

## BANDGAP DETERMINATION IN STRUCTURES COMPOSED BY ORDERED BEAMS CLAMPED TO A PLATE SUBSTRATE

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### ABSTRACT

This work analyses the wave propagation in a kind of phononic crystals composed by vertical beams distributed over a plate substrate. Three different configurations for the arrangement of the beams have been analysed: square, triangular and hexagonal. A dimensional analysis of the problem points out the presence of three dimensionless groups of parameters which control the response of the system. Illustrative examples of the effect of these dimensionless groups are presented.

**Keywords:** phononic crystal, lattice structure, Bloch's theorem, dimensional analysis.

### INTRODUCTION

It is well known that some periodic arrangements of structural elements are able to promote directivity in wave propagation or even to cancel it at certain ranges of frequencies (bandgap). A system with these properties, composed by ZnO nanocrystals clamped to a sapphire substrate (see Fig. 1) was analytically analysed by Eremeyev et al. (2007). It was found that the eigenfrequencies of the system were a combination of the eigenfrequencies of a single nanocrystal and those of the substrate. The above system has potential applications related to nanosensors and gas sensing. Therefore, the control of the location and width of the bandgaps is a key design issue of the phononic crystal structures to achieve a desired performance related to a specific application. In this respect a complete knowledge of the parameters affecting its dynamical behaviour is required. The dynamic behaviour of the system has been analysed considering it as a lattice structure, permitting the use of the Bloch's methodology. Slender beams and small substrate's thickness-to-beam's length ratio has been considered.

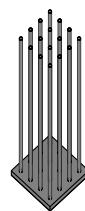


Fig. 1 - Sketch of the studied phononic crystal

In this work, we present a complete dimensional analysis based on the problem formulation of Eremeyev et al. (2010) to uncover the dimensionless groups affecting the dynamical behaviour of the system. Moreover some illustrative examples of the influence of the different groups are presented.

## RESULTS AND CONCLUSIONS

The dimensional analysis of the problem formulated by Eremeyev et al. (2010) reveals three dimensionless groups of variables governing the dynamic behaviour of the lattice (see Table 1). To show illustrative examples of the influence of these groups, a FE model of the unit cell of the square configuration lattice has been analysed. Taking advantage of the Bloch's theorem, the numerical results are shown in Fig. 2 to 4. The band structure, wave number-dimensionless frequency plot, shown in Fig. 2 gives information about the location and width of the bandgap. The directionality of the wave propagation is presented in Fig. 3.

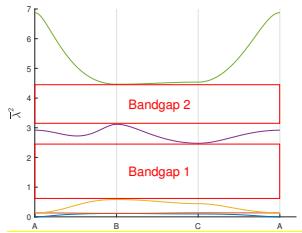


Fig. 2 - Band structure

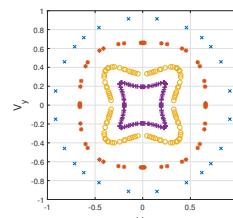


Fig. 3 - Group velocity

Fig. 4 shows the evolution of the mean value and width of the first bandgap for a range of  $A_i$ , which are the dimensionless groups involving geometrical and mechanical properties of the phononic crystal.

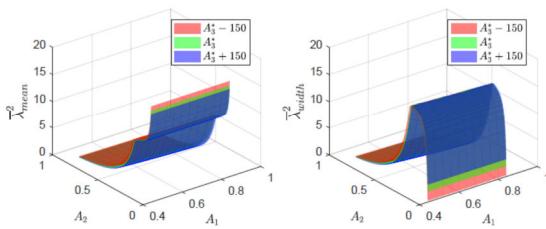


Fig. 4 - Evolution of first bandgap

Table 1 - Dimensionless Groups

$A_1$	$A_2$	$A_3$
$\frac{\rho}{\rho_*}/l$	$a/l$	$C/l/D$

$\rho_*$  ≡ Beam's linear density  
 $\rho$  ≡ Substrate's surface density  
 $l$  ≡ Beam's length  
 $a$  ≡ Substrate's characteristic length  
 $C$  ≡ Beam's stiffness  
 $D$  ≡ Substrate's stiffness

The results show that the most relevant group is  $A_2$ , which corresponds to the ratio characteristic length of unit cell - length of crystal. As  $A_2$  approximates to zero, the width of the bandgap reduces to zero. This is because the limit case  $A_2 \rightarrow 0$  is equivalent to a dense distribution of beams, which prevents the existence of bandgaps. The other configurations studied (triangular and hexagonal) lead to the same qualitative results.

## ACKNOWLEDGMENTS

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