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# VISCO-HYPERELASTIC MATERIALS CHARACTERIZATION BY RHEOLOGICAL MODELS FITTED WITH GENETIC ALGORITHMS

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## ABSTRACT

This work was focused on the characterization of a hyperelastic material to further study a complex case of application. We measured in a simple tensile test a rubber specimen and used the data to define a three-parameter Mooney-Rivlin model for hyperelastic materials. We developed an evolutionary algorithm able to search for the best combination of parameter values that fit the data. The obtained model was implemented into a finite element model to study the characteristics of a complex-shape commercial seal.

***Keywords:*** hyperelasticity, FEM analysis, non-linear analysis, genetic algorithms.

## INTRODUCTION

Hyperelastic materials are widely used in industry and deployed in most fields of production. Despite the large acquired experience in using this type of materials, there are difficulties on applying reliable numerical models to predict their behaviour. Underneath this limitation there are the limits in obtaining the accurate mathematical characterization of the hyperelastic materials. As a straight consequence, the virtual finite element (FE) and other models using hyperelastic materials are quite poor to produce robust and reliable results. The available models for hyperelastic materials require different constants that are not typically found in literature. Additionally those values can be more or less difficult to obtain for any specific case.

The first challenge of this work was to produce and validate an accurate model for a specific material using only simple tests performed with rubber specimens. We performed a uniaxial tensile test to obtain the basic material characteristics (Saso, 2008). The data were used to define a three-parameter Mooney-Rivlin model for hyperelastic materials (Martins, 2006). To achieve the best parameters we implemented a genetic algorithm able to search for the right parameter combination to optimize the model and data correspondence.

Once the characterization was successful the Mooney-Rivlin model was implemented within a FE model to study a complex problem of application of the material to a practical case. The different solutions obtained due to the complexity of the models and the limitations of the available data were finally discussed.

## AN OVERVIEW TO THE MOONEY-RIVLIN MODEL

In classical linear elasticity theory used to model the behaviour of materials at small strains, the constitutive relations between stress and strain remains constant and linear. This assumption was probed successful to describe typical metal materials. However, the observed behaviour of rubbers, foams and biological tissues shows a wider complexity in the relationship between stress and strain. It is the case of elastomers, whose stress-strain curves are strongly non-linear. For these cases the classical elasticity theory is not adequate and a better and appropriate model must be applied.

To model the hyperelastic behaviour it is applied the strain energy density function. It corresponds to the area below the stress-strain curve. In Fig. 1. we plot a typical stress-strain curve for an elastomer. The area beneath the curve is the energy density function.

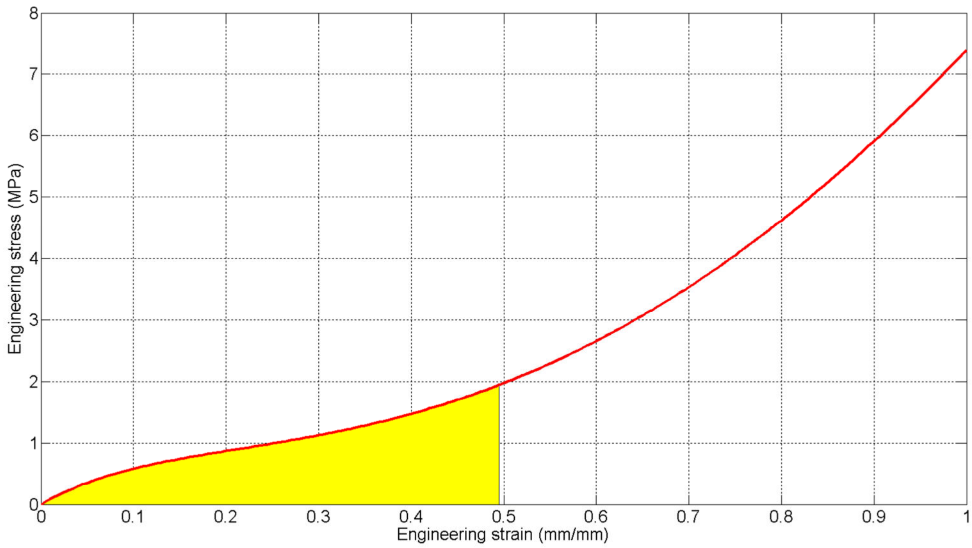


Fig. 1 - Stress-strain curve (solid line) measured for an elastomer. The area beneath the curve (yellow), its integral, defines the strain energy density function.

The models for hyperelastic materials are based in a description or formula to reproduce the strain energy density function. The density function allows for obtaining the constitutive law for each elastomer material by using the large-deformation elasticity theory (Ali, 2010).

The definition of the strain energy density function, or elastic potential, is a function of the invariants of the Cauchy tensor. There exist a large number of hyperelastic models. Each model includes a different number of material constants. The selection may depend on the interest to take into account the different invariants of the Cauchy tensor and the quantity and quality of the data available for each particular material. For rubbers, the most widely deployed models are the so called Mooney-Rivlin models. Among this family, we selected the three-parameter model. It is rather simple, and it takes into account the two first invariants of the Cauchy tensor, as well as the product between them. Models with more constants can be used, however it depends on the available data and their quality to obtain reliable values of the necessary constants and parameters.

Since we aimed first to characterize our rubber material under test, we must use the hyperelastic equation definition according to the descriptions available in the FE software (ANSYS®). Therefore, the strain energy density function was defined as detailed in the user's manual. Once the parameters were obtained, they could be implemented directly into the software. The density (W) was given by:

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + C_{11}(\bar{I}_1 - 3)(\bar{I}_2 - 3) + \frac{1}{d}(J-1)^2$$

Where $I_1$ and $I_2$ are the two first invariants of the Cauchy tensor, and $d$ is the compressibility parameter which we assumed to be zero (incompressible material). $C_{10}$, $C_{01}$ and $C_{11}$ are the constants that the search algorithm must find.

We measured some specimens made of rubber under tensile tests. By using this information we aimed to extract the set of Mooney-Rivlin parameters that best fitted the measured data. For doing so, we implemented a genetic algorithm within MATLAB®.

## EVOLUTIONARY ALGORITHM

The searching for the best parameters for the Mooney-Rivlin model to reproduce the behaviour of a rubber specimen is the most important feature presented in this work. The problem of fitting the real test data was treated as an optimization problem (Tekin, 2004; Karr, 1995), and we used an evolutionary algorithm for the search. The evolutionary computation proposal were inspired in the theory of the evolution of the species (Zitzler, 2002). The individuals better fitted to their environment are able to survive and the genes be transferred to the future generations, contrary to worst fitted. During the evolution process some individuals can suffer mutations generating individuals with different genes, which again can be or not an advantage.

To solve the problem of fitting the measured data, a set of individuals (particular solutions) belonging to the same generation are evaluated. These individuals are characterized by their values for the constants $C_{10}$, $C_{01}$ and $C_{11}$ of our parameterization of the Mooney-Rivlin description. The values of these parameters are within a certain parametric domain space, defining boundary values of the parameters, and within which the algorithm searches the solutions. Each individual reaches a different fitness level with respect to the test data.

For the algorithm it is necessary to provide with an initialization protocol to generate the first generation. During the process, each generation is evaluated, and a selection of genes is done giving preference to individuals with better scores. To transfer their genes, a crossover operator is designed in order to generate new individuals by combining the genes of their progenitors. With low probability, a mutation of one gene may occur. If advantageous for the fitting, it will be transmitted to new generations.

The algorithm evaluates the generations which step by step should produce individuals which are better fitted After enough generations, the algorithm will converge and the quality of the fitness will not increase, providing the best fitted values for the parameters.

## POPULATION INITIALIZATION

The algorithm produces the next step generation genes by using the information of the actual generation, which inherits the genes from previous generations. There must be an initialization phase to create the genes of the first step, whose quality may compromise the convergence of the algorithm.

We used the Latin Hypercube Sampling (LHS) method (Stain, 1987). This method generates a semi-random sample within the parametric domain, where two individuals do not share input parameters after a partition is done. In Fig.2 we show the partition of two possible parameters, and a random selection done with the not-sharing condition. This two-dimensional (square) method can be extended to any number of parameters (hypercube).

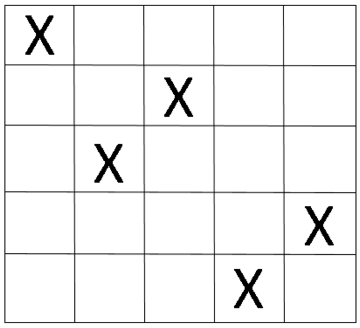


Fig. 2 - Latin Square Sampling graphic model. Two parameters (axis) are partitioned and selections (X) are performed randomly with the not-sharing condition.

## FITNESS EVALUATION AND SELECTION OPERATOR

For each generation the algorithm evaluates the fitness level of the individuals to the measured data, according to the Mooney-Rivlin model. The quality of the fitness was provided by the difference in respect to the measured data. We set values ranging from 0 to 100 according the absolute difference as in Fig.3. For tiny differences the values are close to 100, while increasing differences lead to a zero value fit.

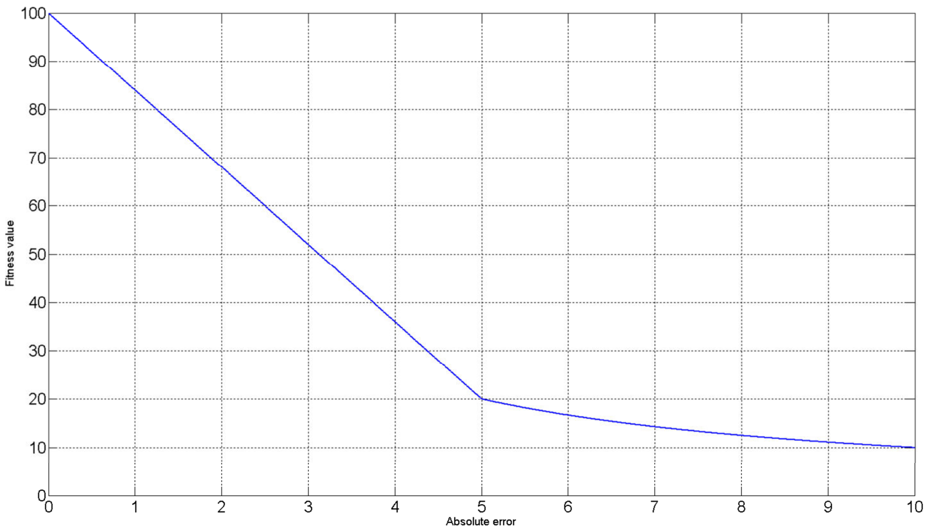


Fig. 3 - Function defined to provide with a fitness quality value according to the absolute difference between the evaluated model and the measured data. The closer the fit, the higher the value within a 0-to-100 range.

The new set of individuals of the next generation was created according a selection of genes based on the level of the fit quality to the data. There are many different ways to carry out the selection process. In our study we used together two methods for selecting the best-suited individuals.

We first used the 'tournament' selection. The individuals of each generation are ranked according their fit quality value and only the genes of the two top candidates were selected.

The second method was a Stochastic Universal Sampling (SUS) method (Baker, 1987). The population is sampled according to the quality fitness values. Therefore a degree of random freedom happens in the selection, while keeping the preference for the better scores. Following this method, we selected four individuals.

Finally we had six individuals selected. Additionally, we allowed that both the predecessors and the offspring competed in the same generation for the next-step genes selection.

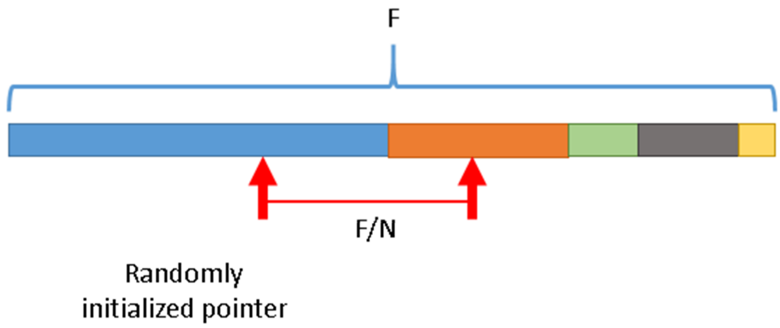


Fig. 4 - SUS graphic example

In Fig. 4 an example of the SUS method is shown. In this method, the colour bars in the figure represents the fitness of different individuals. In this figure, 'F' is the total fitness of all the individuals in the generation and 'N' the number of selected individuals. According to the total fitness of the generation and depending on the number of individuals selected, the distance between pointers can be computed. The first pointer is randomly initialized at any point, once initialized the first pointer, the remaining pointers positions are known and the selection could be carried out. The blue box represents the individual with the best fitness and, on the other hand, the yellow one represents the worst individual. So, the best-fitted individuals are more probably selected than the worst ones.

## CROSSOVER AND MUTATION OPERATORS

After the selection of individuals in a generation is carried out, the recombination of genes must be done. There are many possible options to generate new individuals using some kind of crossover. During the crossover process, two genes are recombined using parts from the parents. The way to combine the genes will make the difference. We used a simple arithmetic combination. In this way, a random number was used to weight the values of the genes from both progenitors. Here we provide an example where two progenitors with genes (22, 10, 4) and (12, 6, 9), corresponding to the parameters ($C_{10}$, $C_{01}$, $C_{11}$), are combined to produce the gens of two members of the offspring:

$$for\ the\ random\ \alpha = 0.1$$
$$\alpha(22, 10, 4) + (1 - \alpha)(12, 6, 9) = (13, 6.4, 8.5)$$
$$(1 - \alpha)(22, 10, 4) + \alpha(12, 6, 9) = (21, 9.6, 5.8)$$

The last operator used to simulate the evolution was the mutation. Mutation is a phenomena which implies the alteration of one or several genes in one individual, with low probability. However it allows that values previously discarded in the selections can happen and maybe add extra value to the fitness process.

In our case we set a 1% probability for mutation of a gen, which could take any value within its domain. The operator was applied always after the selection and crossover operators and it affected to only one of the individuals. If it happens, both individuals, with and without mutation, were kept in the generation. The size of the population is then variable from one generation to another due to mutations.

## RESULTS FOR THE GENETIC ALGORITHM

Once the algorithm is implemented according to all the previous considerations, it was used to fit the experimental curve. The curve included its own uncertainties, which together the parametric model used, prevents from reaching perfect (100%) fit values.

The algorithm was initialized with 70 individuals in a specific parametric domain, and run for 100 generations. We used the results of the first trial to select a narrower parametric domain and run again the algorithm to improve the results we present.

In Table.1 we provide the Mooney-Rivlin parameters obtained for the three top results. We can observe the parameter set for each one of the three candidates with the fitness obtained for each one of them. All three can reproduce properly the curve in the stress-strain domain we measured.

Table 1 - Genetic algorithm results

| Mooney-Rivlin parameter | C10 (MPa) | C01 (MPa) | C11 (MPa) | Fitness |
|---|---|---|---|---|
| Candidate 1 | -3.711803 | 5.169893 | 1.439667 | 50.460517 |
| Candidate 2 | -2.444245 | 3.735415 | 0.995856 | 42.925022 |
| Candidate 3 | -2.855020 | 4.194222 | 1.201878 | 40.508385 |

In Fig.5 we plot the measured data in a stress-strain curve, and the functions corresponding to the three selected parameter sets. The three sets provide with a good overall description of the data, being one or the other closer fitted at some specific strain range.

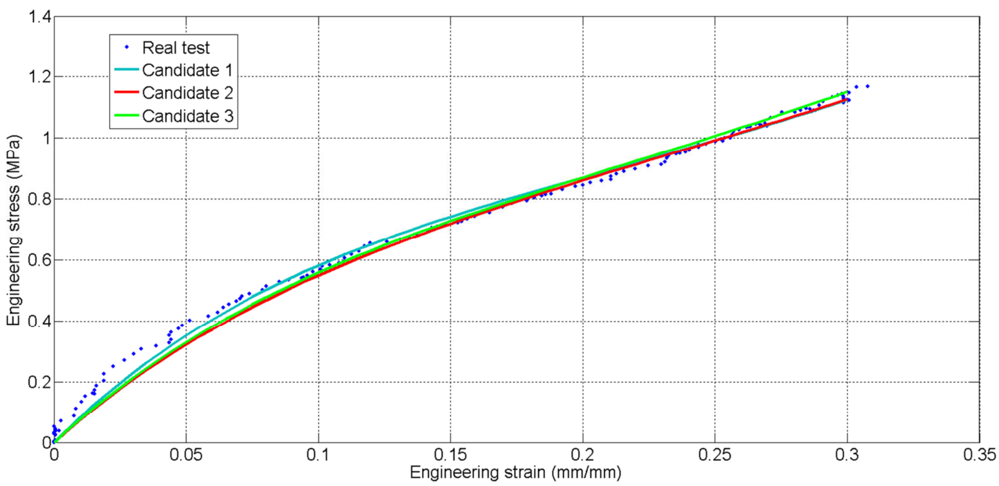


Fig. 5 - Stress-strain curve measured in one rubber specimen. The data (points) and the selected parametric models (lines) are plot in the range of measuremnets of a uniaxial tensile test.

It is important to highlight the fact that we performed a simple uniaxial tensile test, and we reached only about 30% level of the rubber strain limit. In Fig.6 we plot the curves corresponding to the three sets of parameters of the Mooney-Rivlin model we selected, now in an extended strain range. The three descriptions clearly differ at strain values above 0.5. It could be worth to study further the material in order to make an unambiguous model selection. However it is clear that within the strain ranges considered, any parametric set is completely representative of the data.

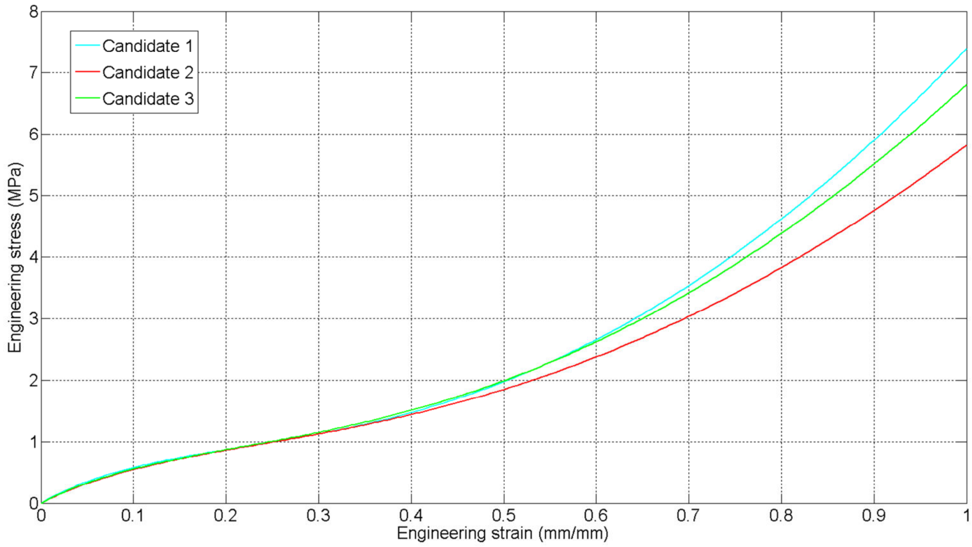


Fig. 6 - Stress-strain curves provided by the three parametric set selected in the study, and corresponding to a uniaxial tensile test. The strain range is extended till 100%.

## FEM ANALYS OF A COMPLEX APPLICATION

We were interested in analysing a case with a complex geometry. We wanted to study the behaviour of a window seal applied to a commercial automobile. In Fig.7 we plot the profile of the seal, reconstructed from images of the actual seal part.

The algorithm we used to define the Mooney-Rivlin parameters of the materials, provided directly with the elastic potential as implemented in the ANSYS® software. The 2D mesh used to discretize the seal had 18846 elements, including both tri- and quad-type linear elements, see Fig.7.

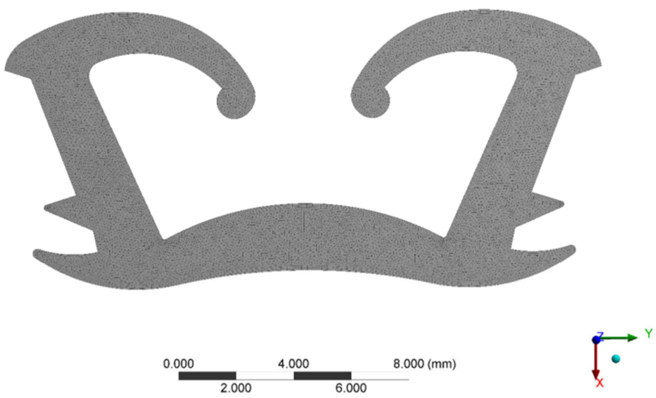


Fig. 7 - Mesh of the model used for the FEM analyses

We performed a 2D analysis to study a complex load state, corresponding to the case in which the rubber lips surround the window glass layer. We used the plane strain hypothesis appropriate to the structure cases with two key dimensions, as we have. In Fig.8, the boundary conditions are displayed.

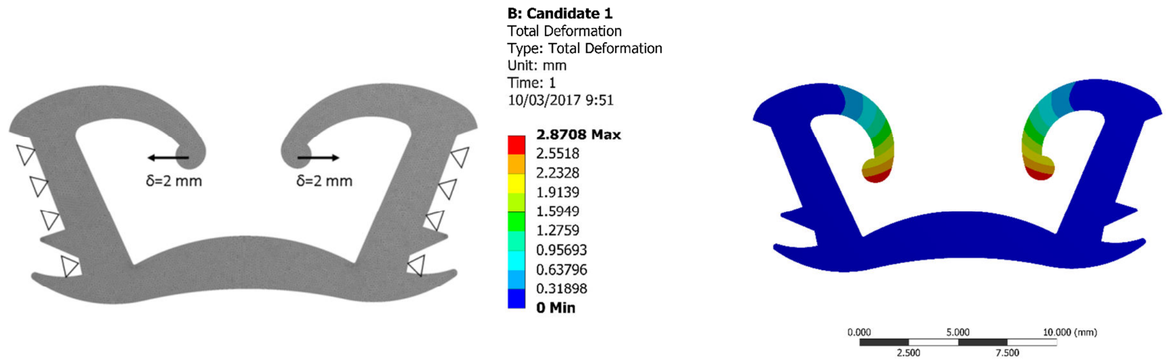


Fig. 8 - (Left) Boundary conditions used in the FE model studied. The lips of the seal in contact with the window layer are displaced (2mm) inwards. The seal is fit inside the window rail, the rail providing a rigid contact at the outer flat surfaces. (Right) Deformed shape of the seal under such boundary conditions

In Fig.8 we plot the result obtained for the deformation of the model under the specified conditions, and corresponding to the maximum considered deformation of the lips. The plot corresponds to the model implemented with the rubber parameters of case-1. All three parameter sets provide a rather similar result, since the calculation was performed with the deformation condition.

In Fig. 9 we plot the reaction force at the seal lips, as a function of the displacement at the lips. The results are for the three different sets of parameters obtained. The higher displacements increase the strain and then the different parametric sets produce results with different rigidities.

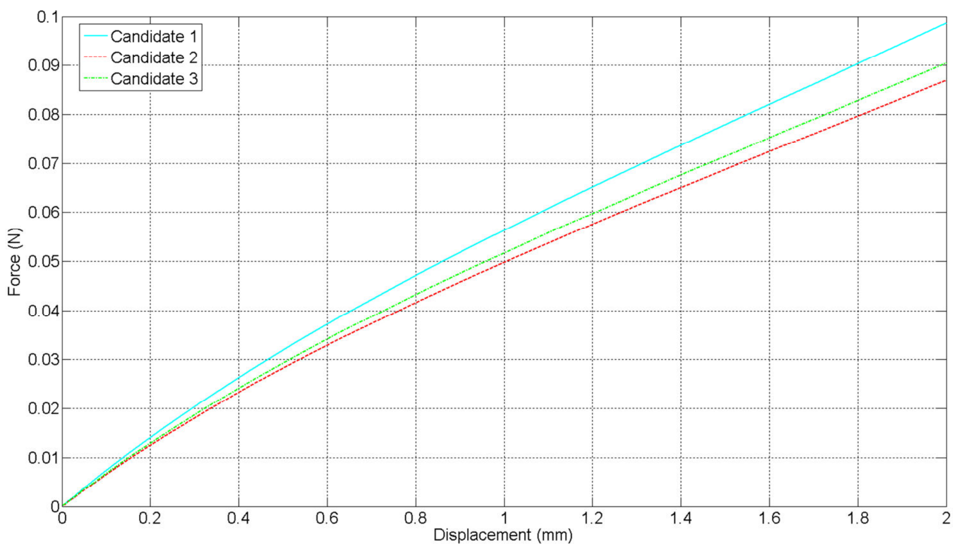


Fig. 9 - Reaction force values at the lips as function of the displacement. The three curves corresponds to the three parametric set studied. The values provided by the models differ up to 10%.

In Fig.10 we plot the first and second invariants of the stress tensor, as function of the displacement of the seal lips. In Fig.11 we show the first principal stress distributions as contours on the seal for the displacement case of 2 mm. The plots correspond to the values of the three parametric sets considered. In all cases we can observe that the different parametric sets provide with different solutions that slowly diverge as the rubber strain increases, as we expected from the behaviour we presented in Fig.6.

In the following figures, all three candidates implemented in the FE code are shown. The candidates follow a three-parameter Mooney-Rivlin model, so although the specific values differ one from one curve from another when high deformation levels are reached, the form of the different curves is very similar. So, as the used model is oriented to reproduce the behaviour of the rubber-like materials, all three candidates generate a realistic response although the particular values differs up to 13%. It means, assuming the implicit error any of the candidates could be used.

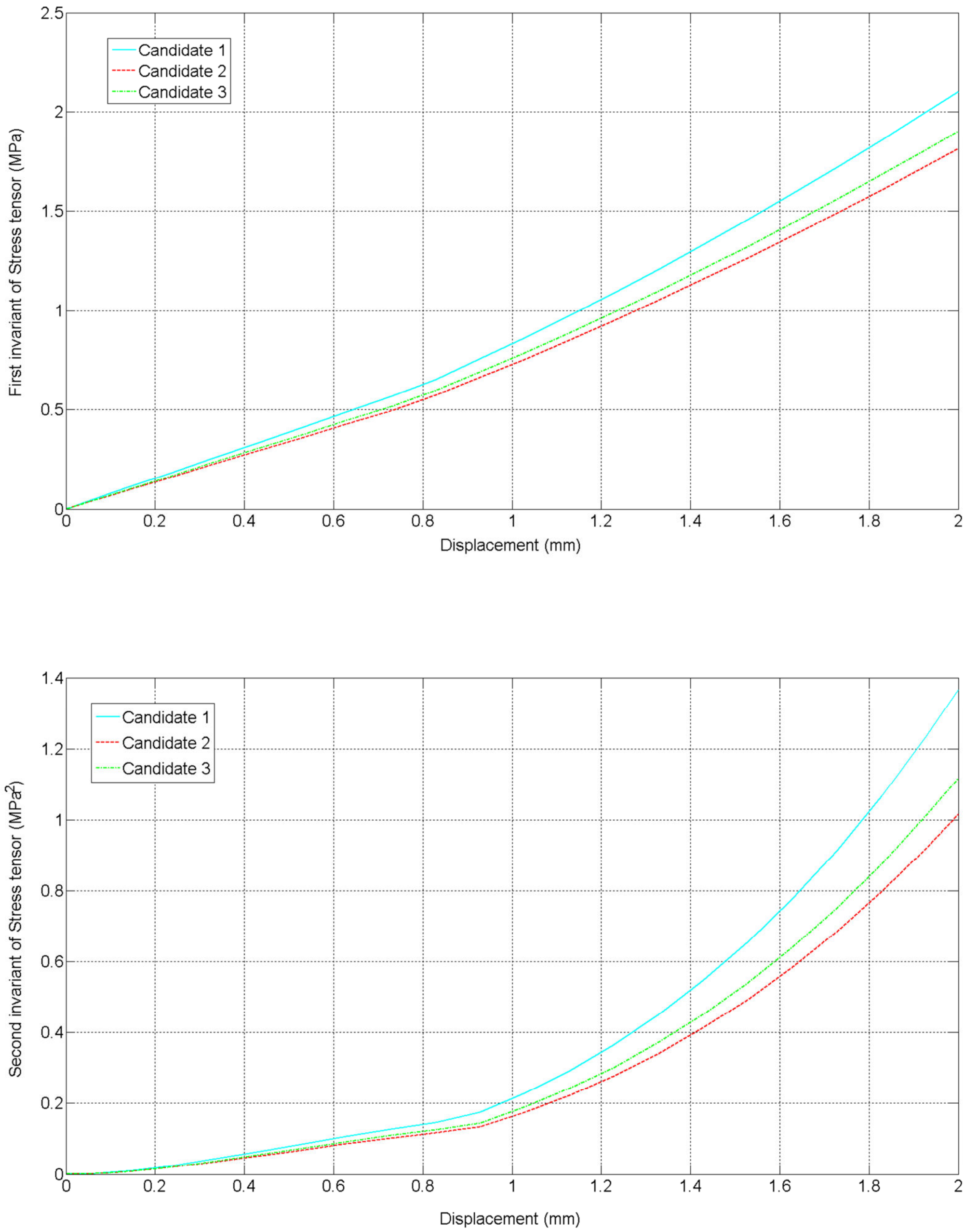


Fig. 10 - First (upper plot) and second (lower plot) invariants of the stress tensor, as a function of the displacement of the seal lips. The curves correspond to the three parametric set obtained in the study.

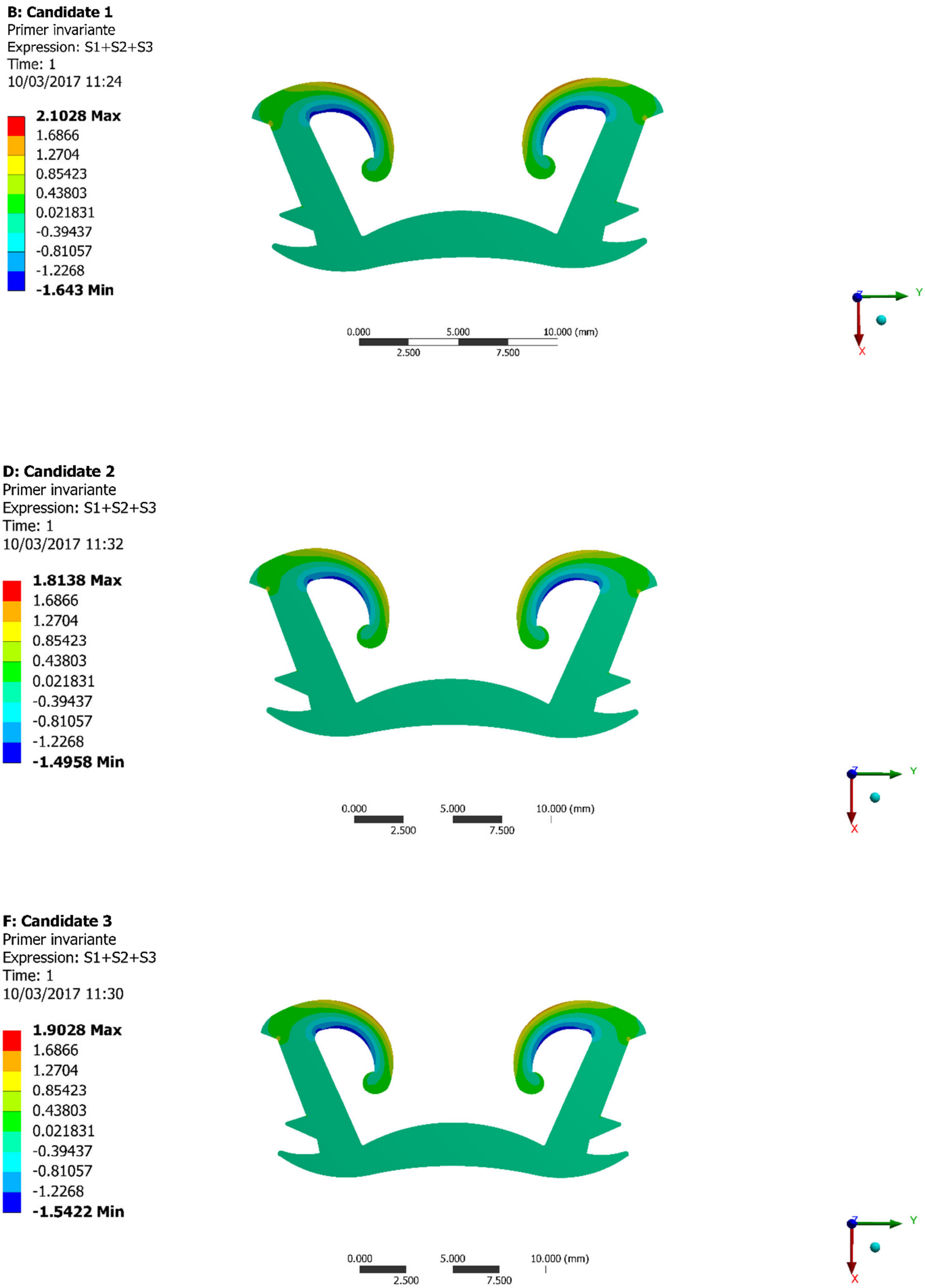


Fig. 11 - Contour plots corresponding to the first principal stress invariant evaluated at the displacement value of 2mm for the seal lips. The three plots are obtained by using each of the three parametric sets for the rubber material.

## CONCLUSIONS

In this study we aimed to present the implementation of hyperelastic materials into complex FE calculations. We used a Mooney-Rivlin models whose parameters were defined from measured data. The procedure for selecting the parameters was based in a genetic algorithm able to efficiently search into the parametric domains to define the best fit of model and data. After two runs of 100 generations we obtained three sets of parameters that described properly the data in the measured range. The obtained sets only differed at much higher strain values than those observed.

The hyperelastic model implemented into a FE element allowed us to study a case of application of the material to a rubber seal of complex shape. The study produced different results depending on the parametric set selected. The situation was simply derived from the limited range of available data to fit the model. However it also shows the demanding characterization that a hyperelastic material requires in order to produce reliable results for simple applications. Our aim was to define a complete and robust procedure for the analysis, from the material characterization, to the FE study, and to highlight the necessity to use data adapted to the study to produce reliable results.

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