NUMERICAL SIMULATION OF BUBBLE PULSATION GENERATED BY DEEP WATER EXPLOSION

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ABSTRACT

Bubble pulsation generated by deep water explosion was calculated with Zamyshlyayev formula and LS-DYNA software. It is shown that the maximum radius of bubble and pulsation period obtained by one-dimensional model of LS-DYNA are close to the test data. Pressure curve of main shock wave gained with LS-DYNA matches well with that of Zamyshlyayev formula. It is shown that one-dimensional spherical symmetric multi-material Euler model in LS-DYNA can be used to predict bubble pulsation in deep water explosion correctly. The research method and conclusion has certain reference significance for the research of deep water explosion.

Keywords: LS-DYNA software, deep water explosion, bubble pulsation, numerical simulation.

INTRODUCTION

When explosive is exploded in the water, it turns into a gas bubble with high temperature and high pressure instantaneously. At the same time, an outward propagating shock front is formed. After some time, because of the overexpansion of the gas bubble, the pressure of the bubble would less than the hydrostatic pressure of the surrounding water. The surrounding water would flow back to the bubble and compress it, until it was over compressed to a high pressure determined by the inward velocity of the water at the equilibrium pressure. Then the bubble would expand again. After some such kind of repetitions, the bubble pulsation phenomenon would form.

The ships would be damaged seriously by the shock wave and the bubble pulsation in the water(Cole, 1948). The shock wave has high peak pressure and short duration, which would result in serious localized damage to hull structures; although the peak pressure of the bubble pulsation is only 10~20% of the peak value of the shock wave, bubble pulsation has long-term effects and greater impulse. Furthermore, the frequency of the bubble pulsation is very close to the first or the second natural frequency of the ship. The pulsating pressure would cause oscillation effects on the ship. The whole structure of the ship would have serious damage, and the shipboard equipment would be damaged and even lose the operational effectiveness. Besides, when the shrinking bubble is close to the ship, it may move toward it. The water jet would form, and the ship would be destroyed again.

Many researchers have studied the bubble pulsation in water explosion with numerical simulation, including boundary element method (Chahine and Duraiswami,1994; Wilkerson,
1988; Wilkerson, 1992), finite volume method (Li et al, 2011), finite difference method (Hu et al, 2009), meshless method (Liu et al, 2003), etc. The two-dimensional adaptive mesh technique of finite element software LS-DYNA can accurately estimate the maximum radius and the first pulsation period of the bubble, but it can’t simulate the collapse of the bubble and the formation of the water jet. The early version of the Euler method in LS-DYNA can’t simulate the bubble pulsation because of lower stability and higher cost (Xin, 2008). Xin Chun-liang (Xin, 2008) studied the bubble pulsation in deep water explosion using the one-dimensional simulation model of the finite difference software AUTODYN. He found that maximum radius and pulsation period of bubble were closer to the test results of Swift (Swift and Decius, 1950), but the simulation curves had obvious numerical noises and the calculation cost was too high — it need several hours to complete the calculation, although one dimensional simulation model was used. LS-DYNA is much faster than AUTODYN. The Euler method of new version LS-DYNA R8.0 has great improvements. This paper would study the Swift bubble pulsation experiment using one-dimensional multi-material Euler method, and the simulation results would be compared with the test data and Zamyslyayev formula (Zamyshlyayev, 1973).

ZAMYSLYAYEV SHOCK WAVE AND BUBBLE PULSATION CALCULATION FORMULA

Zamyshlyayev (Zamyshlyayev, 1973) improved the Cole formula and suggested a semi-empirical formula which divided underwater explosion load into five phases: exponential decay phase, reciprocals decay phase, posterior phase of reciprocals decay, the expansion and contracting phase of bubble, and the pulsation pressure phase. This formula can simulate the whole process of the pressure decay and bubble pulsation during the deep water explosion, and it can be shown as:

\[
P(t) = P_m e^{-t/\theta}, \quad t < \theta
\]

\[
R(t) = P_m 0.368 \frac{\theta}{t} \left[1 - \left(\frac{t}{t_p}\right)^{1.5}\right], \quad t_1 \geq t \geq \theta
\]

\[
R(t) = P^* \left[1 - \left(\frac{t}{t_p}\right)^{1.5}\right] - \Delta P, \quad t_p > t > t_1
\]

\[
P(t) = \frac{10^5}{\rho} \left( \frac{0.686 P_0^{0.96}}{\rho} + 5.978 P_0^{0.62} \frac{1-\xi^2}{\xi^{0.92}} - 30.1 P_0^{0.65} \xi^{0.36} \right) - \frac{1.73 \times 10^{10}}{\rho^4 P_0^{0.43}} \left(1 - \xi^2\right)^{0.1}
\]

\[
T - t_2 \geq t \geq t_p
\]

\[
P(t) = P_{m1} e^{-\left(t - T\right)^2 / \theta^2}, \quad T + t_2 \geq t > T - t_2
\]

Where
\[ P_m = \begin{cases} \left( \frac{W^3}{R} \right)^{1.5} & 6 < \frac{R}{R_0} < 12 \\ 5.24 \times 10^7 & 12 \leq \frac{R}{R_0} < 240 \end{cases} \]

\[ \theta = \begin{cases} 0.45R_0^{-0.45} & \bar{r} < 30 \\ 3.5 \frac{R_0}{C} \sqrt{g \bar{r} - 0.9} & \bar{r} \geq 30 \end{cases} \]

\[ t_d = \frac{R_0}{C} (\bar{r} - m) \]

\[ m = 11.4 - 10.6/r^{0.13} + 1.51/r^{1.26} \quad \bar{r} = \frac{R}{R_0} \quad \bar{t} = \frac{C}{R_0} t \]

\[ \Delta P = 10^5 \left( 563 t - 0.54 - 0.113 P_0^{-1.15} \right) \quad P^* = \frac{7.173 \times 10^8}{(t + 5.2 - m)^{0.87}} \quad t_\rho = \frac{850}{P_0^{0.85} - 20} + m \]

\[ t_1 = \frac{t_0}{(t_1 + 5.2 - m)^{0.87}} = 4.9 \times 10^{-10} P_m \frac{P_0}{R_0} \frac{C}{R_0} \quad P_0 = P_{atm} + \rho g H_0 \quad \bar{P}_0 = \frac{P_0}{P_{atm}} \quad \rho_m = \frac{39 \times 10^6 + 24 P_0}{R_{bc}} \]

\[ \theta_1 = 20.7 \frac{R_0}{P_0^{0.41}} \quad \theta_2 = 3290 \frac{R_0}{P_0^{0.71}} \]

\[ \bar{R}_{bc} = \frac{\bar{R}}{R_0} \]

\[ R_{bc} = \sqrt{R^2 + \Delta H^2 - 2R \Delta H \sin \phi} \quad \Delta H = 13.2 \left( \frac{W^{11/24}}{(H + 10.3)^{5/6}} \right) \]

Where: \( P_m \) is the peak pressure (unit: Pa) of the shock wave; \( \theta \) is the time constant of the shock wave (unit: s); \( W \) is the weight of the TNT (unit: kg); \( R \) is the distance from the explosion center to the gauging point (unit: m); \( R_0 \) is the initial radius of the explosive (unit: m); \( t_d \) is the arrival time of the shock wave (unit: s); \( t_\rho \) is the time of the positive pressure for shock wave (unit: s); \( P_0 \) is the hydrostatic pressure of the explosion center (unit: Pa); \( P_{atm} \) is the atmosphere (unit: Pa); \( C \) is the sound speed in the water (unit: m/s); \( H_0 \) is the initial depth of the explosion (unit: m); \( \rho_m \) is the pressure peak of the second pulsation (unit: Pa); \( \theta_1 \) is the time constant for the second pulsation (unit: s); \( \bar{R}_{bc} \) is the distance from the gauging point to the explosion (unit: m); \( \phi \) is the angle from the connecting line between the explosion center and the gauging point to the horizontal line (unit: s).
THE NUMERICAL SIMULATION OF BUBBLE PULSATION

In the underwater explosion experiment of Swift (Swift, 1950), the depth of the charge is 178.6m, and the organization of the experiment is shown in Fig. 1.

In this experiment, the maximum radius of the bubble is much less than the depth of TNT, and the hydrostatic pressure difference between the top and bottom of the bubble is little. The migration of the bubble caused by the gravity can be neglected. Assuming the pressure is equal for all directions around the explosive. Thus, it can be assumed that bubble pulsation can be simplified as spherical symmetry, and the one-dimensional spherical symmetric model of LS-DYNA can be used to simulate this problem.

Because the non-reflecting and pressure flow out boundary condition can’t be used in one-dimensional simulation model, the size of the water is set to be 100m to prevent the disturbance of the reflected wave on the flow field where we are concerned about. Multi-material Euler method was used for the explosive and the water. The artificial viscosity coefficient was set as the default value. At the initial stage of the explosion, the frequency of the detonation wave and the shock wave in the water is very high. Thus, Xin (Xin, 2008) adopted the fine meshes for the explosive and the water around it, and used coarse meshes for the far-field water. This meshing method balances both the accuracy and the calculation efficiency. In this paper, the meshing method was as follows: uniform meshes were adopted between 0-10m, the size of the beam element was 1mm, and the quantity of the beam element was 10000; between 10-100m, the size of the meshes increased gradually, and the quantity of the beam element is 1000.

The material model of the TNT explosive is *MAT_HIGH_EXPLOSIVE_BURN. JWL equation of state was used to describe the pressure of the explosion products:

\[
P = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-\frac{R_1}{R_2 V}} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-\frac{R_2}{R_2 V}} + \frac{\omega E}{V}
\]

(6)

Where \( P \) is the pressure; \( E \) is the detonation energy per unit volume; \( V \) is specific volume; \( A \), \( B \), \( R_1 \), \( R_2 \), \( \omega \) is constant. The parameters were listed in Table 1(Song, 2013).
Table 1 - Material model and JWL EOS parameters of TNT explosive

<table>
<thead>
<tr>
<th>A(Pa)</th>
<th>B(Pa)</th>
<th>R1</th>
<th>R2</th>
<th>ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.07E11</td>
<td>3.9E9</td>
<td>4.485</td>
<td>0.79</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ρ (kg/m³)</th>
<th>D(m/s)</th>
<th>E₀(Pa)</th>
<th>P_CJ(Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1583</td>
<td>6880</td>
<td>6.62E9</td>
<td>1.94E10</td>
</tr>
</tbody>
</table>

The material model for water was *MAT_NULL without computing deviatoric stress and it is suitable for simulating the fluid material. Gruneisen equation of state was used to describe the relationship between the pressure, density and the specific internal energy.

When the water is compressed (\( u > 0 \)), the pressure is:

\[
\begin{align*}
P &= \frac{\rho_0 C^2 u \left[ 1 + \left( 1 - \frac{\rho_0}{2} \right) u - \frac{a}{2} u^2 \right]}{1 - (S_1 - 1) u - S_2 \frac{u^2}{u + 1} - S_3 \frac{u^3}{(u + 1)^2}} + \left( \gamma_0 + au \right) E \\
&\text{(7)}
\end{align*}
\]

When the water is expanded (\( u < 0 \)), the pressure is:

\[
P = \rho_0 C^2 u + \left( \gamma_0 + au \right) E \tag{8}
\]

Where \( \rho \) is the density of the water; \( C \) is the intercept of the \( u_\rho \) curve; \( S_1, S_2, S_3 \) are the coefficients for the slope of the \( u_\rho \) curve; \( \gamma_0 \) is the Gruneisen coefficient; \( a \) is the first-order volume correction to \( \gamma_0 \). Table 2 lists the parameters for the equation of state shown above. \( E \) is the relative internal energy:

\[
E = \left( \rho gh + P_0 \right) / \left( \rho \gamma_0 \right) \tag{9}
\]

Where \( h \) is the depth of the water, and \( P_0 \) is atmospheric pressure. So the initial specific internal energy for the water 178.6m deep depth is \( E_0 = 6608.29 \text{ J/kg} \).

Table 2 - Material model and JWL EOS parameters of water

<table>
<thead>
<tr>
<th>ρ (kg/m³)</th>
<th>C(m/s)</th>
<th>S₁</th>
<th>S₂</th>
<th>S₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>1025</td>
<td>1520</td>
<td>1.92</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.28</td>
<td>a</td>
<td>E₀(Pa)</td>
<td>P_CJ(Pa)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6608.29</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The physical time for the simulation is 0.05s. The hardware is Intel Xeon CPU with the dominant frequency of 3.47GHz. When single CPU is used, the whole simulation time is 12 minutes, which is obviously faster than AUTODYN.
THE COMPARISON BETWEEN THE SIMULATION RESULTS AND THE EXPERIMENT DATA

The time history of bubble radius was shown in Fig. 2. Comparison of maximum radius and pulsation period was shown in Table 3. It can be seen that the simulation results agree well with the test data. During the contraction of the bubble, the radiation pressure caused additional energy loss. The maximum radius and pulsation period of the bubble are gradually decreasing. As for the maximum radius and pulsation period of the first bubble, the simulation results were slightly less than that of the test data. The reason is as follows: when calibrating the equation of state for explosive products with standard cylinder test, the metal cylinder would fracture quickly in twenty microsecond or so. Thus, the calibrated EOS parameters neglected the energy for the low-pressure gas of explosive products. Both the maximum radius and pulsation period of the second bubble were bigger than the test data. This is due to that the energy loss caused by the material and heat exchange between the bubble and the surrounding water was neglected. Hick (Hicks, 1971) indicated that the deep water bubble was unstable when its radius approached minimum value. The underwater photography showed that a large number of needle-like water jets spurting into the bubble at this time, then the thermal bubble cooling down quickly.

![Time history of bubble radius](image)

**Table 3 - Comparison of maximum radius and pulsation period**

<table>
<thead>
<tr>
<th></th>
<th>The first maximum radius (cm)</th>
<th>The first pulsation period (ms)</th>
<th>The second maximum radius (cm)</th>
<th>The second pulsation period (ms)</th>
<th>The third maximum radius (cm)</th>
<th>The third pulsation period (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test data</strong></td>
<td>39.1</td>
<td>17.85</td>
<td>29.5</td>
<td>13.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Simulation results</strong></td>
<td>38.64</td>
<td>17.61</td>
<td>31.63</td>
<td>14.75</td>
<td>28.58</td>
<td>14.63</td>
</tr>
<tr>
<td><strong>Error (%)</strong></td>
<td>-1.18</td>
<td>1.34</td>
<td>7.22</td>
<td>13.46</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
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Fig. 3 shows the pressure history of water 0.78m (about twice the maximum radius of the bubble) away from explosion center. It can be seen that the pressure history curve is very smooth. There is no obvious numerical noise, while the AUTODYN simulation curve in (Swift, 1950) has much noise.

It can be seen from Table 3 and Fig. 4:

1) As regards to main shock wave. LS-DYNA and Zamyshlyayev formula have very close prediction on the peak value of the overpressure, and the difference is only 1.21%. Two main shock wave curves are approximately coincident.

2) As regards to the first pulsating pressure. The calculated curves have very good symmetric shape. Zamyshlyayev formula has relatively gently ascent stage and descent stage, while the simulation result of LS-DYNA has very sharp curve. The peak value of pulsating pressure obtained by Zamyshlyayev formula is far below that of LS-DYNA, and it is 9.95% of the peak value for the main shock wave. The peak value of pulsating pressure obtained by LS-DYNA is 29.56% of the peak value for the main shock wave.

3) As regards to repeatedly pulsating pressure. The peak value for the repeated pulsating pressure obtained by LS-DYNA decreases progressively, and the wave shape becomes gently little by little.

![Fig. 3 - Pressure history of water 0.78m away from explosion center](image)

Table 4 - Comparison of shock wave and pulsation overpressure calculated with Zamyshlyayev formula, LS-DYNA software

<table>
<thead>
<tr>
<th></th>
<th>Peak value for the shock wave (MPa)</th>
<th>Peak value for the first pulsation (MPa)</th>
<th>Peak value for the second pulsation (MPa)</th>
<th>Peak value for the third pulsation (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation result of the formula</td>
<td>45.34</td>
<td>4.51</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Numerical simulation result</td>
<td>44.79</td>
<td>13.24</td>
<td>4.07</td>
<td>2.16</td>
</tr>
<tr>
<td>Difference(%)</td>
<td>-1.21</td>
<td>193.57</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Fig. 4 shows the impulse history of water 0.78m away from explosion center, and this curve was obtained by the integration on the pressure history shown in Fig. 3. Although the peak value for the first pulsation is far below that of the shock wave, its action time is much longer. So it has great destructive effects on the ship structure and can’t be neglected. The impulse of the first pulsation obtained by the Zamyshlyayev formula is much higher than that of the main shock wave, which does not match with the actual. The peak value for the impulse of the first pulsation obtained by LS-DYNA is slightly lower than that of the main shock wave. The difference between the peak values for the impulse of the first, second or third pulsation is small.

![Impulse history of water 0.78m away from explosion center](image)

Fig. 4 - Impulse history of water 0.78m away from explosion center

Fig. 5 shows the velocity history of water 0.78m away from the explosion center. The maximum velocity for the water particle is 28.4 m/s. It can be gotten by the formula for the maximum velocity of the shock wave front

\[ v_m = \frac{P_m}{(\rho C)} = \frac{45.34 \times 10^6}{(1024 \times 1520)} = 29.1 \text{ m/s} \]

and the difference is only 2.41%. Then the velocity of the water particle decreases immediately to 6m/s. At this time, the velocity of the water particle is affected by the after flow caused by the bubble pulsation. With the continuous expansion and contraction of the bubble, the velocity of the water particle is also vibrating back and forth like whipping.

**CONCLUSION**

1) The one-dimensional spherical symmetric multi-material Euler model of LS-DYNA can accurately simulate the bubble pulsation phenomenon during the deep water explosion. The simulation results of the maximum radius and pulsation period of the first and the second bubble agree well with the test data.

2) The peak value for the main shock wave obtained by LS-DYNA agreed well with Zamyshlyayev formula, while the peak value of the first pulsation is far high than that of the Zamyshlyayev formula. Compared with the Zamyshlyayev formula, the impulse of the water particle obtained by LS-DYNA is closer to the actuals.
REFERENCES


