ON-OFF CONTROL FOR FULL-GAP POSITIONING OF PARALLEL-PLATE ELECTROSTATIC MEMS

Lukas Mol  
Delft University of Technology  
Dept. Microelectronics, Fac. EEMCS  
Mekelweg 4, 2628 CD Delft  
The Netherlands  
+31152785747, L.Mol@tudelft.nl

Luis A. Rocha  
University of Minho  
Department of Industrial Electronics  
Campus de Azurém, 4800-058 Guimarães  
Portugal  
lrocha@dei.uminho.pt

Edmond Cretu  
University of British Columbia  
Dept. Electrical and Computer Eng.  
2332 Main Mall, Vancouver, British Columbia, V6T 1Z4  
Canada  
edmondc@ece.ubc.ca

Reinoud F. Wolffenbuttel  
Delft University of Technology  
Dept. Microelectronics, Fac. EEMCS  
Mekelweg 4, 2628 CD Delft  
The Netherlands  
R.F.Wolffenbuttel@tudelft.nl

ABSTRACT

Electrostatic parallel-plate actuators are classically limited to displacements up to \(1/3\) of the gap due to the pull-in effect [1],[2]. A closed-loop feedback based method presented in [3] is recently introduced to overcome this limitation. Optimized structures are designed to minimize residual position ripple while maintaining bandwidth, effectively reducing the required device size by a factor three.

THE PULL-IN LIMITATION

The \(\mu\)-domain features a strong electromechanical interaction. Electrostatic actuation is commonly used in MEMS (e.g. micromirrors [4], RF switches and tunable capacitors [5], opto-mechanical switches [6]). However, since the electrostatic force from an electric field is inversely proportional to the square of the deflection and the restoring force of an electrostatically actuated beam is, in a first approximation, linear with deflection, an unstable system results and the suspended beam (or rotor) crashes on the stator in case of a deflection, \(\nu\), beyond a critical value, \(\nu_{\text{crit}}\). The pull-in voltage, \(V_{\text{pi}}\), is defined as the voltage that is required to obtain this critical deflection and is determined by the beam material, beam dimensions, residual stress and electrode dimensions [1].

In order to overcome the pull-in limit [1],[1], several techniques have been proposed: geometry leverage [7], series feedback capacitor [8], current drive methods [9],[10] and closed-loop voltage control [11]. Stable displacement over the full available range has not been achieved with these approaches, except for the geometry leverage technique, which is limited by the higher voltage levels and the larger dimensions required. The method recently presented in [3] avoids these difficulties.

STATIC VERSUS DYNAMIC PULL-IN ANALYSIS

A simplified pull-in analysis assumes a quasi-static regime, which reduces the problem to finding the equilibrium between mechanical and electrostatic forces (i.e. the damping is neglected) and results in a sudden pull-in at a well-defined pull-in voltage at a displacement of 1/3 of gap for 1 degree-of-freedom (1-DOF) structures [1]. However, when the changes in the applied voltage are sufficiently fast, the quasi-static regime does not apply and the static pull-in analysis becomes invalid.
The damping forces and mass inertia need to be included in the model for a meaningful study of the dynamic pull-in behavior of the structure [12].

NON-LINEAR DYNAMIC MEMS MODEL

Assuming no external mechanical force applied (no acceleration), the movement of a parallel-plate electrostatic actuator is described by

\[ F_m + F_b + F_k = F_{elec}, \]

where \( F_m \) is the mass contribution (\( m \ddot{x} \)), \( F_b \) is the force caused by the damping, \( F_k \) is the spring force (\( kx \)) and \( F_{elec} \) is the electrostatic force (\( \frac{1}{2} \cdot \pi^2 d^2(x)^2 \)), where \( C'(x) \) is the partial derivative of the capacitance with respect to the displacement \( x \).

THE DAMPING FORCE

For structures in which only the size of the small gap between two plates changes in time, the pressure changes relative to the wall velocity are described by the Reynolds equation [13]:

\[ \frac{d^3 Q_{pr}}{12\eta} \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) = \frac{d}{p_a} \frac{\partial p}{\partial t} + \frac{\partial d}{\partial t}, \]

where \( p_a \) is the ambient pressure, \( \eta \) the gas viscosity, \( Q_{pr} \) describes the relative flow rate coefficient and \( d \) the gap between the surfaces.

An analytical solution for the forces acting on the surfaces can be found if some conditions are assumed [14]: the moving plate is rigid and moves in the direction normal to the surface plane with a harmonic excitation. The solution is frequency dependent and is not suitable for transient analysis.

A more adequate approach is presented in [13] where the damping force can be represented by a network of frequency independent spring-damper elements, which have the same transfer function of the initial solution. Replacing each of the spring-damper elements by its electrical equivalent, allows the modeling of the damping force for large displacements as a series of nonlinear inductances and resistances controlled by the displacement \( x \). The values for the resistors and inductors [15] are:

\[ R_{m,n} = (mn)^2 \left( \frac{m^2}{w^2} + \frac{n^2}{l^2} \right) \frac{\pi^2 d^2(x)^2}{64lwp_a} \]

where \( m \) and \( n \) are odd integers and \( w \) and \( l \) are the width and length of the surfaces (Fig. 1), respectively and \( d(x) = d_0 - x \), with \( d_0 \) being the initial gap). The Knudsen number (that models the rarefaction effects) is also included as displacement dependent parameter \( K_n = \lambda / d(x) \), where \( \lambda \) is the gas mean free path) and accounted for in the model through the relative flow rate coefficient:

\[ Q_{pr} = 1 + 9.638(K_n)^{1.159} \]

In surface-micromachined structures the flow passing the damper circumference has a significant effect on the damping coefficient due to the small thickness dimension, i.e., the length and width of the damper are comparable with the film thickness (gap size) [16]. This is often referred to as the border effect and significantly changes the damping coefficient. For a surface width-to-gap size ratio as high as 20, the damping force is still 35% higher than predicted by Equation (3) [16]. The border effects can be included in the analytically derived squeeze-film model using a modified surface length, \( l_0 = l + \Delta l \), and surface width, \( w_0 = w + \Delta w \). From [17], the effective elongation for a parallel-plate configuration with linear movement is given by:

\[ l_0 = l + 1.3d \]
\[ w_0 = w + 1.3d \]

resulting in a modified length \( l \) and width \( w \) that must be used in (3) to include the border effects in the damping model.

LARGE SIGNAL MODEL

A practical model should include system properties such as hysteresis of the pull-in [2] and the effect of stoppers. Therefore, the various physical parts of the system should be separately specified. As each of the RL sections presented before behaves like a first-order system with variable gain and time constant, the total damping force can be modeled as the sum of several damping forces of the type:

\[ F_{m,n} = x - \frac{\dot{x} - F_{m,n}(x)}{R_{m,n}(x)} \]
Equation (1) can now be rewritten as:

\[ F_m + F_{1,1} + F_{1,3} + F_{3,1} + \ldots + F_{m,n} + F_k = F_{\text{elec}} \]  

(8)

Adopting the notation \( \dot{X} = f(X,V) \) yields for the overall nonlinear system the following equation of motion:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -\frac{1}{m}[(F_{1,1} + F_{3,1} + \ldots + F_{m,n}) + kx - \frac{1}{2} C(x)V^2] \\
\dot{F}_{1,1} &= \frac{y - F_{1,1}R_{1,1}(x)}{L_{1,1}(x)} \\
\dot{F}_{3,1} &= \frac{y - F_{3,1}R_{3,1}(x)}{L_{3,1}(x)} \\
\dot{F}_{m,n} &= \frac{y - F_{m,n}R_{m,n}(x)}{L_{m,n}(x)}
\end{align*}
\]  

(9)

where \( x \) denotes the displacement, \( y \) the velocity and \( \dot{F}_{m,n} \) the time-derivative of the associated \( F_{m,n} \) damping force. For frequencies well below the cut-off frequency the spring force component can be neglected and the equation of motion reduces to:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -\frac{1}{m}\left( \sum_{m,n=\text{odd}} \frac{1}{R_{mn}}y + kx - \frac{1}{2} C(x)V^2 \right)
\end{align*}
\]  

(10)

PHASE PORTRAITS

The equation of motion can be used to construct phase portraits: It is a plot of multiple trajectories in terms of state variables corresponding to different initial conditions in the same phase space [18]. It gives a qualitative view on the behavior of the system. For an electrostatic actuator the state variables are the displacement, \( x \), and the velocity, \( y \). The phase space is two-dimensional (2D) with the displacement in the horizontal axis and the velocity in the vertical axis. Figure 1 graphically shows the dynamics of a structure actuated with 4 volt in such a phase portrait. By changing the applied voltage the phase portrait will change and a shift from a stable an unstable trajectory can be achieved. A state previously outside of the basin of attraction can be inside it after a proper voltage change.

FIGURE 1: PHASE PORTRAIT AT AN ACTUATION VOLTAGE OF 4V.

A timely change of actuation voltage can therefore prevent the pull-in from occurring. This implies that the device must be over damped or critically damped in order not to be in the oscillatory operating regime. As the damping force is non-linear and highly dependent on electrode position, at very small gaps (i.e. large displacements), the damping forces are huge due to the rarefaction effects. These damping forces slow even further the structure displacements, improving the dynamic device response close to the counter electrode when operated with the proposed on-off method.

ON-OFF FEEDBACK CONTROL

It is based on the comparison of the measured momentary actuator displacement with the desired displacement. The voltage applied to the actuator is changed between two values (unlike traditional feedback): between a high level, if the measured displacement is lower than the reference, and a low level, if the actuator displacement is higher than the reference value. Figure 2 shows a block diagram of the used feedback topology. The topology has been realized with a parallel-plate electrostatic actuator featuring a 2.25 \( \mu \)m initial gap. The displacement measurement is implemented with an off-chip capacitive read-out circuit. Figure 3 shows the measured comparator output and the electrode displacement around the set reference level.
REDUCING THE RIPPLE

Due to the feedback topology, there is a finite time delay that results in a small ripple of the electrode displacement. Two important parameters are available for reducing the remaining ripple.

ADJUST THE ACTUATION VOLTAGE LEVEL

Although the actuation voltage levels are not critical for proper operation, adjusting the high and low level will affect the device response and can be used to improve the performance (Fig. 4). Once the structure is oscillating in a stable manner around the set point, the high level can be lowered close to the voltage at which an (unstable) equilibrium exists between the electrostatic and spring forces. And the low voltage level can be raised close to that same voltage.

REDUCE FEEDBACK DELAY

Shortening the feedback loop delay reduces the ripple as well. Each time the comparator switches the movable electrode will have a certain velocity and therefore inertia. This results in a positional overshoot. By reducing the delay time, the effect of inertia and hence positional ripple can be reduced. Figure 5 illustrates the simulated response for two different feedback delays.

FIGURE 2: BLOCK DIAGRAM OF THE FEEDBACK TOPOLOGY USED.

FIGURE 3: MEASURED OPERATIONAL DETAILS OF THE PROPOSED TECHNIQUE.

FIGURE 4: MEASURED RIPPLE REDUCTION DUE TO DYNAMIC VOLTAGE LEVEL ADJUSTMENT.

FIGURE 5: SIMULATED RESIDUAL POSITION RIPPLE FOR TWO DIFFERENT FEEDBACK LOOP DELAYS.
CONCLUSIONS AND FUTURE WORK

A redesign of the actuator is made based on enhanced models for the actuator dynamic behavior. Taking into account the non-linear damping forces, gas type and pressure, and actuation voltage levels. The new devices are expected to feature full-gap dynamic positioning with greatly reduced residual displacement ripple, while maintaining bandwidth and quick response time. Furthermore, a custom IC read-out circuit is designed in CMOS to be combined with the MEMS device to reduce the feedback delay and the effect of parasitics.

The proposed system topology could effectively reduce an existing device size with a factor three for the same operating displacement range.

REFERENCES


