

## ANALYSIS OF THERMO-ELASTIC PROPERTIES OF PARTICULATE NANO-COMPOSITES WITH VARIOUS TYPES OF INTERPHASES

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### ABSTRACT

In this paper a new approach to analysis of thermo-elastic properties of random composites with interphases is outlined and illustrated. It is based on the statistical homogenization method - the method of conditional moments - combined with the recently introduced notion of the *energy-equivalent* inhomogeneity, extended here to include thermal effects. After exposition of its general principles, the approach is applied in the investigation of the effective thermo-elastic properties of materials with randomly distributed nano-particles.

**Keywords:** interphase, energy equivalence, Gurtin-Murdoch model, spring layer model.

### INTRODUCTION

The basic idea of equivalent inhomogeneity (Nazarenko, 2016) is to replace the inhomogeneity and the surrounding interphase by a single *equivalent* inhomogeneity of constant stiffness tensor and coefficient of thermal expansion (CTE), combining elastic and thermal properties of both. The equivalent inhomogeneity is then perfectly bonded to the matrix which enables composites with interphases to be analyzed using techniques devised for composites without interphases. From the mechanical viewpoint, definition of the equivalent inhomogeneity is based on Hill's energy equivalence principle, applied to the problem consisting only of the original inhomogeneity and its interphase. This is illustrated considering spherical particles with two models of interphases, the Gurtin-Murdoch (GM) material surface model and the spring layer model. The resulting equivalent inhomogeneities are subsequently used to determine effective thermo-elastic properties of randomly distributed particulate composites. The effective stiffness tensors CTE's of the considered composites are determined by the method of conditional moments (Nazarenko, 2009). Closed-form expressions for the effective thermo-elastic parameters of a composite consisting of a matrix and randomly distributed spherical inhomogeneities are derived for the bulk and the shear moduli as well as for the CTE's. Dependence of the effective parameters on the interphase properties is included in the resulting expressions, exhibiting analytically the nature of the size-effects in nano-materials.

### RESULTS AND CONCLUSIONS

Numerical results only for composites with GM interphase are presented here. In the first example, the effects of the surface thermo-elastic properties on the effective CTE of solids containing spherical nano-pores (Fig.1a,b) are analyzed. The results are presented for porous aluminum, with  $c_i$  denoting the volume fraction of inhomogeneities (nano-pores in the first

example). The free-surface elastic moduli and CTE are taken the same as in Duan (2007); (A) for surface [100]: the bulk modulus  $K_s = -5.457 \text{ N/m}$  and the shear modulus  $\mu_s = -6.2178 \text{ N/m}$ ; and (B) for surface [111]:  $K_s = 12.932 \text{ N/m}$ ;  $\mu_s = -0.3755 \text{ N/m}$ . The CTE of bulk aluminum is  $\alpha_m = 5.01 \cdot 10^{-5} \text{ K}$  while that of its surface is  $\alpha_s = 5\alpha_m$ . In the second example the epoxy matrix with randomly distributed spherical glass particles is investigated (Fig. 1c). The material parameters are (Duan, 2007): for the particles the Young modulus is  $E_i = 72.4 \text{ GPa}$ , whereas the Poisson ratio  $\nu_i = 0.2$  and CTE  $\alpha_i = 5 \cdot 10^{-6} \text{ K}$ ; for the matrix -  $E_m = 3.45 \text{ GPa}$ ,  $\nu_m = 0.3$ ,  $\alpha_m = 42 \cdot 10^{-6} \text{ K}$ , the interface CTE is  $\alpha_s = 2\alpha_m$ . The effective normalized CTE  $\alpha^*/\alpha_{cl}$  and  $\alpha^*/\alpha_m$  (with the subscript “cl” representing the solution without surface effects) for the material with randomly distributed spherical nano-particles is determined and shown in Figs 1a,b,c. Its dependence on the radius of nano-voids (for fixed volume fraction of nano-pores  $c_i = 0.3$ ) is shown in Fig.1a, on the void volume fraction  $c_i$  (for different radii of nano-pores) in Fig.1b, and on the interface parameter  $q = K_s/(K_i r)$  (for various particle volume fraction  $c_i$ ) in Fig.1c. It is seen that the results obtained using the proposed approach compare well with other theoretical predictions (Chen, 2006; Duan, 2007). So, one can conclude that the new approach is effective and accurate, while rendering a closed-form solution of the problem.

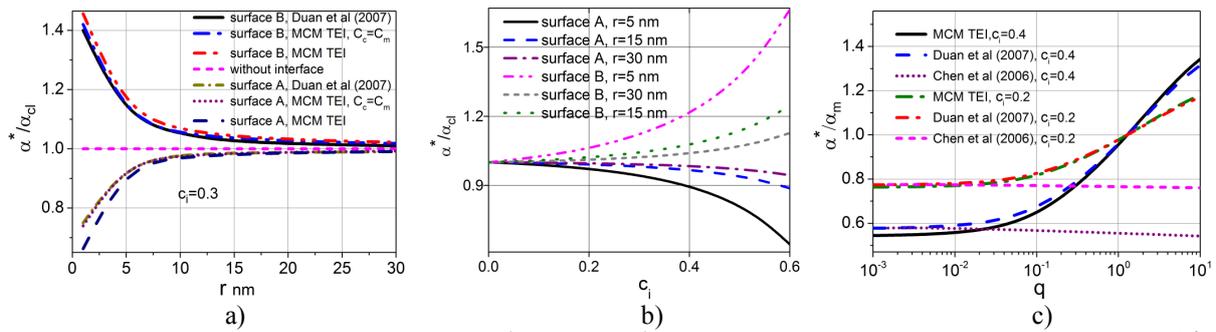


Fig. 1 - Dependence of the normalized CTE  $\alpha^*/\alpha_{cl}$  and  $\alpha^*/\alpha_m$  on a) radius of a spherical cavity (for  $c_i = 0.3$ ); b) on the void volume fraction  $c_i$  for the various radii of a spherical cavity; c) on the interface parameter  $q = K_s/(K_i r)$  for various glass particle volume fractions  $c_i$ .

## ACKNOWLEDGMENTS

LN and HS gratefully acknowledge the financial support by the German Research Foundation (DFG) via Project NA1203/1-1.

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