Proceedings of the 7th International Conference on Mechanics and Materials in Design
Albufeira/Portugal 11-15 June 2017. Editors J.F. Silva Gomes and S.A. Meguid.
Publ. INEGI/FEUP (2017)

PAPER REF: 6518

AN ANALYTICAL SOLUTION FOR TRANSIENT THREE DIMENSIONAL THERMOELASTICITY OF FUNCTIONALLY GRADED CIRCULAR PLATES DUE TO AXISYMMETRIC LOADS

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ABSTRACT

In this paper, an exact solution of transient thermo-elastic problem of three dimensional circular plate made of functionally graded material is developed. The thermal and mechanical loads on upper and lower surfaces are assumed to apply axisymmetric and a simply supported boundary conditions on the lateral surface of the plate is considered, where no limiting assumption is used. The material properties, except Poisson's ratio, are assumed to varying exponentially across the thickness direction. A full analytical method is used, and an exact analytical solution is presented.

Keywords: functionally graded, thermo-elasticity, circular plate, analytical methods.

INTRODUCTION

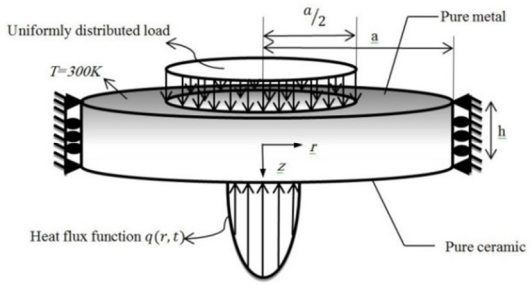
The purpose of this paper is to develop a full analytical method for the thermo-elasticity field in the heterogeneous functionally graded circular plate. The properties of the material are assumed to be exponential functions of the thickness coordinate z. Using the basic equations of axisymmetric problems in thermo-elasticity, the Navier equations are derived and the solution of displacement field is proposed based on appropriate Bessel functions. Simply supported mechanical boundary condition of the plate together with the equations resulting from substitution of displacement field in the Navier equations will determine the Bessel function properties in terms of its arguments. Transient thermal boundary conditions are taken into consideration and the corresponding time dependant temperature field is obtained, which is used as the particular solution of Navier partial differential equations. Also, mechanical boundary conditions on the upper and lower surfaces of the plate are used to determine the unknown coefficients and consequently complete displacement field as a general solution for the Navier equations are obtained. Using the analytical method as well as considering the general form of boundary conditions, any combination of thermal and mechanical loads can be taken without any limitation.

RESULTS AND CONCLUSIONS

Four examples considering different combinations of thermal and mechanical loads have been solved to highlight the peculiarity of the analytical solution obtained. In one example (case 1), a solid circular plate is considered with material properties and boundary conditions shown in figure 1, also in another example (case 2) a comparison is made between behaviors of a plate

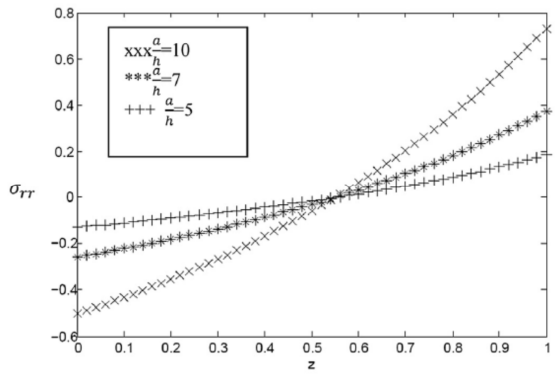
of homogeneous and five different FG materials in terms of displacement and stress components. The following results have been presented as samples.

Metal: Ti-6Al-4V (upper surface)	Ceramic:Zro2 (lower surface)
$E_m = 66.2(Gpa)$ $\nu = 0.322$ $\alpha_m = 10.3 \times 10^{-6}\left({}^{1}/_{k}\right)$ $\rho_m = 4.41 \times 10^{3}\left({}^{kg}/_{m^3}\right)$ $k_m = 18.1\left({}^{W}/_{mK}\right)$ $c_m = 808.3\left({}^{J}/_{kgK}\right)$	$E_c = 117(Gpa)$ $\nu = 0.322$ $\alpha_c = 7.11 \times 10^{-6}\left({}^{1}/_{k}\right)$ $\rho_c = 5.6 \times 10^{3}\left({}^{kg}/_{m^3}\right)$ $k_c = 2.036\left({}^{W}/_{mK}\right)$ $c_c = 615.6\left({}^{J}/_{kgK}\right)$



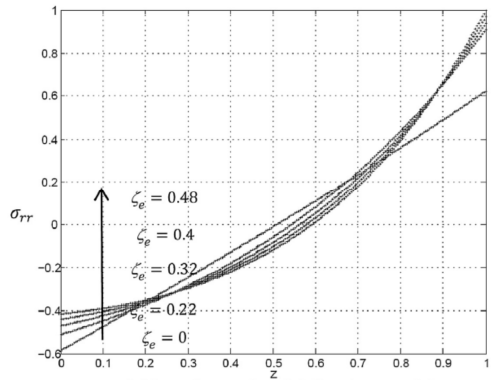
Some results are as following graphs:

(case 1)



Normal stress in axial direction at r=0.3

(case 2)



Normal stress in axial direction at r=0.3

Fig. 1 - Some examples considering different combinations of thermal and mechanical loads

This study shows that:

- The neutral axis and plane of the FG plate will not fall in the mid-surface, but for different thicknesses its location will be the same.
- Displacements in isotropic materials are more than those of FG plates with same boundary condition and loading conditions.
- There is a zero point for circumferential stress in radial direction which is near the 0.8 of the dimensionless radius of the plate.

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