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PARAMETRIC INVESTIGATION OF FREE VIBRATION OF DOUBLE LAP COMPOSITE JOINTS

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ABSTRACT

This paper deals with a modal parametric study for fixed-free double-lap adhesively bonded laminated plates using a mathematical model based on the Hamilton's principle and solved numerically using the finite element discretization method programmed on MATLAB. Validation was carried out through ANSYS Workench.
The following parameters were examined: adherents fibers' fraction volume, adhesive's thickness, overlap length, adherents' thicknesses and adhesive Young's Modulus. The results have shown fibers fraction volume and thicknesses as well as overlap length affect remarkably the natural frequencies of free vibration while adhesive thickness was found to have no effect. Finally, in the low margin of adhesive's Young's moduli, the natural frequencies were found to increase remarkably with the adhesive Young's modulus, while for higher Young's moduli the natural frequencies increase very slightly.

Keywords: free vibration; bonded joints; finite element method.

INTRODUCTION

It is known that composites are very sensitive to disruption such as holes creation; hence one of the best techniques to assemble composites with avoiding the traditional ways of assembling such as screwing or riveting, is adhesively bonding. Mechanical behavior of such complex structures was a main concern of many researchers. An important aspect to be investigated is the vibrational behavior. Many geometries of specimens were proposed especially the single lap joint. The studies were divided between analytical, experimental and numerical.

One can cite the work of (He, 2001) who applied the finite element technique on a cantilevered single lap joint to investigate the effect of Young's modulus and Poisson's ratio of the adhesive on the natural frequencies of transverse vibration. Two numerical approaches to study the influence of geometrical parameters of single lap joints on natural frequencies were the main concern of (Gunes, 2007). The same authors (Gunes, 2010) extended the previous work for ceramic and metallic adherends where the influence of the thickness was investigated. Moreover, in the paper of (He, 2014) a detailed numerical approach of torsional vibration of single lap joints was developed. A full experimental study of a bonded metallic mean was the main interest of (Garcia-Baruetabeña, 2014): they examined the effect of geometrical parameters on the resonant frequencies, amplitudes of vibration and loss factors. The paper of (Du, 2014) has focused on the relationship between vibrating fatigue cycles and modal frequencies; in addition they studied numerically the effect of adhesive modulus of

elasticity and contact area. An analytical model describing wave propagation in a single lap composite joint was established in (Samaratunga, 2015) where a numerical validation on ABAQUS was carried out. ANSYS Workbench was the numerical tool of (Zeaiter, 2016) to investigate the effect of materials substrates of a double-lap joint on the first ten resonant frequencies where five different metals were considered.

On the other hand, fewer are the works that considered adhesively bonded joints under external harmonic excitation. (Vaziri, 2001) applied an out-of plane harmonic force on a voided-single-lap joint; a closed-form analytical model was established to find the dynamic response. An almost similar work was accomplished in (Vaziri, 2002) for an axial harmonic force applied on a voided-tubular joint. Moreover, a closed form analytical model to calculate dynamic shear in the glue layer was derived in (Challita, 2012) on a double-lap metallic joint with elastic adhesive subjected to harmonic axial force; the model was based on the modified shear-lag model. (Al-Mitani, 2016) has extended the previous work for composite substrates and visco-elastic adhesive,

In the present work, the double-lap composite joint will be considered, an energy-based-formulation will be proposed to evaluate the resonant frequencies and many parameters will be investigated to show their influence on those frequencies.

MATHEMATICAL MODEL

Specimen geometry

The specimen's geometry adopted in this study is the double lap joint shown in Fig. 1. The structure is constituted of three rectangular plates of length L and width w, bonded together. The middle plate of thickness t_2 is shifted horizontally with respect to the two other cantilevered plates of thickness t_1 with an overlap length a. The other end of the middle plate is free. Bonding is applied between the upper and the middle plates and between the middle and the lower plates with a thickness t_a.

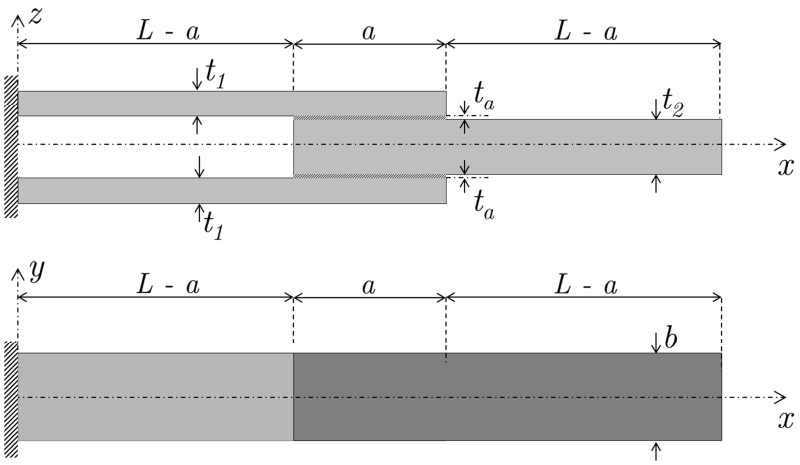

Fig. 1 - Double-lap joint geometry and dimensions

Assumptions

In this study, the following assumptions are considered: (1) Small deformations, (2) the adhesive is under the anti-plane stress state,(3) all materials used are homogeneous and linearly elastic, (4) the peel and shear stresses along the thickness direction in the adhesive

layer are assumed uniform and (5) the transverse normal modulus of adhesive is much lower than that of the adherents so that the transverse normal deformation of the adherents is negligible in comparison with that of the adhesive.

Energy formulation

According to the Hamilton's principle,

$$\int_{t_1}^{t_2} \delta(T - V + W)dt = 0$$

Where$T$ and $V$are the kinetic and strain energy of the system. W is the total work done on the system by the external forces, not considered in the modal analysis.
The kinetic and strain energy of each plate and each adhesive layer are expressed as function of the deformation vector.

Finite Element discretization

The problem described in the continuous domain is discretized into isoparametric, 8-noded serendipity elements with first order shear deformation for each plate and each adhesive layer, using the finite element method (FEM), by transforming the physical coordinates into local coordinates as shown in Fig. 2 and Fig. 3. Shape functions developed for 8-noded serendipity elements are used to discretize the domain. Stiffness and Mass matrices of each plate and each adhesive layer are then derived and assembled on the whole structure; total mass and stiffness matrices are obtained and the following Eigen-equation can be solved for determining the natural frequencies:

$$(-\omega^2[M] + [K])\{\Delta\} = 0$$

The natural frequencies and the mode shapes are then obtained and the modal problem is solved.

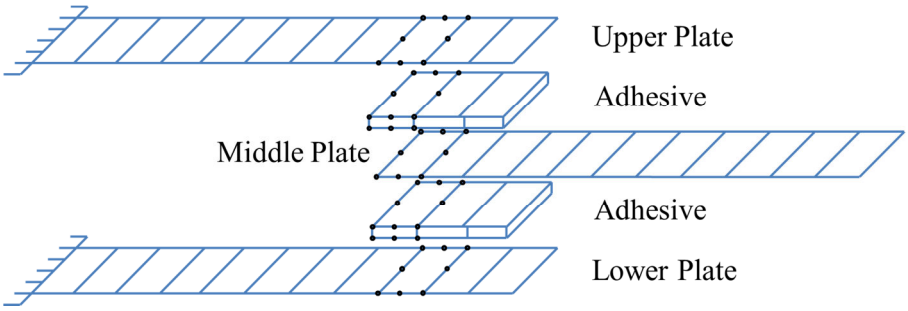

Fig. 2 - Finite Element discretization

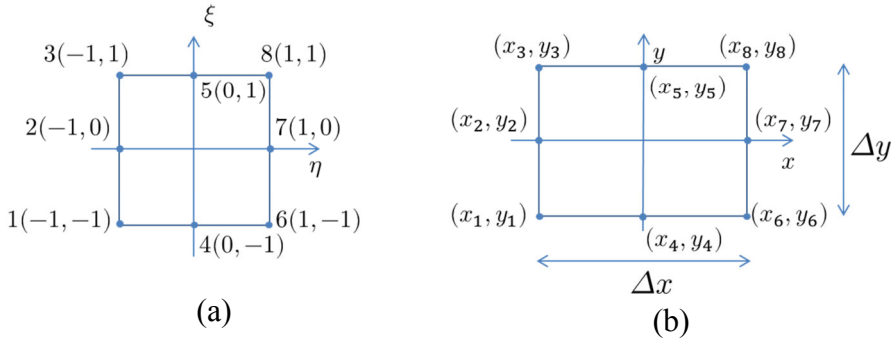

Fig. 3 - 8-noded serendipity element (a) local coordinates (b) global coordinates

NUMERICAL MODEL

The following assumptions are to be considered: only linear elastic material behavior is valid, small deflection theory is used, and no nonlinearities are included, damping matrix $[C]$ is not present, so damping is not included, applied forces vector $\{F\}$ is not presentsince free vibration is considered. The following steps must be respected in ANSYS in order to create the model and collect the results; draw geometry, assign material properties, define contact regions(bonded), define mesh controls: In this case, automatic mesh generation, include supports: 2 fixed ends and one free end, request frequency finder results, set frequency finder options, solve the model, review results.

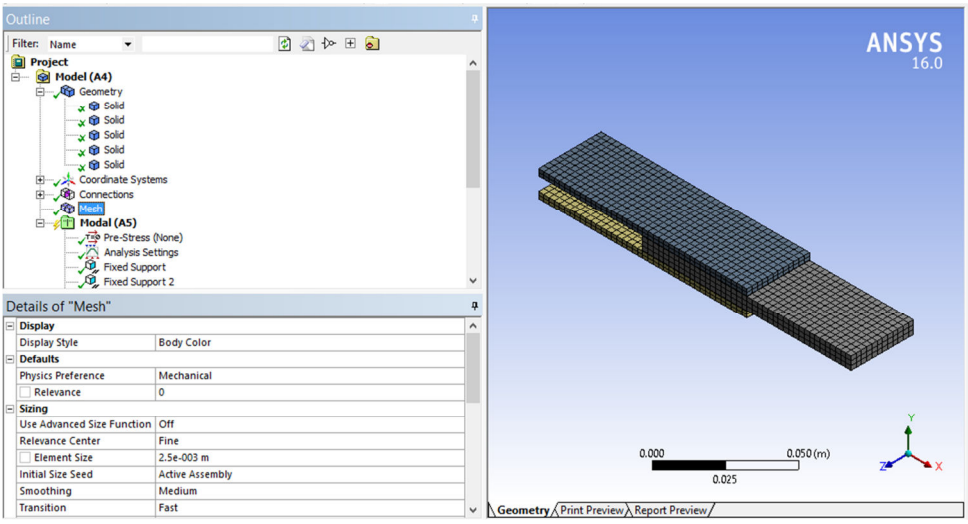

Fig. 4 - Mesh generation on ANSYS Workbench

VALIDATION

The mathematical model developed earlier is set and solved on MATLAB and the collected results are compared and validated with the ones obtained from numerical model simulation on ANSYS Workbench with a model based on the finite element method; the mesh considered is an hexahedron element as shown in Fig. 4, and the mesh size is automatically generated, giving the best results with optimized time of resolution.

The considered substrate is glass fiber- epoxy laminate with $(0^{\circ}/90^{\circ})$ unidirectional layers' direction and $0.5mm$ thickness. The mechanical properties of one 0°layer are given in Table 1. Adhesive between substrates is epoxy/resin, and its mechanical properties are given in Table 2.

Table 1 - Mechanical properties of unidirectional glass fiber-epoxy laminate

Properties	Symbol	Value
Longitudinal Young's Modulus (MPa)	E_L	46,000
Transverse Young's Modulus (MPa)	E_T	10,000
Longitudinal Shear Modulus (MPa)	G_{LT}	4,600
Longitudinal Poisson's Ratio	υ_{LT}	0.31
Density (kg/m^3)	ρ	2,100

Table 2 - Isotropic properties of epoxy adhesive

Properties	Symbol	Value
Young's Modulus (MPa)	E_a	500
Poisson's Ratio	υ_a	0.35
Density (kg/m^3)	ρ_a	1,595

Fig. 5 shows a natural frequencies' comparison between the analytical model solved on MATLAB and the numerical model of ANSYS Workbench for composite substrate. Maximum error of 8.53% is reached at mode 7. The relative error related to the fundamental frequency is 3.53%. The model is validated for composite substrate.

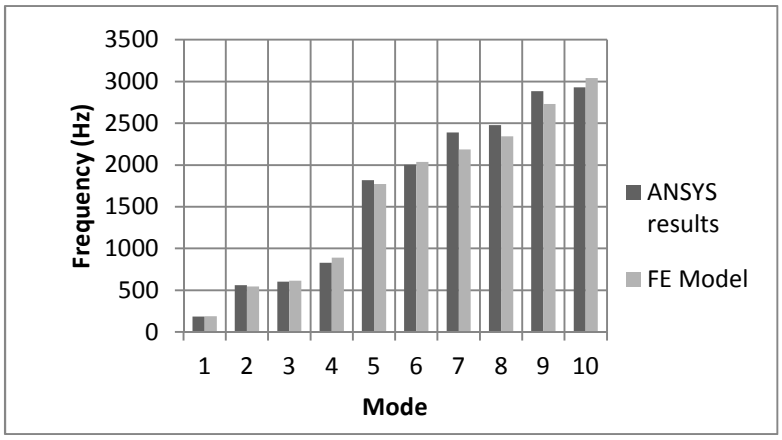

Fig. 5 - Natural Frequencies comparison between ANSYS results and mathematical model for DLJ glass-epoxy composite.

PARAMETRIC STUDY

Fixed and variable parameters

In this study, some parameters are considered as constant in the structure's design and do not interfere in the parametric study: plates length: $L = 100mm$, plates width: $w = 30mm$, plates' material glass/polypropylene $0°/90°$ with varying fibers fraction volume, adhesive's Poisson ratio $\upsilon_a = 0.3$, adhesive's density $\rho_a = 1595$ Kg/m^3.

Table 3 - Mechanical and geometric parameters variation

Fibers fraction volume (%)	Adhesive's thickness (mm)	Adhesive Young's Modulus (MPa)	Plates' thickness (mm)	Overlap (mm)
20	0.05	200	1	10
30	0.1	500	1.5	20
40	0.15	**1000***	**2***	**30***
50	**0.2***	2000	2.5	40
60*	0.25	5000	3	50
	0.3	10000		60
				70
				80
				90

*Values in bold are considered as "reference values" for the corresponding parameter.

Geometric and mechanical properties of the structure affect the vibrational behavior of the system. Hence, a parametric study is presented for double-lap adhesively bonded, laminated plates, by varying the different parameters as shown in Table 3: Fibers' fraction volume, Adhesive's thickness, Overlap Length, Adherents' thicknesses, Adhesive Young's Modulus. The results are then discussed and analyzed to study the effect of each parameter on the vibrational behavior, leading to a better design of the structure.

Variation of the Fibers' fraction volume

The Fibers fraction volume in a laminated structure affects the mechanical properties of the material, such as the Young's Modulus, the Poisson Ratio and the Shear Modulus in every direction. The variation of the engineering constants and density of the plates with the fibers fraction volume are presented in Table 4.

Table 4 - Variation of engineering constants and density of the plates with the fibers fraction volume

V_f (%)	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	υ_{12}	υ_{13}	υ_{23}	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ρ (kg/m^3)
20	5.185	5.185	2.873	0.126	0.167	0.167	0.654	0.806	0.806	1.098
30	7.063	7.063	2.117	0.1	0.163	0.163	0.743	0.91	0.91	1.197
40	8.976	8.976	2.443	0.086	0.1572	0.1572	0.86	1.05	1.05	1.296
50	10.947	10.947	2.891	0.0794	0.1496	0.1496	1.02	1.242	1.242	1.395
60	13.02	13.02	3.542	0.073	0.1412	0.1412	1.253	1.523	1.523	1.494

The influence of the fraction volume of fibers on the resonant frequencies is graphically plotted in Fig. 6. The results show an increase of the frequency by increasing the fibers' fraction and hence improving the stiffness matrix.
This increase is weak for the first mode. In fact, between 20% and 60% of fibers fraction volume, the natural frequency varies between 83.209Hz and 108.5Hz, equivalent to 0.63Hz increase per 1% of fibers fraction volume increase. The increase becomes strong for higher modes: approximately 2Hz for mode 2, 2.86Hz for mode 3, 2.38Hz for mode 4, 6.10Hz for mode 5 and 7.46Hz for mode 6 per 1% of fibers fraction volume increase.

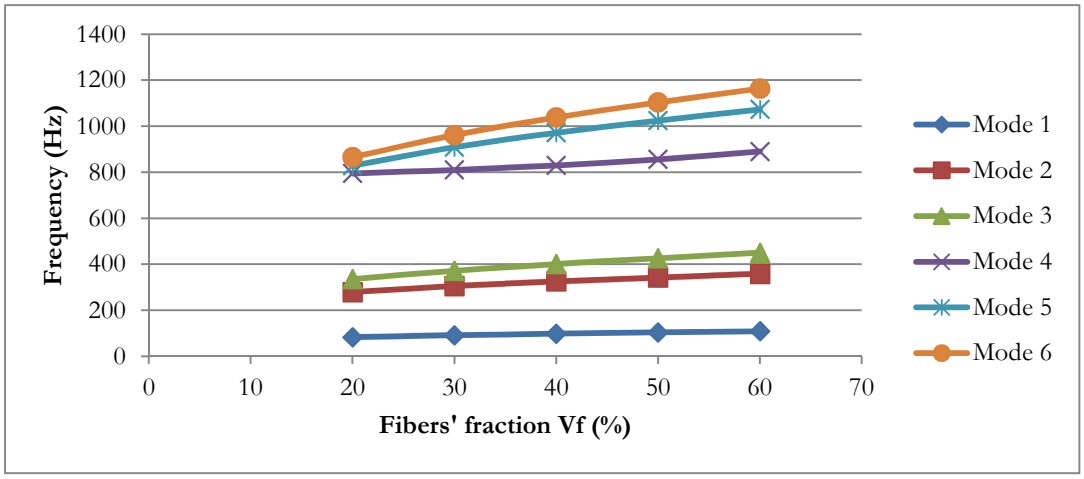

Fig. 6 - Graphical representations of the first 6 modes resulting from variation of fibers fraction volume with respect to the reference model

Variation of the adhesive's thickness

The variation of the adhesive's thickness effect on the free vibration is presented in this part. The natural frequency variation with the adhesive's thickness is plotted in Fig. 7. Results show that the increase of the adhesive's thickness affects slightly the increase of the natural frequencies for modes 1, 2, 3, and 5: between 0.05mm and 0.3mm adhesive's thickness, the frequency increases with a rate 5% for mode 1, 7% for mode 2, 0.6% for mode 3, 5.5% for mode 5 and 0.03% for mode 6. For mode 4, a more important increase in the natural frequencies can be noticed, especially for low adhesive's thickness: between 0.05mm and 0.15mm, the frequency increases from 639.6Hz to 847.3Hz. This can be explained by the fact that mode 4 is a torsional mode around X axis where the adhesive thickness affects the vibrational behavior of the structure.

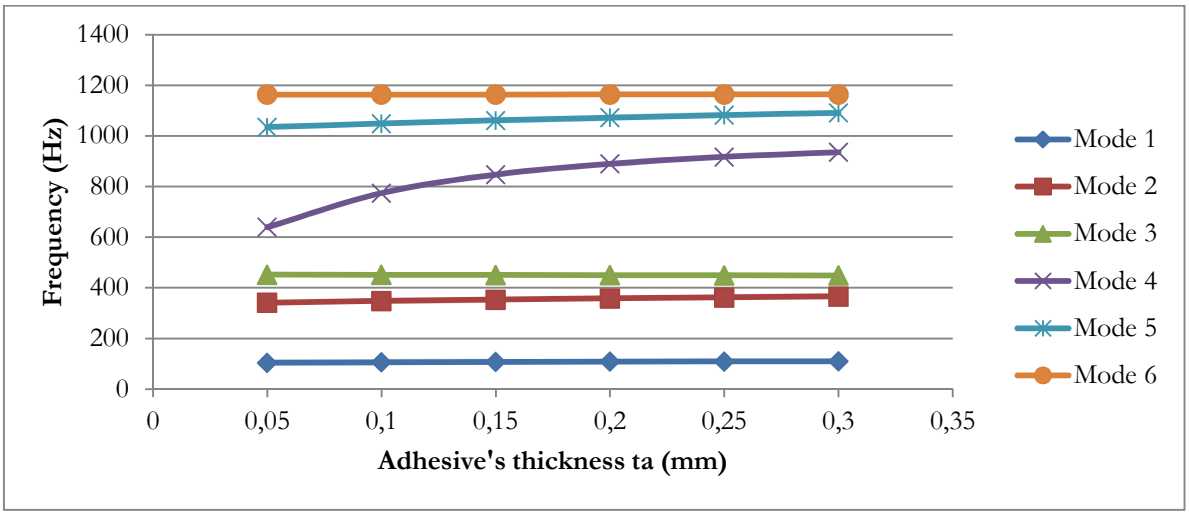

Fig. 7 - Graphical representations of the first 6 modes resulting from variation of Adhesive's thickness with respect to the reference model

Variation of the overlap length

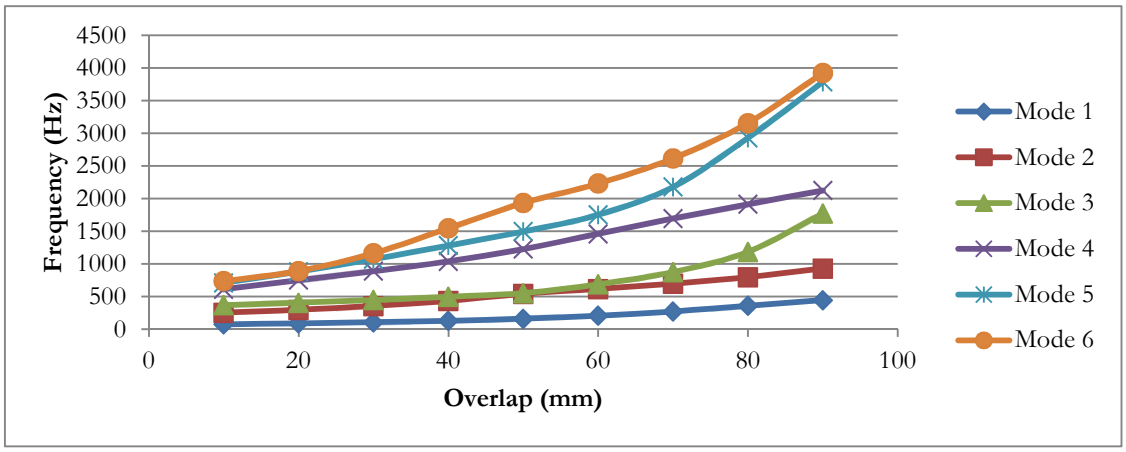

Fig. 8 - Graphical representations of the first 6 modes resulting from variation of the overlap length with respect to the reference model

The overlap length variation effect is now studied. The results are shown in Fig. 8 and show the remarkable influence of the overlap length on the natural frequency; an increase in the overlap length leads to an important increase in the natural frequency. The mean frequency increasing ratio with the overlap between 10 mm and 90 mm is approximately 4.

The rate of the natural frequency increment with the overlap length is: 4.6Hz/mm, 8.4Hz/mm, 17.5Hz/mm, 18.9Hz/mm, 38.4Hz/mm, and 39.8Hz/mm for modes 1, 2, 3, 4, 5 and 6 respectively.

It can also be noticed that for modes 5 and 6 and for an overlap between 10 mm and 30 mm, and between 70 mm and 90 mm, the corresponding natural frequencies values are very close.

Variation of the Adherents' thicknesses

The variation effect of the adherents' thickness on the vibrational response is presented in this part. The results are shown in Fig. 9. It is noticed that for low modes, the natural frequency increase with the plates' thickness increase is weak: 32Hz/mm for mode 1, 81.2 Hz/mm for mode 2. And the natural frequency for mode 3 between 1 mm and 2.5 mm is quite constant. For higher modes, the increase of the natural frequency with the plates' thickness becomes higher: 154.4Hz/mm for mode 4, 302.4Hz/mm for mode 5 and 352.27Hz/mm for mode 6.

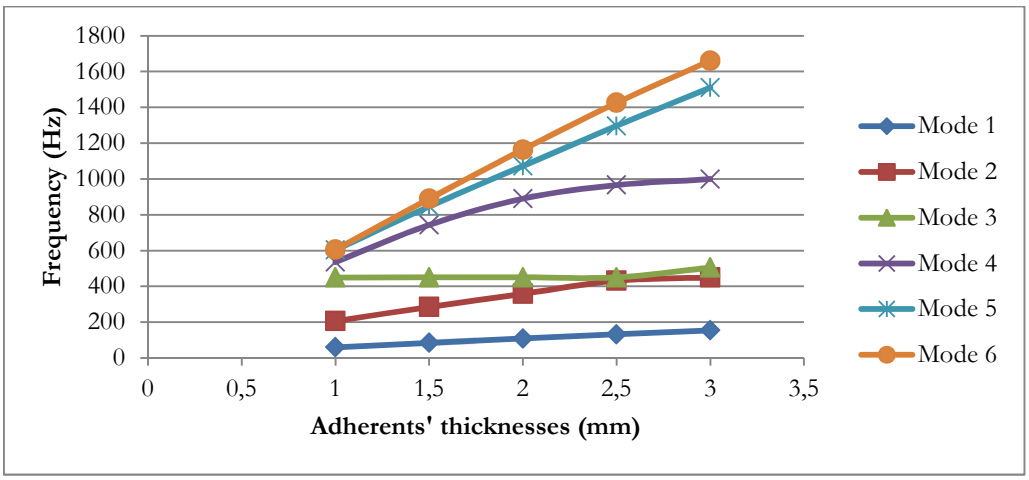

Fig. 9 - Graphical representations of the first 6 modes resulting from variation of the adherents' thicknesses with respect to the reference model

Variation of the adhesive Young's modulus

The variation effect of the adhesive Young's modulus on the natural frequency is presented in this part, with a variation between 200 MPa and 10000 MPa. The results are shown in Fig. 10. It is noticed that for low Young moduli, between 200MPa and 1000MPa, and especially for modes 4 and 5, the natural frequency increases with the adhesive Young modulus. But for higher Young's moduli, it is well noticed for the 6 modes that the natural frequency varies slightly between 2000 MPa and 10000 MPa: 0.74Hz/1GPa for mode 1, 4.18Hz/1GPa for mode 2, 0.065Hz/1GPa for mode 3, 8.37Hz/GPa for mode 4, 7.5Hz/1GPa for mode 5 and 2.63Hz/1GPa for mode 6.

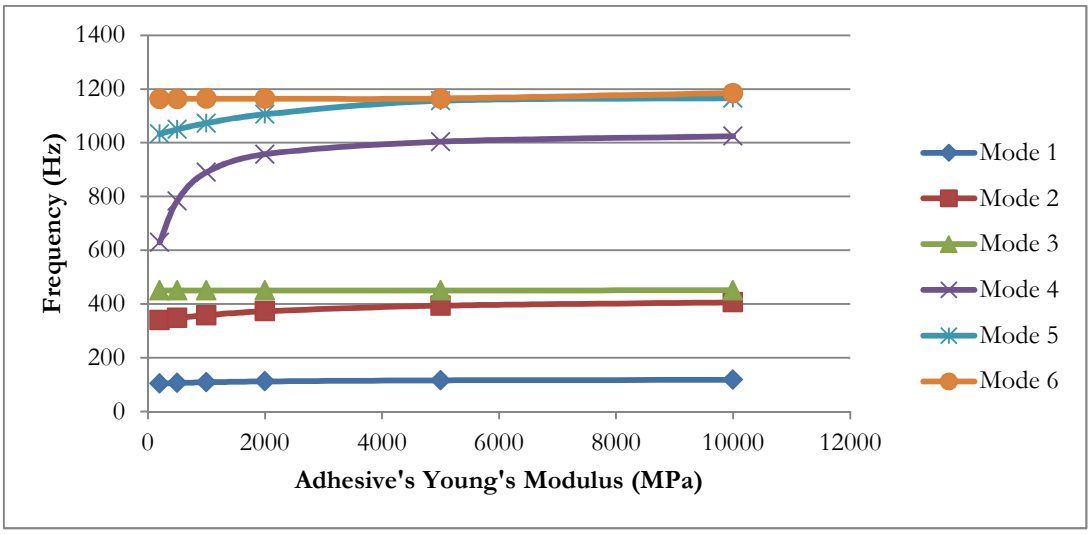

Fig. 10 - Graphical representations of the first 6 modes resulting from variation of adhesive Young's modulus with respect to the reference model

CONCLUSION

A parametric study on the adhesively composite double-lap joints is presented in this paper. First, a finite element mathematical model of the Double-lap adhesively bonded structure, based on Hamilton's principle, was presented. Next, the model is compared with numerical model on ANSYS Workbench. The results have shown good agreement between both models for composite. Then, the parametric study is established and the results have shown that substrates' fibers fraction volume and thicknesse, as well as the overlap length affect remarkably the natural frequencies of free vibration while adhesive thickness does not to have any effect, except for the torsional mode (mode 4). Finally, in the low margin of adhesive's Young's moduli, the natural frequencies were found to increase remarkably with the adhesive Young's modulus, while for higher Young's moduli the natural frequencies increase very slightly.

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