# HYSTERESIS IN THE PULL-IN OF MICROSTRUCTURES

**L.A. Rocha, E. Cretu* and R.F. Wolffenbuttel**
*Delft University of Technology, ITS/Et, DIMES,*
*Mekelweg 4, 2628 CD Delft, The Netherlands*
**E-mail: L.Rocha@its.tudelft.nl**
**Also at: Melexis, Transportstr. 1, Tessenderloo, Belgium*

### *Abstract*

*The hysteresis in the pull-in voltage of a single side clamped free-standing beam, under lateral deflection, has been investigated for application as a DC voltage reference. Proper operation depends on a mechanical stopper located between the deflection at pull-in and the full width of the gap between beam and counter electrode. This stopper introduces hysteresis, which depends on the position of the stopper and the mode of operation. The effect is analysed by simulation and verified by experimental results. A beam dimensioned for a 9 V pull-in typically shows a 1 V hysteresis.*

## I. INTRODUTION

Recent research has demonstrated that the pull-in voltage of a micromechanical structure can be used as an on-chip DC voltage reference [1,2]. The basic phenomenon is the loss of stability at the equilibrium position, where the elastic forces equilibrate the electrostatic ones. The basic pull-in device is a simple mechanical spring, in the form of a single-side clamped beam, with an electrostatic actuation voltage applied. The characteristic displacement vs. voltage curve of the microbeam shows a square root relation of displacement with voltage up to the pull-in voltage at d/3. As the electrostatic force in a vertical field is inversely proportional to the square of the deflection and the restoring force of the beam is, in a first approximation, linear with deflection, an unstable system results in case of a deflection, v, beyond a critical value, $v_{crit}$. The pull-in voltage, $V_{pi}$, is defined as the voltage that is required to obtain this critical deflection [3]. For a stable equilibrium deflection the second derivative of the potential energy of the system to deflection should be positive: $\partial^2 U_p/\partial v^2 > 0$, thus $V_{pi}$ results from $\partial^2 U_p/\partial v^2 = 0$ and is uniquely determined by the beam material, the beam dimensions and the residual stress.

The key performance specification in the intended application is the reproducibility in terms of both a uniquely determined static characteristic and long-term stability. This property is affected by the hysteresis in a MEMS structure with a stopper. In this paper we will indicate the necessity of such a stopper and the effect in hysteresis.

## II. HYSTERESIS IN THE PULL-IN STRUCTURE

An important aspect of a movable micro-electromechanical system like the one presented here is that it requires a stop position, which in turn causes mechanical hysteresis [4]. In our case, this phenomenon is important, as it seriously complicates the design of a feedback system required to control the structure.

It should be emphasized that the hysteresis in such a MEMS device is not due to a parasitic or practical device limitation, such as sticking. Rather it is fundamental to the basic device operation. The stopper should be positioned somewhere between the deflection at pull-in, $x_n=1/3$ and $x_n=1$ to prevent the beam from hitting the counter electrode and thus compromising reliability and short-circuiting the capacitor.

The hysteresis phenomenon can be demonstrated using the one degree of freedom case of a parallel plate structure. Computing both the electrostatic and mechanical force for such a case we can see the evolution of the equilibrium position with increasing voltage (Fig. 1). Pull-in occurs when the mechanical force no long

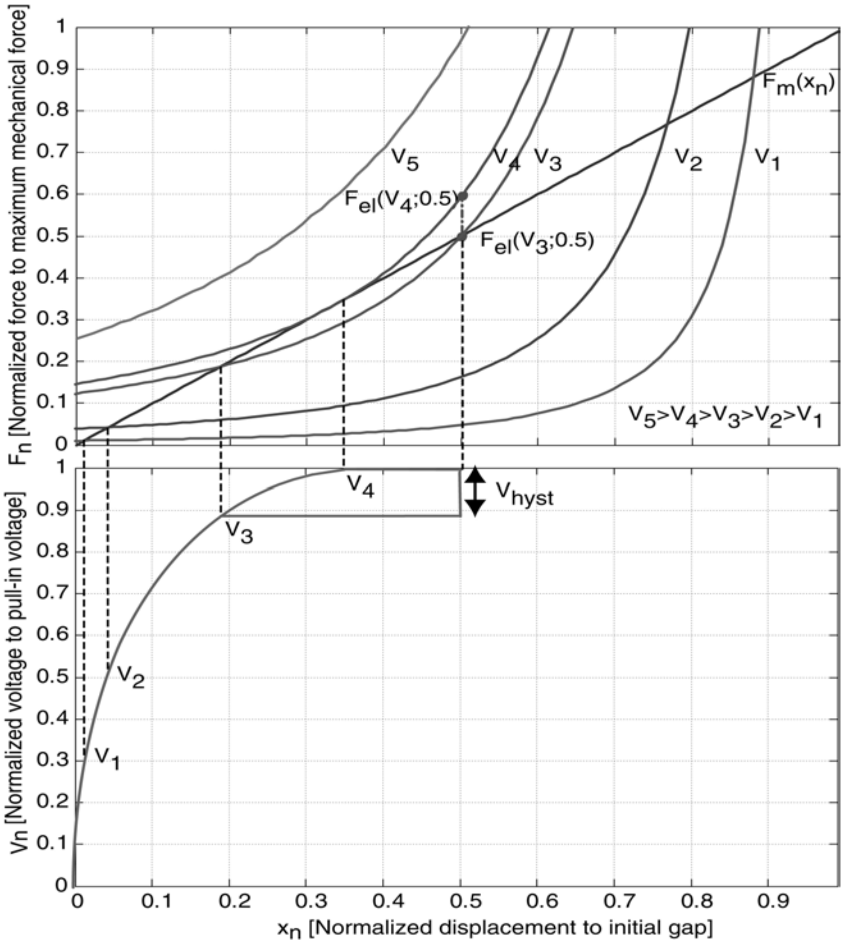

*Figure 1. Explaining hysteresis*

equilibrates the electrostatic one ($V=V_4$). The last equilibrium position occurs at $x_n=1/3$. After pull-in the structure will stop at the designed stopper position (in this example in the middle of the initial gap), where the electrostatic force equals the sum of the mechanical force with the reaction force of the stopper.

When reducing the voltage applied, we would like the system to return to the same stable position taken at the threshold of pull-in, but that is not the case. Rather, after pull-in the electrostatic force increases as the gap decreases ($F_{el}(V_4;0.5)$ in Fig. 1). Because the electrostatic force is now larger than it was when it collapsed, a lower voltage is required to reach balance between the electrostatic force and the mechanical force ($F_{el}(V_3;0.5)$ in Fig. 1). There are two distinct solutions for the position where the mechanical force equilibrates the electrostatic force (the reaction of the stopper becomes zero). As the higher displacement solution gives an unstable position, the structure returns to the stable zero position. Consequently, the hysteresis depends on the stop position. Designing a structure with a stop position closer to the deflection at pull-in results in a smaller value of the hysteresis. Therefore if we can calculate the hysteresis and the pull-in value from device dimensions, we can fully analytically describe the static behavior of the structure.

## III. MODELLING THE HYSTERESIS

Modelling is based on the variational approach, in which the equilibrium points and their stability are determined by studying the variations of the total energy:

$$U(w_1,\varphi_1,V)=U_{elastic}(w_1,\varphi_1)+U_{electric}(w_1,\varphi_1,V) \quad (1)$$

The pull-in voltage can be found analytical by solving the determinant equation:

$$\left| \frac{\partial^2 U(w_1, \varphi_1; V)}{\partial \vec{x}^2} \right| = 0 \,, \tag{2}$$

in variable V, and the state variables ($w_1$, $\varphi_1$) correspond to the equilibrium position determined from :

$$\left[ \frac{\partial U}{\partial \vec{x}} \right] (\vec{x}_{eq}; V) = \begin{bmatrix} \dfrac{\partial U}{\partial w_1} \\ \dfrac{\partial U}{\partial \varphi_1} \end{bmatrix} (w_{1eq}, \varphi_{1eq}; V) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3}$$

The energy balance is evaluated using two parameters ($w_1$, $\varphi_1$) to fully determine the configuration (Fig. 2).

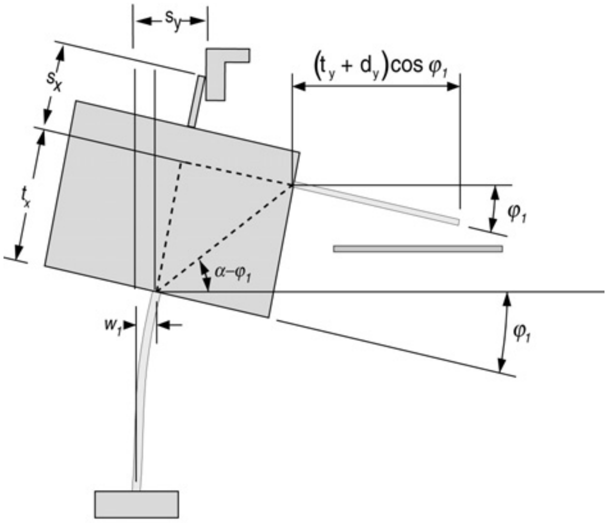

*Figure 2. Identification of the state variables used in the model.*

The goal of the hysteresis model is to find the voltage $V_{hyst}$, for which the mechanical force equals the electrostatic one at the stop position. For our 2D model this can be done by solving (3) in variable V, while the state variables ($w_1$,$\varphi_1$) correspond to the equilibrium position given by:

$$s_y = w_1 + (t_x + s_x)\sin(\varphi_1) \tag{4}$$

The system was solved numerically ($s_y$=2 µm, $t_x$=100 µm and $s_x$=120 µm), and the predicted behaviour is presented in Fig. 3. The hysteresis results were then verified in a finite element model. By using contact elements to simulate the stoppers, the full behaviour of the structure can be analysed using 2D FEM and the simulation results shown in Fig. 4 are in agreement with those in Fig. 3.

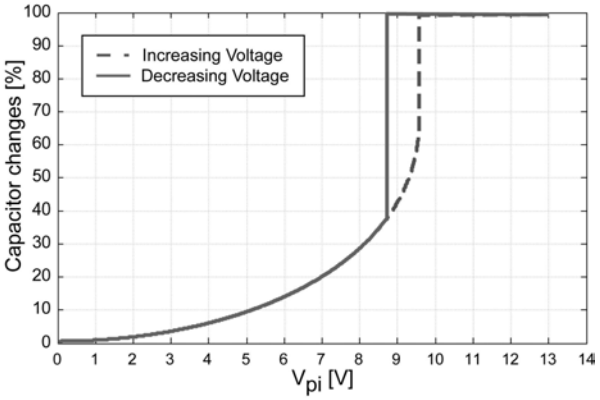

*Figure 3. Predicted capacitance changes.*

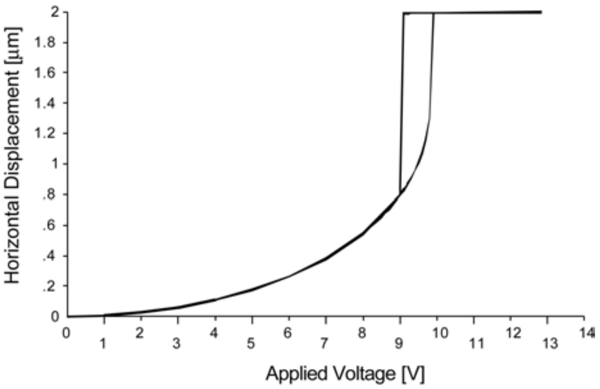

*Figure 4. FEM of the hysteresis.*

## IV. MICROSTRUCTURE FABRICATION

An epi-poly process was used for the fabrication of 11 µm thick, single-side clamped 100 µm long free-standing structures with electrode structures at the tip [5,6]. A fabricated pull-in device is shown in figure 5.

The device is basically a free-standing lateral beam anchored at one end (the base) only. The beam can be deflected by electrostatic actuation in the plane of the wafer using a voltage applied across parallel plate capacitors composed of two sets of electrodes located alongside the free-standing tip, with counter electrodes anchored to the substrate. The deflection can be measured using the differential sense capacitor located directly on top of the substrate and aligned with the square-shape electrode at the tip of the beam. These buried polysilicon electrodes are electrically isolated from the substrate and placed symmetrically on either side of a guard electrode directly underneath the axial direction of the beam. Finally, there are electrically isolated stoppers to limit the lateral motion. The electrodes beneath the movable structure are used for capacitive detection of the pull-in voltage. The readout circuit is based on a capacitor bridge with two active arms.

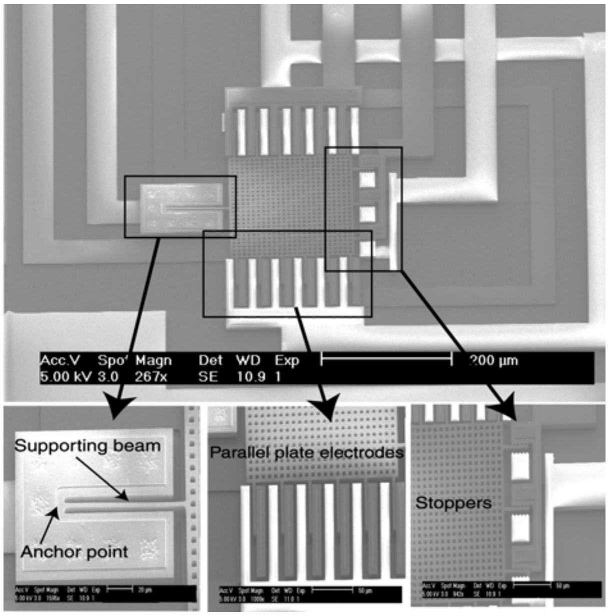

*Figure 5. Fabricated microstructure.*

## V. EXPERIMENTAL RESULTS

Figure 6 presents the behaviour of a sample over a complete cycle (with increasing and decreasing voltage) When comparing the results with those obtained from the modeling (Fig. 3), a reasonable agreement is observed. The measured hysteresis gap is about 1V. The small shift in position is mainly due to the simplifications in the model and non-idealities, such as stray capacitances and fringe fields.

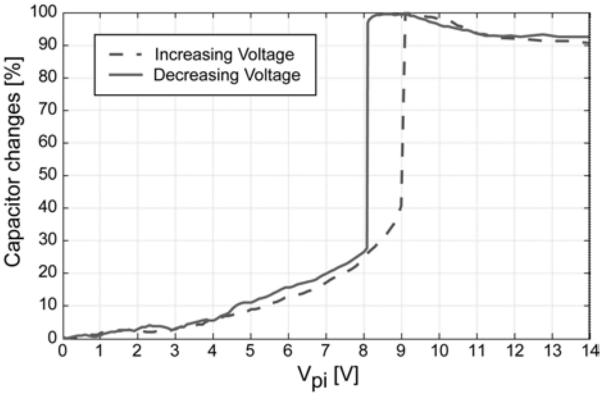

*Figure 6. Hysteresis in a pull-in microstructure.*

## VI. ACKNOWLEDGEMENTS

This work is supported by the Netherlands Technology Foundation (STW) under grant DEL55.3733.

## VII.REFERENCES

[1] J. Kyynäräinen, A.S. Oja and H. Seppä, "Stability of MEMS devices for electrical metrology, *IEEE Transactions on Instr. and Meas.*, Vol. 50, N°6, pp. 1499-1503, 2001.

[2] E. Cretu, L.A. Rocha and R.F. Wolffenbuttel, "Micromechanical voltage reference using the pull-In of a beam", *IEEE Trans. on Instr. and Meas.* Vol. 50, N°6, pp. 1504-1507, Dec. 2001.

[3] H.A.C. Tilmans, and R. Legtenberg, "Electrostatically driven vacuum-encapsulated polysilicon resonators, Part 2, Theory and performance", Sensors and Actuators A 45(1994) 67-84.

[4] J.R. Gilbert, G.K. Ananthasuresh, S.D. Senturia "3D modeling of contact problems and hysteresis in coupled electro-mechanics", *in Proceedings MEMS '96*, 1996, pp. 127 -132.

[5] http://www.vdivde-it.de/mst/imsto/ Europractice/Bosch/default.html

[6] M. Offenberg, F. Lärmer, B. Elsner, H.Münzel and W.Riethmüller, "Novel process for an integrated accelerometer", *in Proc. Transducers 95*, Stockholm, Sweden, June 25-29, Vol.1, pp. 589-593.