MATHEMATICAL MODEL FOR PREDICTING THE DURABILITY OF COMPOSITE MATERIALS IN FATIGUE TESTS

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ABSTRACT

Assessments of the durability of products made from metallic materials are carried out on the basis of safe durability model and/or on the basis of the concept of safe development of crack. Extensive operating experience and accumulated volume of experimental data give sufficient confidence to designers when metal products designing with the required durability. Unlike metals, composite materials represent a heterogeneous and anisotropic medium. The inherent structure of composite materials considerably complicates the nature of development of technological defects and operational damages from the moment of crack starting until the carrying capacity of the product. As a result, all this leads to a revision of the guidelines (see, for example, [1]), as the concepts of safe development of crack, and models a safe life. In the present work the changing of residual strength of composite material at cyclic testing is studied within the safe life model and its relation with the characteristics of static strength. At this the cyclic stress amplitude may be both constant with asymmetry coefficient \( R \) to zero and valuable.

Keywords: fatigue, composite material, mathematical model, testing, construction.

INTRODUCTION

It is assumed that the residual strength \( \sigma_r \) is a random function of load cycles number \( n \). Each of its implementation could be presented as a linear function of the static tensile strength \( \sigma_b \), amplitude \( \sigma \) cyclic stress and power function of quotient of the current number of cycles \( n \) to the previous \((N-1)\) of \( N \) number of loading cycles

\[
\sigma_r = \sigma_b + (\sigma - \sigma_b) y^\alpha, \ y = n/(N-1)
\]

The possibility of random implementation describing with formula (1) is confirmed comparison with experimental data, cited in [2],

Moreover it is assumed that the static strength \( \sigma_b \) of composite material obeys the three parameter Weibull distribution

\[
F(\sigma_b) = 0, \text{ if } \sigma_b < \sigma_{b0} \text{ and } F(\sigma_b) = 1 - \exp\{1 - [(\sigma_b - \sigma_{b0})/\delta]^\gamma]\}, \text{ if } \sigma_b \geq \sigma_{b0}
\]

As a result of the analysis of distribution laws of residual strength \( \sigma_r \) and normalized residual strength relative to the amplitude of cyclic stress \( x = \sigma_r/\sigma \) is shown that the normalized residual strength \( x \) also distributed by Weibull law

\[
P(\Xi < x) = 1 - \exp\{-[(x-x_0)/\delta_x]^\gamma\} \quad (3)
\]
with threshold value $x_0 = y^\alpha + (1 - y^\alpha)\sigma_{b0}/\sigma$, scale parameter $\delta_x = (1 - y^\alpha)\delta/\sigma$ and Weibull modulus $\gamma_x = \gamma$.

Case, where residual strength $\sigma_r$ turns equal amplitude $\sigma$ cyclic loading ($x = 1$), was adopted for the exhaustion of the carrying capacity of the sample. In case $x = 1$ the value (3) corresponds to the probability of fracture when load numbers, ranging from 1 to $N$. Number of cycles of loading $N$, in which the probability of event $x = 1$ is the maximum is considered as limiting. This condition leads to dependence

$$\sigma = \sigma_{b0} + \delta (N)^{1/\gamma}, \quad (4)$$

which connects the amplitude cyclic stress $\sigma$ with the number of cycles to failure $N$ ($S/N$ curve). Noteworthy, that parameters $\sigma_{b0}$, $\delta$ and $\gamma$ in ratio (4) coincide the parameters of static strength $\sigma_b$ distribution of composite material (see relation (2)).

RESULTS AND DISCUSSION

The resulting ratio (4) provides a very convenient and easy way to predict the durability of composite materials, especially in the early stages of development of the material. Adopted consistent assumptions and made strict conversions are justification for the verification of the developed mathematical model of durability

The dependence

$$\sigma = \delta (N)^{-1/\gamma}, \quad (5)$$

as well as dependencies (4), is received in [3] by a somewhat different way and in assuming that static strength $\sigma_b$ of composite material obeys two parameter Weibull distribution. In researches [3] much attention paid to reviewing the adequacy of the prediction results by (5) in order to assess the durability of various composite materials with different ways of loading, including the construction of diagrams Goodman. It is shown that the results of the predictions from equation (5) vary from very good to poor. In [3] the efforts have been made to improve predictions by integrating the stress amplitude changes during cyclic loading

In our researches the suitability of the resulting ratio (4) for predicting the durability of composites is established by comparison with both own experimental results and published in the available literature data. The justification (comparisons with test results) was performed with adopted the uncertainties, especially regarding the establishment of the threshold $\sigma_{b0}$. Thus the resulting generalized ratio (4) is recommended to use for assessment of the durability of composite materials only on the preliminary stages of designing

REFERENCES

