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GAUSSIAN HYPERMODEL APPLIED TO FAILURE ANALYSIS OF LAMINATED COMPOSITES

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ABSTRACT

This work deals with the use of a Gaussian Hyper model to recover the interface thermal contact conductance between two materials within the Bayesian framework. Such function is assumed to be piecewise smooth with discontinuities of unknown location and size. In this work, we propose the use of a Gaussian smoothness prior, where the scaling parameter appearing in the prior was treated as an unknown hyperparameter. This parameter was estimated as part of the inference problem in a hierarchical model, with a hyperprior density in the form of a Rayleigh distribution. The feasibility of the approach was evaluated with simulated temperature measurements.

Keywords: Gaussian hypermodel, hyperparameter, thermal contact conductance, infrared thermography.

INTRODUCTION

The use of infrared thermography and other non-intrusive techniques are of great importance in many areas, such as electronics, telecommunications, aviation, defense, and oil industries, among many others. Abreu *et al*. (2014) showed that it is possible to estimate the location of contact failures between the layers of a laminated composite by solving an inverse heat conduction problem using thermographic images as input data. In this context, a heat flux is imposed on one surface of the composite material, while temperature measurements are obtained on the same surface, through an infrared camera, in a non-intrusive manner. By using the measured temperatures, the thermal contact conductance between the layers can be estimated (Colaço and Alves, 2013; Abreu *et al*., 2014; Abreu *et al*., 2016; Padilha *et al*., 2016), which can provide a quantitative analysis of the failures on interface.

This problem was investigated previously by the authors (Abreu *et al*., 2014) using a total variation prior information, which is suitable for piecewise regular solutions with sparse gradients (Kaipio and Somersalo, 2005). However, this prior information involves an additional parameter that needs to be specified for the application of Markov Chain Monte Carlo Methods (MCMC). The specification of a value for such parameter is a difficult task that can be accomplished by running several numerical experiments. Appling the MCMC method, this work proposes the use of a Gaussian smoothness prior where the scaling parameter appearing in the prior can be treated as an unknown hyperparameter and estimated as part of the inference problem in a hierarchical model, with a hyperprior density in the form of a Rayleigh distribution (Kaipio and Somersalo, 2005; Calvetti and Somersalo, 2007; Calvetti and Somersalo, 2008).

PROBLEM FORMULATION AND SOLUTION METHODOLOGY

Consider a plate composed of two layers subjected to convective heat losses at the surface Γ_{oo} and where a heat flux is applied. The opposite surface, Γ_o, is subjected to convective heat losses (see Fig. 1) and at the interface between the layers, Γ_c, we consider a spatially distributed contact resistance by defining a contact conductance.

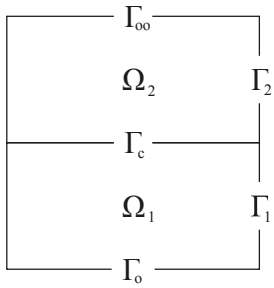


Fig. 1 - Geometry for a two-dimensional composite medium.

The plate is assumed to be initially at a uniform temperature, the heat transfer through the lateral surfaces, Γ_1 and Γ_2, are supposed negligible and the physical properties of each layer are assumed homogeneous and not dependent on temperature. The dimensionless mathematical formulation of this physical problem is given by (Ozisik, 1993):

$$\frac{1}{\alpha_{1,2}^*}\frac{\partial\theta_{1,2}(X,Y,Z,\tau)}{\partial\tau}=\nabla^2\theta_{1,2} \quad \text{in} \quad \Omega_{1,2} \quad \text{for} \quad \tau>0 \tag{1.a}$$

$$-k_1^*\frac{\partial\theta_1}{\partial Z}+Bi_0\theta_1=0 \quad \text{on} \quad \Gamma_o \quad \text{for} \quad \tau>0 \tag{1.b}$$

$$k_1^*\frac{\partial\theta_1}{\partial Z}=Bi_c(X,Y)[\theta_2-\theta_1] \quad \text{on} \quad \Gamma_c \quad \text{for} \quad \tau>0 \tag{1.c}$$

$$k_2^*\frac{\partial\theta_2}{\partial Z}=Bi_c(X,Y)[\theta_2-\theta_1] \quad \text{on} \quad \Gamma_c \quad \text{for} \quad \tau>0 \tag{1.d}$$

$$k_2^*\frac{\partial\theta_2}{\partial Z}+Bi_2\theta_2=q^*(X,Y,\tau)+Bi_2\theta_\infty^* \quad \text{on} \quad \Gamma_{oo} \quad \text{for} \quad \tau>0 \tag{1.e}$$

$$\frac{\partial\theta_1}{\partial X}=\frac{\partial\theta_2}{\partial X}=0 \quad \text{on} \quad \Gamma_1 \quad \text{for} \quad \tau>0 \tag{1.f}$$

$$\frac{\partial \theta_1}{\partial Y} = \frac{\partial \theta_2}{\partial Y} = 0 \qquad \text{on} \quad \Gamma_2 \quad \text{for} \quad \tau > 0 \tag{1.g}$$

$$\theta_1^* = \theta_2^* = 0 \qquad \text{in} \quad \Omega_{1,2} \quad \text{for} \quad \tau = 0 \tag{1.h}$$

where the dimensionless groups are defined as:

$$\theta(X,Y,Z,\tau) = \frac{T(x,y,z,t) - T_o}{T_o} \qquad q^*(X,Y,\tau) = q(x,y,t)\frac{c}{k_{ref}T_o} \tag{2.a,b}$$

$$k^* = \frac{k}{k_{ref}} \qquad \alpha^* = \frac{\alpha}{\alpha_{ref}} \tag{2.c,d}$$

$$X = \frac{x}{c} \quad Y = \frac{y}{c} \quad Z = \frac{z}{c} \quad Z_1 = \frac{c_1}{c} \quad A = \frac{a}{c} \quad B = \frac{b}{c} \tag{2.e-j}$$

$$\tau = \frac{\alpha_{ref} t}{c^2} \qquad Bi_c(X,Y) = \frac{h_c(x,y)c}{k_{ref}} \tag{2.k,l}$$

This direct problem was solved by using a hybrid analytical-numerical approach, using the same code used in Abreu *et al.* (2014), which is based on the Generalized Integral Transform Technique (GITT) and finite-differences (Cotta, 1993; Anderson *et al.*, 2013).

INVERSE PROBLEM

In this work the objective is to analyze and compare the performance of the Markov chain Monte Carlo (MCMC) method used in Abreu *et al.* (2014), where the Total Variation (TV) prior information was used, with an approach using a Gaussian Smoothness prior density. For this last prior density, the parameters are not fixed, but can also be estimated as part of the inference problem in a hierarchical model. This approach cannot be made when the Total Variation priori information is used, besides the fact that the specification of a value for such parameter is a difficult task that, in general, can only be accomplished by running several numerical experiments (Calvetti and Somersalo, 2007; Calvetti and Somersalo, 2008).

The quality of the adhesion at the interface between two materials can be indirectly analyzed, using the information about the Biot at the interface, $Bi_c(X,Y)$, which is a nondimensional representation of the thermal contact conductance (TCC). The TCC value tends to zero in surface regions where the contact is not perfect and its value is large in regions with perfect contact. For the cases studied here, the dimensionless thermal contact conductance was adjusted to be near 20 on regions with perfect contact. So, the inverse problem is defined by the identification of the $Bi_c(X_I,Y_J)$ on a discrete superficial grid points X_I and Y_J, where $X_I = I\Delta X$, $Y_J = J\Delta Y$, $I = 1, \ldots, I_f$, $J = 1, \ldots, J_f$, and with grid spacing given by $\Delta X = A/I_f$ and $\Delta Y = B/J_f$. So, consider the vector of unknown parameters (Abreu *et al.*, 2014):

$$\mathbf{P}^T = \left[Bi_{c1}, Bi_{c2}, ..., Bi_{cM}\right] \tag{3}$$

We suppose that some temperature measurements are available on surface Γ_{oo}. Considering that the vector containing the measured temperatures is written as:

$$\mathbf{Y}^T = \left(\vec{Y}_1\ ,\ \vec{Y}_2\ , ...,\ \vec{Y}_{k_{\max}}\right) \tag{4.a}$$

where $\vec{Y}_k$ contains the measured temperatures of each of the M grid elements at time t_k, $k = 1, ..., k_{max}$, that is,

$$\vec{Y}_k\ = \left(Y_{k1}\ ,\ Y_{k2}\ \ , ...,\ Y_{kM}\right) \tag{4.b}$$

we have $N_{meas} = (M \cdot k_{max})$ measurements in total. By the use of the Bayes' theorem, the new information (measurements) is combined with the previously available information (prior). Considering the Bayes' theorem (Kaipio e Somersalo, 2005):

$$\pi_{posterior}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi(\mathbf{P})\pi(\mathbf{Y}|\mathbf{P})}{\pi(\mathbf{Y})} \tag{5}$$

where $\pi_{posterior}(\mathbf{P})$ is the posterior probability density, $\pi(\mathbf{P})$ is the prior density, $\pi(\mathbf{Y}|\mathbf{P})$ is the likelihood function and $\pi(\mathbf{Y})$ is the marginal probability density of the measurements, which plays the role of a normalizing constant. In this work we considered Gaussian smoothness priori information (Kaipio e Somersalo, 2005; Orlande, 2012):

$$\pi(\mathbf{P}) \propto \exp\left[-\frac{1}{2}\gamma\left\|\mathbf{D}\left(\mathbf{P}-\tilde{\mathbf{P}}\right)\right\|^2\right] \tag{6.a}$$

where $\|.\|$ denotes the L_2 norm. The constant γ is a parameter associated with uncertainties in the prior and $\mathbf{P}$ is a reference value for $\tilde{\mathbf{P}}$. The matrix $\mathbf{D}$ is such that each line of $\mathbf{D}(\mathbf{P}-\mathbf{P})$ involves the parameter corresponding to that line and its neighbors (Kaipio and Somersalo, 2005; Calvetti and Somersalo, 2007; Mota et al, 2010; Orlande, 2012). In this work we wrote this matrix as:

$$\mathbf{D} = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix} \qquad \text{with size}\,(M-1)\times(M) \tag{6.b}$$

The parameter γ appearing in the prior function in equation (6.a) is treated as hyperparameter, and it is estimated as part of the inference problem in a hierarchical model (Kaipio and Somersalo, 2005). The hyperprior density for this parameter is taken in the form of a Rayleigh distribution.

By assuming that the measurement errors are Gaussian random variables, with zero means and known covariance matrix **W** and that the measurement errors are additive and independent of the parameters **P**, the likelihood function can be whiten like a Gaussian multivariate function (Kaipio e Somersalo, 2005; Orlande, 2012; Abreu *et al.*, 2014). Therefore, the posterior distribution information can be written as:

$$\pi(\gamma, \mathbf{P}|\mathbf{Y}) \propto \frac{\gamma}{\gamma_0^2} \exp\left\{ -\frac{1}{2}[\mathbf{Y} - \mathbf{\Theta}(\mathbf{P})]^T \mathbf{W}^{-1}[\mathbf{Y} - \mathbf{\Theta}(\mathbf{P})] - \frac{1}{2}\gamma \left\| \mathbf{D}\left(\mathbf{P} - \tilde{\mathbf{P}}\right) \right\|^2 - \frac{1}{2}\left(\frac{\gamma}{\gamma_0}\right)^2 \right\} \tag{7}$$

where $\gamma_o > 0$ is the center point of the Rayleigh distribution (Kaipio and Somersalo, 2005). This value is obtained from some empirical tests.

RESULTS

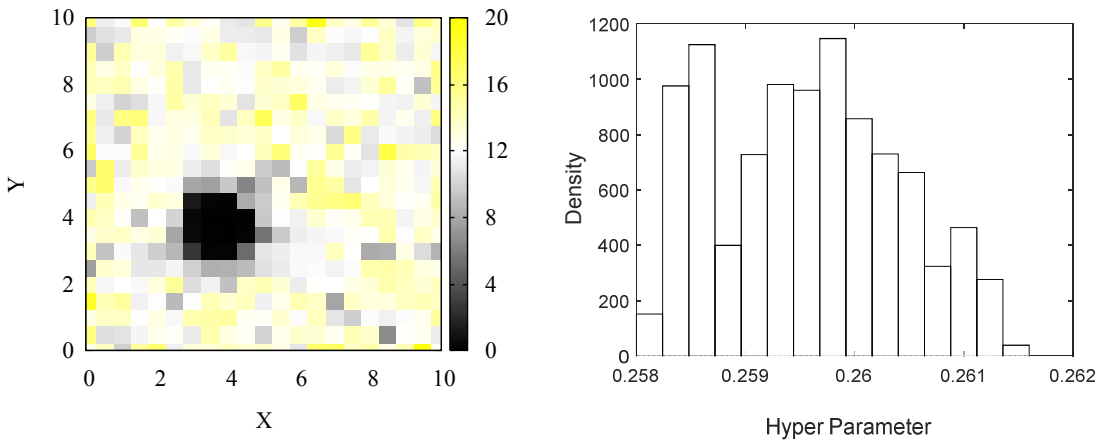
In this work, we considered the same materials and properties used in Abreu *et al*. (2014), in order to compare the results with a previous implemented method. A heat flux of of 25 kW/m^2 was applied at the top surface. The materials considered for the layers were titanium (k = 21.9 W/mK and α = 9.32 $\times 10^{-6}$ m^2/s) and epoxy with graphite fibers - 25% vol (k = 0.87 W/mK and α = 0.66 $\times 10^{-6}$ m^2/s).

Simulated measurements with a standard-deviation of 0.05 ^{o}C are used in the inverse analysis to recover the square contact failure showed in results and the inverse problem was solved with a Markov chain Monte Carlo (MCMC) method implemented through the Metropolis-Hastings' algorithm (Kaipio e Somersalo, 2005; Orlande, 2012; Abreu *et al.*, 2014). The measurements were assumed available with a frequency of 10 Hz and the duration of the experiment was taken as 10s.

A result for a typical case where the thermal contact conductance varies spatially is shown in Fig. 1.a, for a test case where there is a failure in the region between the dimensionless positions $3.5 < X < 5$ and $3.5 < Y < 5$. The values obtained for the estimated function in these places go to zero, and represents lack of contact between the layers. For the contact region, the dimensionless thermal contact conductance is more than 12.

Considering **P*** as the candidate point and $\mathbf{P}^{(t-1)}$ as the current state of the Markov chain, the proposal density for all results was taken as $\mathbf{P}^{*} = \mathbf{P}^{(t-1)} + w(2\mathbf{U} - 1)$, where $w = 0.05$ and **U** is a vector of random numbers uniformly distributed in (0,1), (Abreu *et al*., 2014).

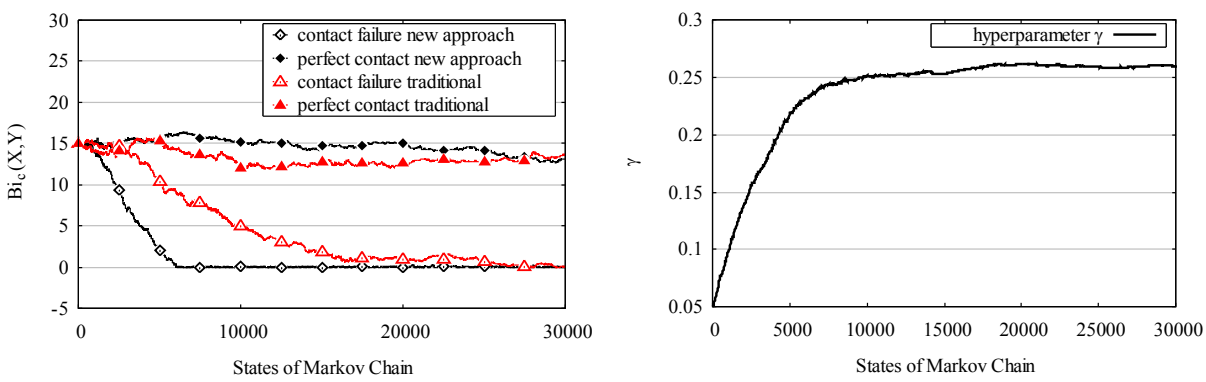
Figure 1.b represents a histogram of the hyperparameter estimate by using the proposed approach. With this value, Figure 2.a shows that the acceptance ratio and the Markov Chains have a faster convergence to equilibrium state than using traditional approaches. The Figure 2.b shows the convergence of the Markov Chain for the hyperparameter, using a random walk like proposal for γ. The centre point for the Rayleigh distribution was $\gamma_o = 0.05$.



(a) (b)

Fig. 1 - Tensile test results

The acceptance ratios of the proposed states in the Markov chains ranged from 30% to 40% for the case studied here and the Markov chains were simulated with 30000 states. The burn-in period was taken as the first 10000 states for the result using the proposed new approach and the Fig. 2 revel that there not convergence for traditional approach with 30000 states. For the case examined, a perfect contact was assumed for the initial states of the Markov chains in the whole interface, using Bi_c=15, while the exact value used for the perfect contact was Bi_c=12 to generate the simulate measurements.



(a) (b)

Fig. 2 - States of the Markov chains for the failure point at X=4, Y=4 and for contact point at X=8, Y=8.

CONCLUSIONS

This study shows that there is a substantial improvement when it is possible to estimate the hyperparameter, because the acceptance ratio is automatically adjusted in this approach.

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