maximizing - Two Markets II

- Market model w/o transmission limitation

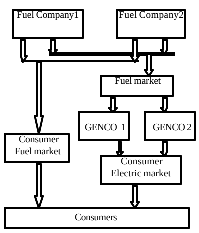
$$\begin{bmatrix} q_{11}(t+1) \\ q_{12}(t+1) \\ \hat{q}_{21}(t+1) \\ \hat{q}_{22}(t+1) \\ q_{21}(t+1) \\ q_{22}(t+1) \\ \hat{q}_{11}(t+1) \\ \hat{q}_{12}(t+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & * & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & * & 0 \\ 0 & * & 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} q_{11}(t) \\ q_{12}(t) \\ \hat{q}_{21}(t) \\ \hat{q}_{22}(t) \\ q_{21}(t) \\ q_{22}(t) \\ \hat{q}_{11}(t) \\ \hat{q}_{12}(t) \end{bmatrix} + \begin{bmatrix} * \\ * \\ 0 \\ 0 \\ * \\ * \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} -b_1 & 0 & 0 & 0 & -b_1 & 0 & 0 & 0 \\ 0 & -b_2 & 0 & 0 & 0 & -b_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{11}(t) \\ q_{12}(t) \\ \hat{q}_{21}(t) \\ \hat{q}_{22}(t) \\ q_{21}(t) \\ q_{22}(t) \\ \hat{q}_{11}(t) \\ \hat{q}_{12}(t) \end{bmatrix} + \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

- Market model with transmission limitation

Energy Market Dynamics Model

- Problem for consumer
  - Heating problem
- Fuel company decision
  - Direct to consumer?
  - Indirect through power plant?
- GENCOs decision
  - Value added



Energy Market Dynamics Model II

- Fuel inventory
- Market output

$$\begin{bmatrix} p_e(t) \\ p_f(t) \\ p_{fc}(t) \\ q_{fc}(t) \\ q_{ec}(t) \\ q_{f\,\mathrm{int}}(t) \end{bmatrix} = \begin{bmatrix} -b_e & 0 & -b_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -b_e & 0 & 0 & 0 & -b_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -b_e & 0 & 0 & 0 & -b_e & 0 & 0 \\ * & 0 & * & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\ * & 0 & * & 0 & 0 & * & 0 & 0 & 0 & * & 0 & 0 \\ -h_1 & 0 & -h_2 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{e1}(t) \\ \hat{q}_{e2}(t) \\ q_{e2}(t) \\ \hat{q}_{e1}(t) \\ q_{f1}(t) \\ q_{fc1}(t) \\ \hat{q}_{f2}(t) \\ \hat{q}_{fc2}(t) \\ q_{f2}(t) \\ q_{fc2}(t) \\ \hat{q}_{f1}(t) \\ \hat{q}_{fc1}(t) \end{bmatrix} + \begin{bmatrix} * \\ * \\ * \\ * \\ * \\ * \\ 0 \\ 0 \\ * \\ * \\ * \\ 0 \\ 0 \end{bmatrix}$$

Dynamic Simulation Summary

- Different expectation models
- Different market situations
- Conclusion from simulation
- Application of models and method developed

Preliminary Analysis

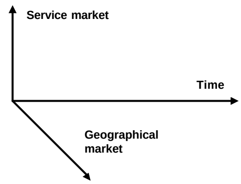
- GENCO decision making in one market in one period
- GENCO decision making in one market in two successive periods
  - Effects of ramp rate
  - Effects of start-up/shut-down cost

Preliminary Analysis II

- GENCO decision-making in two markets in one period
  - Different price strategies
  - Different market conditions
- GENCO decision-making in two markets in two successive periods
  - A nonlinear optimization

Market-based Scheduling

- A three–dimension problem
  - generation scheduling along time line
  - in multiple geographical markets
  - in multiple service markets



Market-based Scheduling II

- Objective : profit maximization

$$p = \sum_i p_i = \sum [p_i q_i - c_i(q_i) - TC(q_{i-1}, q_i)]$$

- Constraints
  - capacity constraints for generators
  - generator ramp rate constraints
  - minimum up time and down time
  - some conventional constraints for UC are not valid

Market-based Scheduling III

- Solving market-based generation scheduling using Dynamic Programming
- Definition of state and stage
  - Stage: hour for day ahead market
  - State: the combinations of maximum generation provided by all units in different working modes during and after startup processes.
- Formulation $f_M = \max(p_i - TC(i-1, i) + f_{M-1})$

Market-based Scheduling IV

- A numerical example
  - two units with quadratic cost functions
  - 12 hours
- Running time comparison
  - Dynamic programming found the optimal solution
  - Dynamic programming is efficient

Market Based Scheduling - Uncertainty

- Assumptions for Demand and competitor
- Uncertainty between different time are independent

$$f_M = \max E[(p_M - TC(State(M-1), state(M)) + f_{M-1})]$$

- Expected profit maximizing problem in auction at each state
  - Based on different pricing strategies

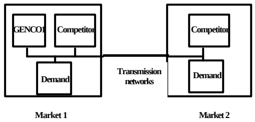
Market Based Scheduling – Uncertainty II

Step 1. Problem initialization.
Step 2. Compute expected profit from the initial state i_0 to technically feasible state i : f_1= TC (i0, i) +.
Find the optimal generation output and bidding decisions from the initial state to the first stage.
Store the best expected profit and the best state transition path (generation output for each unit and pricing decisions).
Step 3. Find the optimal generation output and pricing decisionsfor each technically feasible state of the current stage j using optimal results from the previous stage j -1.
Step 4. If $j = M$, go to step 6.
Step 5. $j = j + 1$, go to step 3.
Step 6. Trace the optimal state transition path. Output generation amount for each unit and price in all stages.

- A numerical example
  - Two units, probabilistic estimate of competitor
  - 6 hours

Market-based Scheduling – Multiple Geographical Markets

- Market scheme



- Objective

$$\max p = \sum_i p(i) = \sum_i [p_1(i)q_1(i) + p_2(i)q_2(i) - c_i(q_1(i) + q_2(i))$$
$$- TC(q_1(i-1) + q_2(i-1), q_1(i) + q_2(i)) - TFC(q_2(i))]$$

- Constraints
  - capacity constraints , ramp rate constraints,
  - minimum up time, minimum down time,
  - transmission constraints

Market-based Scheduling – Multiple Geographical Markets II

- Problem formulation

$$f_M = \max(p_i - TC(i-1, i) + f_{M-1})$$

St.

- Profit from the market

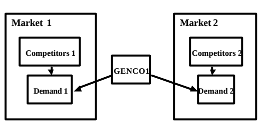
$$p_i = \max(p_{i1}, p_{i2}, p_{i3}, p_{i4})$$

- An example of possible profit

$$p_1(i) < p_{c1}(i), q_1(i) \leq q_{d1}(i), p_2(i) < p_{c2}(i), q_2(i) \leq q_{d2}(i), q_2(i) \leq ATC(i)$$
$$p_{i1} = \max(p_1(i)q_1(i) + p_2(i)q_2(i) - c(q_1(i) + q_2(i)) - TFC(q_2(i)))$$
$$q_{min} \leq q_1(i) + q_2(i) \leq q_{max}$$

Market-based Scheduling - Multiple Service Markets

- Market scheme



- Objective

$$p = \sum_i p(i) = \sum_i [p_1(i)q_1(i) + p_2(i)q_2(i) - c_i(q_1(i), q_2(i))$$
$$- TC(q_1(i-1), q_2(i-1), q_1(i), q_2(i))]$$

Market-based Scheduling - Multiple Service Markets II

- Problem formulation

$$f_M = \max(p_i - TC(i-1,i) + f_{M-1})$$

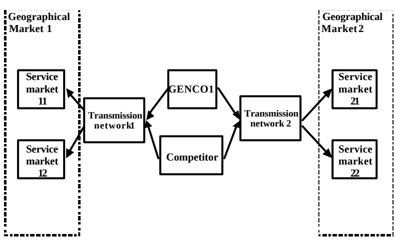
- Profit from the market

$$p_i = \max(p_{i1}, p_{i2}, p_{i3}, p_{i4})$$

- An example of possible profit

$$p_{i1} = \max(p_1(i)q_1(i) + p_2(i)q_2(i) - c(q_1(i), q_2(i)))$$
$$p_1(i) < p_{c1}(i), q_1(i) \le q_{d1}(i), p_2(i) < p_{c2}(i), q_2(i) \le q_{d2}(i)$$
$$q_{\min} \le q_1(i) + q_2(i) \le q_{\max}$$

Market-based Scheduling – Multiple Service Markets III



Market-based Scheduling - Multiple Geographical And Service Markets

- Objective

$$p = \sum_i p(i) = \sum_i [p_{11}(i)q_{11}(i) + p_{12}(i)q_{12}(i) + p_{21}(i)q_{21}(i) + p_{22}(i)q_{22}(i)$$
$$- c(q_{11}(i), q_{12}(i), q_{21}(i), q_{22}(i))$$
$$- TC(q_{11}(i-1), q_{12}(i-1), q_{21}(i-1), q_{22}(i-1), q_{11}(i), q_{12}(i), q_{21}(i), q_{22}(i))$$
$$- TFC(q_{11}(i), q_{12}(i), q_{21}(i), q_{22}(i))]$$

- Formulation for dynamic programming

$$f_M = \max(p_i - TC(i-1,i) + f_{M-1})$$

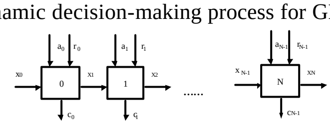
Market-based Scheduling - Multiple Geographical & Service Markets II

- Optimal procedure for GENCO

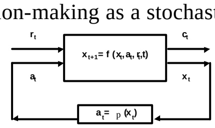
Step 1. Problem initialization.
Step 2. Compute expected profit from the initial state i_0 to technically feasible state i :
$$f_1 = TC (i0, i) +.$$
Find the optimal generation output and bidding decisions from the initial state to the first stage for each geographical market and each service market. Store the best expected profit and the best state transition path (generation output for each unit and pricing decisions).
Step 3. Find the optimal generation output and pricing decisionsfor each technically feasible state of the current stage j using optimal results from the previous stage j -1.
Step 4. If $j = M$, go to step 6.
Step 5. $j = j + 1$, go to step 3.
Step 6. Trace the optimal state transition path. Output generation amount for each unit and price for each geographical market and service market in all stages.

Market-based Scheduling - Optimizing Control Form

- Dynamic decision-making process for GENCOs



- GENCO Decision-making as a stochastic control problem



Overview GENCO Decisions

- Similarity between short-term and long-term
  - to maximize gains in the market.
  - limited by economic and technical constraints
  - influenced by demand and competitors' action.
  - influenced by fuel and other markets.
  - sequential decision making and dynamic
- Different constraints for short-term and long-term
  - Economic , technical, demand properties

Overview: Long-term Decisions

- Evolving of Generation long-term decision
  - To minimize cost to reliably meet demand …
  - To maximize net worth of the company …
  - To maximize the expected utility to manage market risk
- Models for short -term market dynamics simulation are applicable to long-term study with modifications
- Real options: a good method to manage risk efficiently for decision makers

Summary

- Electric market dynamic problems do exist, as seen in practical markets and simulation results. The market may experience different transition processes, even if the final steady state is the same.
- The electric market can be modeled as a control system. GENCOs' decision-makings can be seen as control processes. Modeling market and decision-making of GENCOs using control theory provide lots of unique information in market.

Summary II

- Interactions between GENCOs are important to decisions and market performance.
- Different expectations of GENCOs lead to different decisions and market properties.
- Market administration should factor into rules interactions between market participants to avoid dynamic problems.
- Decision analysis/decision theory should be used to systematically solve decision problems in deregulated markets with constraints under certainty and uncertainty.

Summary III

- Optimal decision problem in short-term market is a three-dimension problem: to develop market-based probabilistic generation schedule and make bidding decisions for each service market in each geographical market.
- Optimal decision-making procedures have been established with consideration of market rules, technical constraints, market conditions (competitor actions and demand properties), and uncertainty.

Summary IV

- Dynamic programming is one way to solve market-based generation scheduling problems.
- Stochastic Dynamic Programming should be used when there is uncertainty in market
- Long-term decision-making (market-based generation expansion) is different from short-term decisions but same method has be used to study long-term dynamics.
- Interactions between market participants must be included in long-term decision models

Future Examples

- Develop schemes for market administrators to control market properties when necessary
- Design the decision-making model to find the best overall return for GENCOs
  - Decisions in both physical and financial markets
  - technical constraints, market conditions, and financial constraints
  - Value at risk
    - Market-based probabilistic generation scheduling model

Summary

- Decision Analysis provides strategies
- Subjective Probability
- Estimated Benefits (profit)
- Estimated Impacts (costs)
- Logical, consistent, defendable bids
- Value of Information
- Value of Research and Development

Recent Publications

- Yang, W.; Sheblé, G.B. Modeling generation company decisions and electric market dynamics using control theory, *Proceedings of IEEE Power Engineering Society Summer Meeting 2002* p1385-1391
- Yang, W.; Sheblé, G.B. Discrete generation decisions simulation including market dynamic interactions, *Proceedings of the 34th North American Power Symposium*, p387-392
- Yang, W.; Sheblé, G.B. Market based probabilistic generation scheduling for GENCOs, presented in *Probabilistic Methods Application in Power Systems 2002*
- Yang, W.; Sheblé, G.B. Discrete generation decisions simulation with market dynamic interactions, *Proceeding of the 15th Conference on Systems Engineering*, p470-476
- Yang, W.; Sheblé, G.B. Power market stability under discrete price expectation models, *Proceedings of the 32nd North American Power Symposium 2000, P9-22~27*

Recent Publications II

- Yang, W.; Sheblé, G.B. Modeling generation company decisions-making and electric market dynamics (To be published in *System Science*)
- Yang, W.; Sheblé, G.B. An integrated generation scheduling and bidding scheme to maximize profit for generation companies in competitive market (submitted to *Electric Power System Research*)
- Yang, W.; Sheblé, G.B. Cournot-like models for generation dynamics in electric market (Submitted to *Energy Journal*)
- Yang, W.; Sheblé, G.B. Modeling Electric market and GENCO decisions in new environment using control theory (to be submitted to *Electric Power System Research*)
- Yang, W.; Sheblé, G.B. Market based generation scheduling problems for GENCOs in new deregulated electric market (to be submitted to *Electric Power System Research*)
- Yang, W.; Sheblé, G.B. Short-term GENCOs decision model in the new deregulated electric market (to be submitted to *Electric Power System Research*)

Questions?

- More interesting long term includes financial instruments