Proceedings of the 7th International Conference on Mechanics and Materials in Design
Albufeira/Portugal 11-15 June 2017. Editors J.F. Silva Gomes and S.A. Meguid.
Publ. INEGI/FEUP (2017)

PAPER REF: 6551

FLUTTER OF TOW-STEERED COMPOSITE LAMINATES UNDER SUPERSONIC FLOW

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ABSTRACT

Based on the Hamilton principle, the dynamics and the flutter characteristics of three-dimensional variable-stiffness composite laminates under supersonic flow were investigated. The von-Karman large deflection strain-displacement relationship and the piston theory were employed to account for the structural and aerodynamic nonlinearities, respectively. The natural frequency and the flutter of composite laminates were computed by finite element method, and the impacts of temperature, boundary conditions and ply orientations of curvilinear fibers on flutter were discussed. The results indicate that thermal load and boundary condition have a significant effect on the flutter boundary. It also turned out that the dynamics and flutter of variable-stiffness composite panels can be modified by varying the fiber orientation angle.

Keywords: thermal flutter, tow-steered variable stiffness composite laminate, curvilinear fiber path.

INTRODUCTION

Traditionally, straight fiber placement (Fig. 1) [1] is used in fiber reinforced composites manufacturing technology, which means the fiber orientation angle within one ply is constant. However, with the advancement of modern fabrication technology, such as automated fiber placement techniques, it is technically feasible and economically affordable to produce laminates with continuously varying fiber orientation angle. The fiber stiffness properties within a ply can be varied as fiber orientation varying continuously. Variable-stiffness composites (Fig. 2) have drawn much attention for their outstanding designability, weight reduction and cost saving.

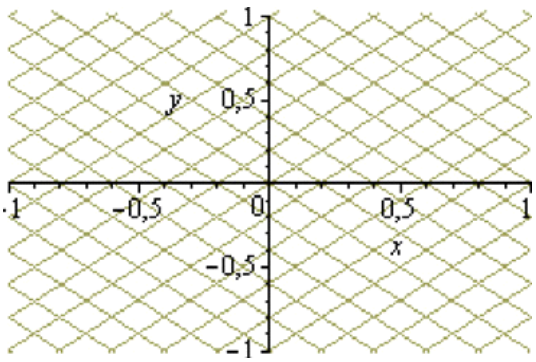


Fig. 1 - Composite laminate with straight fibers

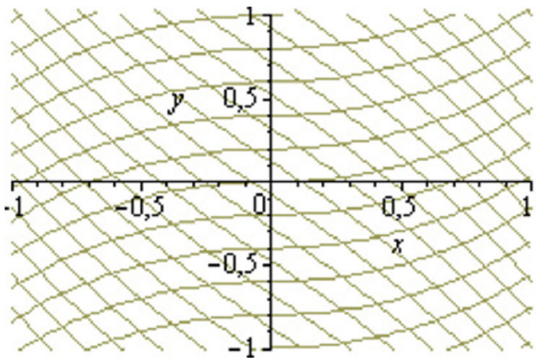


Fig. 2 - Composite laminate with curvilinear fibers

As for mechanical properties of tow-steered variable-stiffness composite laminates, most studies mainly focus on buckling. The curvilinear fiber format was first put forward by Hyer[2,3] to substitute for straight fiber in order to improve the mechanical properties of plate with holes. Subsequently, Gürdal [4-7] proposed the concept of tow-placed variable-stiffness composites and took the residual thermal stresses produced by solidification into account. The results showed that compared with the traditional composite laminates, the buckling behaviors of variable-stiffness ones were much better. Wu [8, 9] analyzed the buckling and post buckling of curvilinear fiber composites with numerical simulation method.

The researches on aeroelastic about tow-steered variable-stiffness composites have received relatively little attention. Stodieck [10, 11] studied aeroelastic behavior of rectangular wing. Tow-steered composites were used to tailor the aeroelastic behavior, and they showed a good performance over traditional unidirectional composite laminates. Stodieck [12] assessed the potential wing weight savings of a full-size aeroelastically tailored wing. It turned out that the mass reductions of optimized tow-steered laminates were much better than optimized straight-fiber composites. Haddadpour [13] investigated the aeroelastic design of composite wings with curvilinear fiber which were modeled as thin-walled beams, and optimized the wing with a linear spanwise variation of the fiber orientation to maximize aeroelatic instability speed purpose. It indicated that aeroelastic stability of variable-stiffness wings were improved compared with conventional, constant-stiffness ones. Stanford [14] studied aeroelastic tailoring of a cantilevered flat plate in low-speed flow, located the Pareto front between static aeroelastic stresses and dynamic flutter boundaries using a genetic algorithm and compared curvilinear fiber steering with straight ones.

Researches on panel flutter were first launched in 1950s. Theoretical analyses were firstly carried out by Miles [15] and Jordan [16]. Xue [17, 18] analyzed nonlinear flutter of two and three dimensional composite panels with arbitrary temperatures. Kouchakzadeh[19] examined nonlinear aeroelasticity of a general laminated composite plate in supersonic air flow, and studied the effects of in-plane force, static pressure differential, fiber orientation and aerodynamic damping on the nonlinear aeroelastic behavior of the plate. Culler [20] analyzed panel flutter of von Karman panel considering fluid-thermal-structural coupling. It indicated that including elastic deformations in the aerodynamic heating computations resulted in nonuniform heat flux, and it also impacts flutter boundary predictions and nonlinear flutter response.

This paper focuses on aeroelasticity of tow-steered variable-stiffness composite laminates. The natural dynamic and flutter under supersonic flow were studied. The von-Karman large deflection strain-displacement relationship and the piston theory were employed to account for the structural and aerodynamic nonlinearities, respectively. The natural frequency and the flutter of composite laminates were computed, and the impacts of various parameters, such as temperature, boundary condition and ply orientations upon the thermal flutter performance of variable-stiffness composite laminates were quantified.

REFERENCE PATH OF CURVILINEAR FIBER

The ply orientations of each ply in tow-steered variable-stiffness composite laminates vary as the position varying continuously. It is assumed that fiber orientation within a layer varies linearly from the center of a rectangular plate, with length a and width b, which locates the origin of the Cartesian coordinate system, Fig. 3. The notation of $<T_0|T_1>$ defines a fiber orientation like Eq.(1).

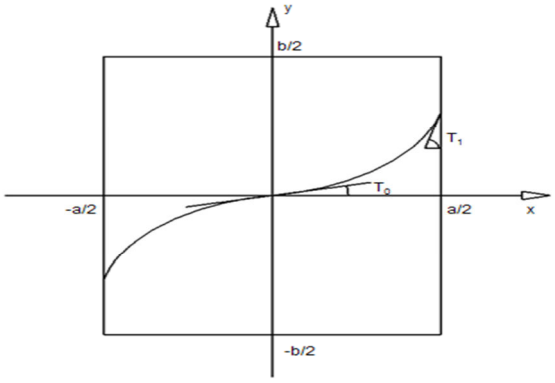


Fig. 3 - Definition of curved fiber

$$\theta(x) = \frac{2(T_1 - T_0)}{a}|x| + T_0 \tag{1}$$

where T_0 and T_1 are the angle between the fiber and the x axis in the center and vertical edges of the plate, respectively. The reference path of curvilinear fiber is

$$y(x) = \frac{ax}{2(T_1 - T_0)|x|}\left\{\ln[\cos T_0] - \ln\left[\cos\left(\frac{2(T_1 - T_0)}{a}|x| + T_0\right)\right]\right\} \tag{2}$$

1. THERMAL FLUTTER EQUATION OF COMPOSITE LAMINATES

Based on von-Karman's assumption, the large deflection strain-displacement relationship is

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix} + z\begin{Bmatrix} \dfrac{\partial \theta_y}{\partial x} \\ \dfrac{\partial \theta_x}{\partial y} \\ \dfrac{\partial \theta_y}{\partial y} + \dfrac{\partial \theta_x}{\partial x} \end{Bmatrix} \tag{3}$$

$$= \{\varepsilon_m\} + \{\varepsilon_{mb}\} + z\{\kappa\}$$

$$\{\gamma\} = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w}{\partial y} \\ \dfrac{\partial w}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} \tag{4}$$

where u_0, v_0, w_0 are the displacements of the mid-plane in the x, y and z directions, respectively; θ_x and θ_y are rotation in the yz and xz planes respectively.

The stress-strains relations with the consideration of thermal effect are as follows:

$$\begin{aligned} \{\sigma\} &= [\overline{Q}]\{\varepsilon_E\} = [\overline{Q}]\left(\{\varepsilon\} - \{\overline{\alpha}\}\Delta T\right) \\ \{\tau\} &= [\overline{Q_S}]\{\gamma\} \end{aligned} \tag{5}$$

where $\left[\overline{Q}\right]$ and $\left[\overline{Q_S}\right]$ are the reduced stiffness matrix; ΔT is temperature change, and $\left\{\overline{\alpha}\right\}=\left[T_\varepsilon\right]\left\{\alpha\right\}$ is coefficient of thermal expansion.

For $\left\{\varepsilon^0\right\}=\left\{\varepsilon_m\right\}+\left\{\varepsilon_{0_{mb}}\right\}$, the constitutive relations are

$$\begin{aligned}&\begin{Bmatrix}\{N\}\\ \{M\}\end{Bmatrix}=\begin{bmatrix}[A] & [B]\\ [B] & [D]\end{bmatrix}\begin{Bmatrix}\{\varepsilon^0\}\\ \{\kappa\}\end{Bmatrix}-\begin{Bmatrix}\{N_{\Delta T}\}\\ \{M_{\Delta T}\}\end{Bmatrix}\\ &\{R\}=\left[A_S\right]\{\gamma\}\end{aligned} \tag{6}$$

$$\begin{aligned}&\left([A],\ [B],\ [D]\right)=\sum_{k=1}^{N_L}\int_{Z_{k-1}}^{Z_k}\left[\overline{Q}\right]\left(1,\ z,\ z^2\right)dz\\ &\left[A_S\right]=\sum_{k=1}^{N_L}\int_{Z_{k-1}}^{Z_k}\left[\overline{Q_S}\right]dz\\ &\left(\{N_{\Delta T}\},\ \{M_{\Delta T}\}\right)=\sum_{k=1}^{N_L}\int_{Z_{k-1}}^{Z_k}\left[\overline{Q}\right]\left\{\overline{\alpha}\right\}\Delta\mathrm{T}\left(1,\ z\right)dz\end{aligned} \tag{7}$$

where [A], [B] and [D] are stiffness matrices; $[A_S]$ is shear stiffness matrix; $\{N_{\Delta T}\}$ and $\{M_{\Delta T}\}$ are thermal in-plane load and moment vectors; $\{N\},\{M\},\{R\}$ are in-plane load, moment, and transverse load vectors, respectively.

The aerodynamic load is assumed to be that of two-dimensional quasi-steady supersonic piston theory

$$p-p_\infty=-\frac{2q}{\beta}\left\{\frac{\partial w}{\partial x}\right\}-\frac{2q}{\beta}\left(\frac{Ma^2-2}{Ma^2-1}\frac{1}{v_\infty}\left\{\frac{\partial w}{\partial t}\right\}\right) \tag{8}$$

where $\beta=\sqrt{Ma^2-1}$; $q=\dfrac{\rho_a v_\infty^2}{2}$ is the dynamic pressure; ρ_a is the air density; v_∞ is the air-stream velocity and Ma is Mach number.

Based on principle of virtual work, the equation of motion can be obtained

$$\begin{aligned}&[\mathrm{M}]\{\ddot{\mathrm{w}}\}+[C]\{\dot{w}\}+([K_a]\\ &+[K_L]-[K_{\Delta T}]+\frac{1}{2}[N_1]+\frac{1}{3}[N_2])\{w\}=\{P_{\Delta T}\}\end{aligned} \tag{9}$$

where $[M]$ is mass matrix; $[C]$ is aerodynamic damping matrix; $[K_a]$ is aerodynamic stiffness matrix; $[K_L]$ is linear elastic stiffness matrix; $[K_{\Delta T}]$ is thermal stiffness matrix; $[N_1]$ and $[N_2]$ are the first and second order nonlinear stiffness matrices due to large deflection, respectively. $\{w\}$ is the displacement vector; $\{P_{\Delta T}\}$ is thermal load vector.

Generally, thermal panel flutter analysis is based on the following three assumptions: 1) the static deformation of the panel does not affect temperature distributions; 2) the response time of temperature field variation is much less than that of flutter response, hence, the temperature field is considered as constant during the thermal flutter analysis; 3) the effect of temperature on mechanical properties of materials is out of consideration.

Based on these assumptions, thermal panel flutter can be proceeded as follows: for purpose of introducing thermal stiffness produced by thermal stress into equivalent stiffness matrix, thermal loads are included in finite element analysis; and a time domain approach is used to perform flutter analysis of heated plates under supersonic flow afterwards.

THERMAL FLUTTER ANALYSIS OF COMPOSITE LAMINATES

Material properties of composite laminate in this study are given in Table 1. The dimensions of the plate are given as: b*a=0.3m *0.2m. The plate is comprised of 8 layers, and lamination scheme is $[0/90/<T0|T1>/<-T0|-T1>]_s$. The composite plate is clamped at all edges.

The non-dimensional aerodynamic pressure λ and frequency ω^* are introduced as follows [21]:

$$\lambda = \frac{2qa^3}{D\sqrt{Ma^2 - 1}}, \omega^* = \omega\sqrt{a^4 \rho t / D} \tag{10}$$

Where D is rigidity of panel ($D = E_2 t^3$).

Table 1 - Material properties of composite laminate

E1	155GPa	**α1**	-0.07×10-6℃-1
E2	8.07GPa	**α2**	30.1×10-6℃-1
G12	4.55GPa	**ρ**	1550kg/m3
μ12	0.22	**Ply thickness**	0.15mm

NATURAL MODES OF VIBRATION

In this section natural frequencies are given because of the close relationship between dynamics and flutter characteristics. The lamination scheme is $[0/90/<T0|T1>/<-T0|-T1>]_s$, and two kinds of fiber orientation are considered as examples: 1) T_1=45 is constant, T_0 is in the ascending order and the value of grads is constantly 5 from 0 to 45 degree; 2) T_0=45 is constant, T_1 is in the ascending order and the value of grads is constantly 5 from 45 to 90 degree. The effects of fiber orientations on natural frequency are shown in Table 2-3.

Table 2 - Natural frequency of composite laminates with various fiber orientation angles at the root (T_0)

T1=45	**T0=0**	**T0=5**	**T0=10**	**T0=15**	**T0=20**	**T0=25**	**T0=30**	**T0=35**	**T0=40**	**T0=45**
Mode 1	269.81	269.62	269.34	268.98	268.56	268.09	267.59	267.08	266.57	266.09
Mode 2	348.24	348.80	349.43	350.12	350.89	351.71	352.59	353.52	354.50	355.50
Mode 3	517.81	519.01	520.46	522.18	524.17	526.45	529.02	531.89	535.04	538.43
Mode 4	710.86	709.05	707.06	704.94	702.71	700.41	698.08	695.75	693.43	691.18
Mode 5	756.53	755.88	755.32	754.82	754.33	753.79	753.13	752.30	751.23	749.92
Mode 6	775.36	776.95	778.94	781.39	784.34	787.85	791.95	796.67	801.99	807.85

Table 3 - Natural frequency of composite laminates with various fiber orientation angles at the tip (T_1)

T0=45	T1=45	T1=50	T1=55	T1=60	T1=65	T1=70	T1=75	T1=80	T1=85	T1=90
Mode 1	266.09	265.15	264.33	263.63	263.05	262.58	262.20	261.89	261.63	261.40
Mode 2	355.50	355.20	354.91	354.65	354.41	354.20	354.01	353.85	353.72	353.61
Mode 3	538.43	539.14	539.81	540.45	541.07	541.67	542.25	542.83	543.42	544.03
Mode 4	691.18	688.33	685.80	683.62	681.79	680.26	679.00	677.96	677.09	676.36
Mode 5	749.92	747.60	745.44	743.46	741.67	740.06	738.63	737.34	736.21	735.22
Mode 6	807.85	809.59	811.29	812.96	814.61	816.24	817.85	819.46	821.08	822.71

The effect of different T_0 or T_1 on natural frequency of tow-steered variable stiffness composite laminates can be seen from Table 2-3. As T_1=45 is constant and T_0 is in the ascending order, the first, fourth and fifth order natural frequency is descending while the second, third and six order natural frequency is ascending. As T_0=45 is constant and T_1 is in the ascending order, the first, second, fourth and fifth order natural frequency is descending while the third and six order natural frequency is ascending. The results indicate that the natural modes of vibration of variable-stiffness composite laminates can be changed by varying fiber orientations.

MODE COALESCENCE IN TOW-STEERED COMPOSITE LAMINATES

In this section mode coalescence in tow-steered composite laminate with different ply orientations is given. Fiber orientation angles are as follows: $[0/90/<0|15>/<0|-15>]_s$, $[0/90/<0|30>/<0|-30>]_s$, $[0/90/<0|45>/<0|-45>]_s$, $[0/90/<15|45>/<-15|-45>]_s$, $[0/90/<30|45>/<-30|-45>]_s$, $[0/90/45/-45]_s$. Fig.4 shows the first and the third mode coalesce both with straight fibers and curvilinear fibers. The mode coalescence changes little as the change of fiber orientation angles in tow-steered composite laminates as shown in fig.4.

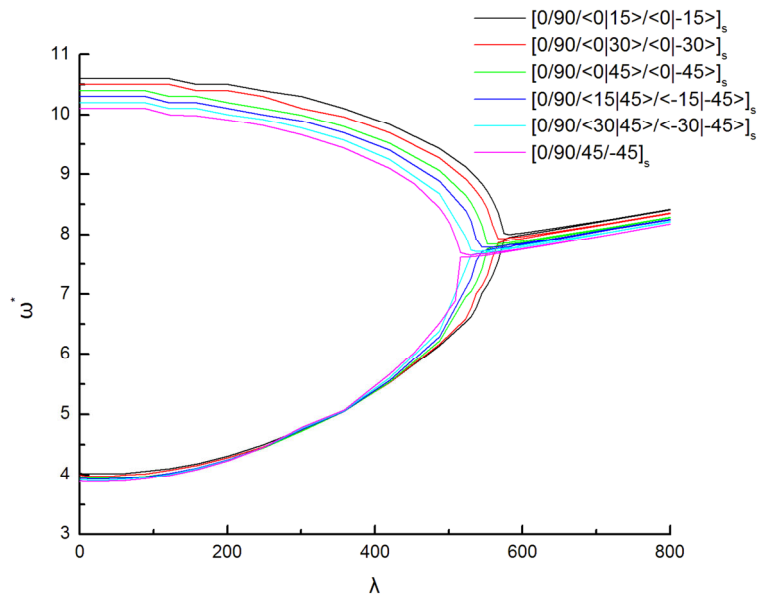


Fig. 4 - Mode coalescence in flutter analysis of composite laminates with curvilinear fibers

3.3 EFFECT OF TEMPERATURE

Effect of temperature on the flutter dynamic pressure with both curvilinear fibers $[0/90/<30|45>/<-30|-45>]_s$ and straight fibers $[0/90/\pm45]_s$ is presented in Fig.5. Accordingly, the effect of temperature on non-dimensional dynamic pressure is significant both in composite laminates with curvilinear fibers and straight fibers. The increasing thermal loads result in the direct decrease of non-dimensional dynamic pressure as shown in Fig. 5.

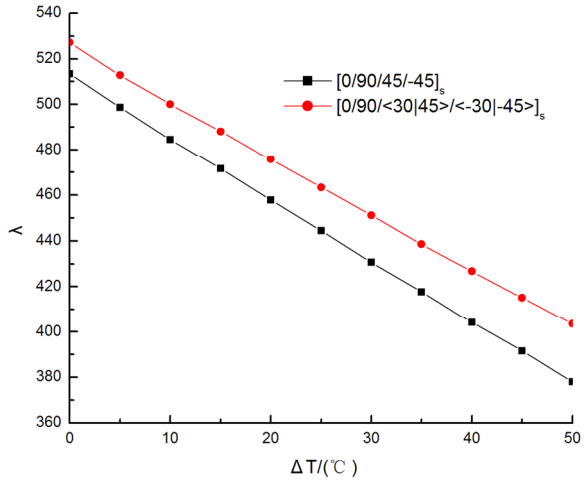


Fig. 5 - Effect of temperature on non-dimensional flutter dynamic pressure

3.4 EFFECT OF BOUNDARY CONDITION

The effect of boundary condition on flutter dynamic pressure is investigated in this example. Fig. 6 shows the non-dimensional flutter dynamic pressure for two different boundary conditions: clamped and simply supported at all edges. For all fiber orientations, the boundary conditions result in significant changes of flutter boundary. The composite laminate with more restrained boundary condition is more stable as expected. The non-dimensional flutter dynamic pressure of clamped plate is much higher than that of simply supported plate as shown in Fig. 6.

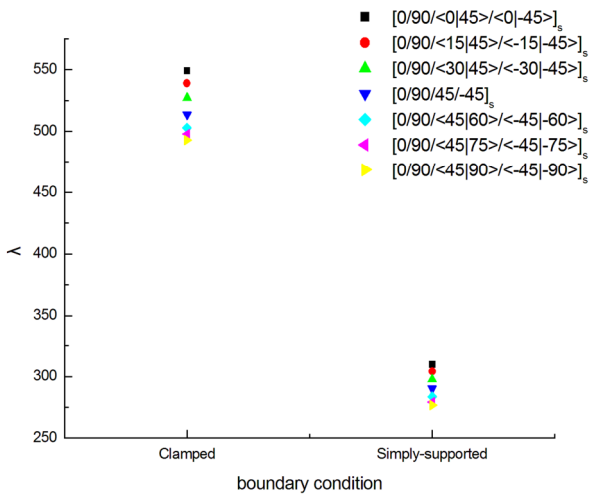
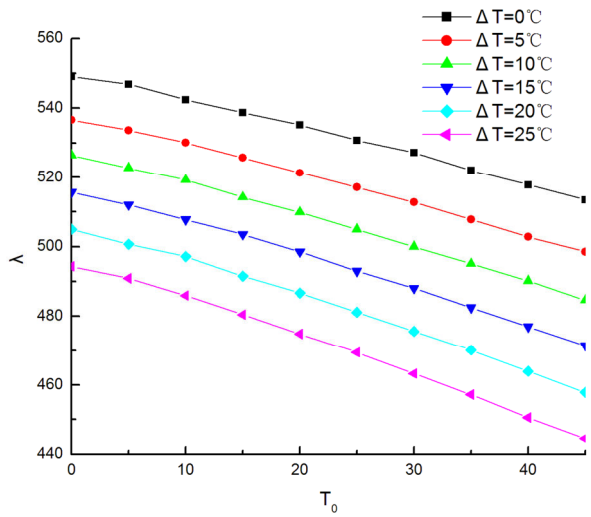


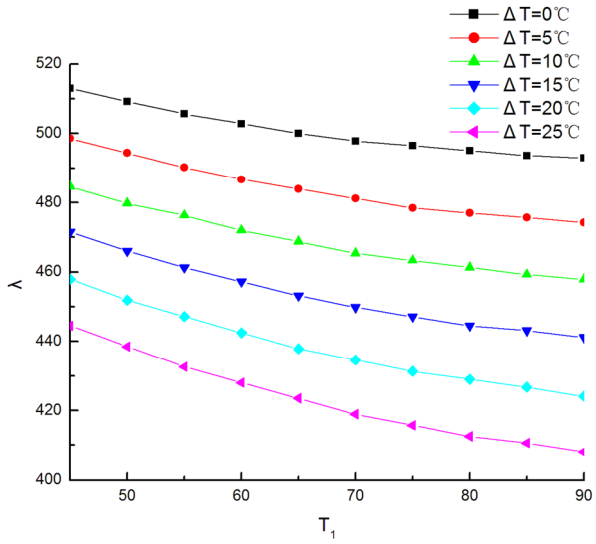
Fig. 6 - Effect of boundary condition on non-dimensional flutter dynamic pressure

3.5 EFFECT OF PLY ORIENTATION

The effect of ply orientation on flutter dynamic pressure is studied in this section. The lamination scheme of the plate is [0/90/<T0|T1>/<-T0|-T1>]$_s$ and two kinds of fiber orientation are considered as examples: 1) T_1=45 is constant, T_0 is in the ascending order and the value of grads is constantly 5 from 0 to 45 degree; 2) T_0=45 is constant, T_1 is in the ascending order and the value of grads is constantly 5 from 45 to 90 degree.



(a) Non-dimensional flutter dynamic pressure with various fiber orientation angles at the root (T_0)



(b) Non-dimensional flutter dynamic pressure with various fiber orientation angles at the tip (T_1)

Fig. 7 - Effect of fiber orientation angle on non-dimensional flutter dynamic pressure of composite laminates with curvilinear fibers

Effect of fiber orientation on non-dimensional dynamic pressure of curvilinear fiber composite laminates with various fiber orientation angles is shown in Fig.7. As T_1=45 is constant and T_0 is in the ascending order, the non-dimensional flutter dynamic pressure is descending. As T_0=45 is constant and T_1 is in the ascending order, the non-dimensional flutter dynamic pressure is descending. The results indicate that the non-dimensional flutter dynamic pressure of variable-stiffness composite laminates can be changed by varying fiber orientations.

RESULTS AND CONCLUSIONS

A supersonic flutter analysis of variable stiffness composite laminated plates with curvilinear fibers was performed. The von-Karman large deflection strain-displacement relationship and the piston theory were employed to account for the structural and aerodynamic nonlinearities, respectively. The effects of using variable-stiffness composite laminates instead of traditional composite laminates on natural modes and panel flutter were investigated and impacts of temperature, boundary conditions and ply orientations of curvilinear fibers on flutter were discussed. The following points can be concluded:

(1) The natural frequency of tow-steered composite laminates varies as the T_0 or T_1 varying. The natural modes of vibration of variable-stiffness composite laminates can be changed by varying fiber orientations.

(2) The effect of fiber orientation angles in tow-steered composite laminates on mode coalescence is insignificant.

(3) The increasing thermal loads result in the direct decrease of non-dimensional dynamic pressure both in composite laminates with curvilinear fibers and straight fibers.

(4) For all fiber orientations, the non-dimensional flutter dynamic pressure of clamped plate is much higher than that of simply supported plate.

(5) The flutter dynamic pressure of tow-steered composite laminates varies as the T_0 or T_1 varying. The non-dimensional flutter dynamic pressure of variable-stiffness composite laminates can be changed by varying fiber orientations.

The results show that thermal loads and boundary conditions result in significant influence on flutter boundary under supersonic flow. The designability of curvilinear-fiber composites is better than that of straight-fiber panels. The flutter speed of variable-stiffness composite panels can be modified by varying the fiber orientation angle.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the funding by the International Exchange Program for Graduate Students, Tongji University

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