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# A RBDO APPROACH FOR THE RELIABILITY ASSESSMENT OF COMPOSITE STRUCTURES

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## ABSTRACT

The introduction of reliability assessment methods in safety analysis of composite structures increased the complexity and the efficiency of failure evaluation proposed methodologies. Here, an approach based on surrogate modelling of first ply failure (FPF) to obtain the reliability of the whole composite structure is proposed and analysed for reliability-based design (RBDO). This approach overcomes the expensive costs associated with exhaustive local reliability evaluation. However, it can drive an underestimation of the structural reliability index. The identification of different failure modes and its importance in reliability analysis are outlined and discussed.

***Keywords:*** composite structures, surrogate modelling, reliability, multiple failures, RBDO.

## INTRODUCTION

Composite plates and shells structures develop interactions in the physical response between ply, laminate and structure levels inducing failure events at different scales and competing failure paths. As a result of the associated analysis and design complexity the structural response becomes highly unpredictable since uncertainties in geometry, loading, or material properties can completely change the failure path. Composite structures introduce many failure modes and exhibit failure responses across various length scales (Boyer et al., 1997; Carbillet et al., 2009; Conceição António and Hoffbauer, 2009). In the presented paper approximate representations of failure events were introduced aiming to obtain an improvement of the efficiency in reliability analysis. The errors introduced by some approximations used in structural reliability calculations of composites are studied.

## RELIABILITY ANALYSIS REVIEW

In the reliability analysis of composite structures, the basic random variables, which are assumed to be uncorrelated, define the vector $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_n)$. Their mean values and variances describe their statistical nature. In this case, the random variables are the elastic and strength properties of the composite laminates of the structure. If the boundary surface of the safety domain is written as

$$z = \varphi(\pi_1, \pi_2, ..., \pi_n) = 0 \tag{1}$$

the values of $\boldsymbol{\pi}$ belonging to the failure domain will satisfy the inequality:

$$z = \varphi(\boldsymbol{\pi}) < 0 \tag{2}$$

The probability of failure is defined as

$$P_f = P[\varphi(\boldsymbol{\pi}) < 0] = \int_{\Omega} f(\boldsymbol{\pi})\, \mathrm{d}\boldsymbol{\pi} \tag{3}$$

where $f(\boldsymbol{\pi})$ is the joint probability density function of $\boldsymbol{\pi}$, $\Omega$ is the failure region, and $\varphi(\boldsymbol{\pi})$ is the so-called limit state function that separates the design space into failure ($\varphi(\boldsymbol{\pi}) < 0$) and safe ($\varphi(\boldsymbol{\pi}) > 0$) regions. The distribution of the basic variables, $\pi_i$, and the limit state surface, $\varphi(\boldsymbol{\pi})$, are known, and the probability of failure can be employed as a measure of reliability. However, Eq. (3) cannot be evaluated analytically for realistic composite structures because the calculation of the integral is too difficult. To avoid this problem, the moment reliability theory is used in this work, namely, the so-called Hasofer-Lind reliability index (Hasofer and Lind 1974, Melchers 1999). The Hasofer-Lind method is performed in two steps. The first one consists of projecting Equation (1) into the space of standardised variables:

$$u_{\mathrm{i}} = \frac{\pi_i - \overline{\pi}_i}{\sigma_{\pi_i}} \tag{4}$$

where $\overline{\pi}_i$ and $\sigma_{\pi_i}$ are, respectively, the mean values and the standard deviations of the basic random variables. The second step measures, in this space, the minimum distance $\beta$ of the transformed surface

$$\varphi(u_1, u_2, ..., u_{\mathrm{n}}) = 0 \tag{5}$$

to the origin of the axes. A design is considered reliable, at the $\beta_a$ level, prescribed by an appropriated code provision, if $\beta \geq \beta_a$. On the other hand, considering that the probability density in the standard normal space decays exponentially with the distance from the origin, the point with the maximum probability of failure on the limit-state surface is the point of minimum distance to the origin. From the operational point of view, the search for this point can be formulated as a constrained optimisation problem

$$\text{Minimise: } \beta(\mathbf{u}) = \left(\mathbf{u}^T\, \mathbf{u}\right)^{1/2} \tag{6}$$

$$\text{Subject to: } \varphi(\mathbf{u}) = 0$$

where $\mathbf{u}$ is the vector of the standardised variables defined through Eq. (4), and the solution, $\mathbf{u}^*$, is referred in technical literature as the design point or the MPP. The assumption that the minimum distance $\beta$ obtained from the solution of the minimisation problem in Eq. (6) is a measure of reliability is equivalent to the discretization at one single point of the safety domain boundary, expressed in the space of the standardised variables. This corresponds to the substitution of the hypersurface by the hyperplane passing through the point defined by $\mathbf{u}^*$ (Hasofer and Lind 1974). By formally introducing a normal probability distribution function, $\Phi$, the first-order approximation to $P_f$ can be written as

$$\mathrm{P_f} \approx \Phi(-\beta) \tag{7}$$

where $\beta$ is known as the reliability index, i.e., the minimum distance from the origin to the limit-state surface. The design point or MPP, $\mathbf{u}^*$, is obtained using an iterative scheme of the Hasofer-Lind method (Hasofer and Lind 1974, Conceição António 1995, António et al. 1996, Melchers 1999) and based on gradients evaluated by the adjoint variable method (Conceição António 1995).

## STRUCTURAL RELIABILITY ASSESSMENT APPROACH

Defining the *Tsai number,* $R_k$ , as a strength/stress ratio (Tsai 1987), it can be used together the interactive quadratic failure criterion of Tsai-Wu at the *k-th* point of the structure, where the stress vector is evaluated, by solving equation

$$1-\left(F_{ij}\, s_i\, s_j\right) R_k^2+\left(F_i\, s_i\right) R_k=0 \tag{8}$$

where $s_i$ are the components of the stress vector, and $F_{ij}$ and $F_i$ are the strength parameters associated with unidirectional reinforced laminate.

Laminate composite material structures are parallel systems. Indeed, when a point of the structure reaches the first ply failure (FPF) there are internal loads sharing and the structure can not fail. If there is residual strength the timing correspondent to the ultimate structural failure is associated with the last ply failure (LPF) (Tsai 1987). The analysis of LPF is very complex and costly when applied to shell composite structures. Furthermore, the first ply failure is the beginning of material degradation and the elastic material behavior can not be accepted. This way a conservative rule based on first ply failure is preferable mainly taking into account the weak matrix properties of the composite materials. Since the safe region is related to $R_k > 1$, the most critical *Tsai number* associated with first ply failure (FPF) is established as

$$\overline{R}=Min(R_1,\ldots,\, R_k\,,\ldots,R_{N_s}\,) \tag{9}$$

being $N_s$ the total number of points where the stress vector is evaluated. The location of the points where the stresses are evaluated depends on the post-processing methodology used in the structural analysis. In this approach the finite element method is used for structural analysis and the stress vector is evaluated at the Gauss points of numerical integration. In this particular case, the formulae presented in Eq. (8) and Eq. (9) for the limit state function can be written as

$$\varphi(\boldsymbol{\pi})=\overline{R}-1 \tag{10}$$

From the deterministic point of view the structural failure analysis is associated to the most critical Tsai number defined in Eq. (9) and the composite structure fails at $\overline{\mathrm{R}}=1$ . However, in reliability analysis a limit state function must be considered for each point (for example at the Gauss integration points) on structure where the Tsai number is evaluated as defined in Eq. (8). This means that $N_s$ limit state functions should be considered and so, reliability analysis becomes very expensive and unpractical. To overcome this difficulty only the limit state associated to the most critical Tsai number $\overline{\mathrm{R}}$ will be considered in structural reliability analysis as proposed in Eq. (10). Following the formulae proposed in the previous section the reliability index of the composite structure, $\beta_s$, is obtained.

Since the stress vector is evaluated at discrete points, the measure of structural reliability depends on local conditions and on the shape of the failure envelope and so the approximation considered in Eq. (9) and Eq. (10) is valid as a *first design approach*. The most real scenario for the reliability assessment is to consider a reliability index $\beta_k$ for each $R_k$ in Eq. (8) as follows,

$$\beta_k=F(R_{\mathrm{k}}) \tag{11}$$

and so, the system reliability index of the composite structure is defined as

$$\beta_s = Min(\beta_1, \ldots, \beta_k, \ldots, \beta_{N_s}) \tag{12}$$

Although the first ply failure concept is commonly associated to a deterministic analysis of laminate composite material failure it can not be dissociated from the physical phenomenon of failure and consequent material degradation and it must be considered in reliability analysis. In the proposed approach the physical phenomenon of FPF is considered together with a probabilistic failure analysis considering the uncertainty in mechanical properties of the ply material. This is the reason why the failure probability of the system is equal to the largest failure probability among the components (plies) as defined in Eq. (12).

It is troublesome to evaluate the reliability index associated with each *k-th* limit state function in an explicit way for complex structures, composed of many elements and then to calculate the reliability of the structural system, $\beta_s$. However, the structural reliability index $\beta_s$ evaluated from Eq. (9) and Eq. (10) can be very different when compared with the realistic values obtained from Eq. (12).

So, the influence of multiple failures $R_k$ on the surrogate model in Eq. (10) must be analysed on reliability assessment. To simplify the problem a linear loading case applied to angle-ply composites is considered. The study is driven for the maximum load capability of the composite structure for a target reliability index, $\beta_a$.The optimal maximum load $\lambda$ for $\beta_a$ is obtained over the ply angle design variable, *a,* as follows,

$$\underset{\lambda,\ a}{\text{Minimise}}\ \left[\beta_s(\lambda,\ a) - \beta_a\right]^2$$

$$\text{subject to} \quad \beta_s(\lambda, a, \boldsymbol{\pi}) \geq \beta_a \quad \text{and} \quad 0 \leq \text{a} \leq \tfrac{\pi}{2} \tag{13}$$

where $\boldsymbol{\pi}$ is the random variable vector. This is a conventional RBDO target optimisation problem and the reliability condition, $\beta_s(\lambda, \text{a}, \boldsymbol{\pi}) = \beta_a$, is automatically satisfied (Conceição António and Hoffbauer 2009). To solve the target problem in Eq. (13), an optimisation algorithm based on gradients or evolutionary methods can be implemented. However, for the case with two design variables, load factor $\lambda$ and ply angle *a*, a decomposition of the problem is considered, and the following algorithm (Conceição António and Hoffbauer 2009) is proposed:

**Do** $a := 0$
1st Step: Choice of initial solution (n=0) for load factor, $\lambda_{\text{n}}$;
2nd Step: The Hasofer-Lind method and the appropriate iterative scheme are applied to evaluate the structural reliability index, $\beta_s$;
3rd Step: Check for convergence, $(\beta_s - \beta_a) \to 0$.
**If** it does not converge, **then**
go to the 4th Step
**else** go to the 5th Step;
4th Step: $n := n + 1$
Use the bisection method to estimate the load factor, $\lambda_{\text{n}+1}$
Go back to 2nd Step.
5th Step: If $a \leq \frac{\pi}{2}$ then

$a := a + \Delta a$, go to 1st Step
**else Stop**;
**End Do**

The Hasoffer-Lind iterative scheme is based on the method developed by Conceição António (Conceição António 1995, António et al. 1996). The procedure is a SORM technique based on a quadratic approximation of limit state surface (Conceição António 1995). A gradient-based method (Conceição António 1995) is used to find the most probable failure point (MPP) and the gradients are obtained using the adjoint variable method (Conceição António 1995).

## RESULTS

To solve the RBDO inverse problem in Eq. (13), an optimisation algorithm is implemented and applied to a clamped cylindrical shell laminated structure. Nine vertical loads of mean value $P_k$ are applied along the free linear side (AB) of the structure. This free linear side (AB) is constrained in the y-axis direction. The structure is divided into four macro-elements, grouping all elements, and there is one laminate per each macro-element. The laminate distribution of the structure is shown in Fig.1a).

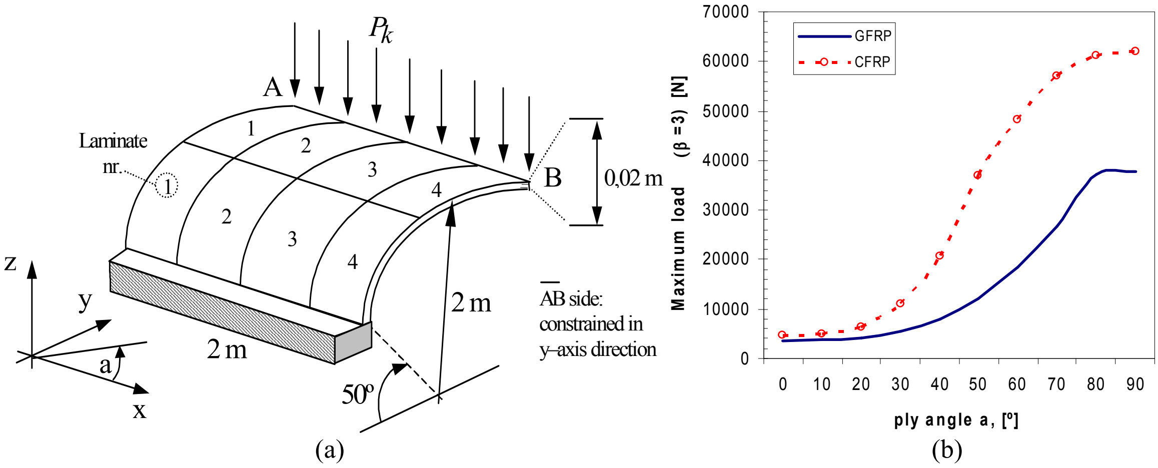


Fig. 1 – (a) Cylindrical shell and composite laminates distribution, and (b) loading capability of structural system: Maximum load for $\beta_a = 3$ (surrogate model Eq. (9) and Eq. (10))

Two composite systems, the E-glass/epoxy (Scotchply 1002) and the Carbon/epoxy (T300/N5208) (Tsai 1987), are used in the presented analysis. The balanced angle-ply laminates with five layers and the stacking sequence $[-a/+a/0^\circ/+a/-a]$ are considered. The structural analysis of laminated composite structures is based on the shell finite element model developed by Ahmad (1969). This shell element is obtained from a 3-D finite element using a degenerative procedure. It is an isoparametric element with eight nodes and five freedom degrees per node based on the Mindlin shell theory.
The mean values of the elastic and strength properties of the ply material used in the laminate construction of the composite structure are presented in Table 1. The elastic constants of the orthotropic ply are the longitudinal elastic modulus $E_1$, the transversal elastic modulus $E_2$, the in-plane shear modulus $G_{12}$, and the in-plane Poisson's ratio $\nu_{12}$. The ply strength properties are the longitudinal strength in tensile, X, and in compression, X', the transversal strength in tensile, Y, and in compression, Y', and the shear strength, S.

Table 1 - Mean value of mechanical properties of composite layers

| Material | $E_1$ [GPa] | $E_2$ [GPa] | $G_{12}$ [GPa] | $\nu_{12}$ |
|---|---|---|---|---|
| T300/N5208<br>E-glass/epoxy (Scotchply 1002) | 181.00<br>38.60 | 10.30<br>8.27 | 7.17<br>4.14 | 0.28<br>0.26 |
| | X; X' [MPa] | Y; Y' [MPa] | S [MPa] | ρ[kg/m3] |
| T300/N5208<br>E-glass/epoxy (Scotchply 1002) | 1500; 1500<br>1062; 610 | 40; 246<br>31; 118 | 68<br>72 | 1600<br>1800 |

To investigate the multiple failure influence on reliability analysis of the composite structures, the mechanical properties denoted by $\boldsymbol{\pi}$ are the considered random variables. All random variables are non-correlated following a normal probability distribution function defined by their respective mean and standard deviation. The mechanical properties vector, $\boldsymbol{\pi}$ includes the following random variables: longitudinal Young's modulus $E_{1,j}$, transversal modulus $E_{2,j}$, transversal tensile strength $Y_j$, and shear strength $S_j$, where subscript j denotes the laminate number. Sixteen mechanical properties are considered as random parameters with uncertainty in this analysis: $E_{1,j}$, $E_{2,j}$, $Y_j$, $S_j$, j=1, … 4. The present study can be extended to other random variables. The coefficients of variation, $\mathrm{CV}(\boldsymbol{\pi}) = 6\%$ of mechanical properties $\boldsymbol{\pi}$ are tested.

The constrained target optimisation problem given in Eq. (13) is solved using the algorithm proposed in previous section. The maximum allowable load, $\overline{P}(\beta_a)$, on the composite structure, considering the prescribed reliability index of $\beta_a = 3$, is obtained. This maximum load, as function of the ply angle design variable, *a*, is plotted in Fig. 1b). The maximum load for Carbon/Epoxy composite system (CFRP) is higher than for E-glass/ Epoxy composite system (GFRP) and the difference increases with ply angle from 0º to 90º. The maximum load presented in Fig. 1b) is obtained using the surrogate modelling defined by Eq. (9) to Eq. (10).

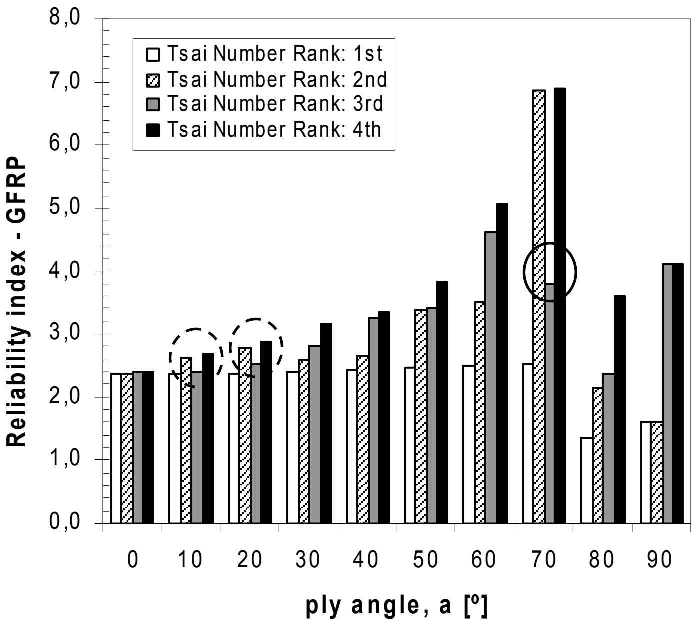


Fig. 2 - Structural reliability assessment: Influence of multiple failure points.
Applied load=1.05 × maximum load for $\beta_a$=3 (Scotchply 1002, 1st case study)

In order to study the influence of multiple failures on deviations of the proposed surrogate modelling relatively to the real reliability index of the laminated composite structure, an applied load higher than 5% of maximum load $\overline{P}(\beta_a)$ is considered in the following examples. The first studied case is performed for the E-glass/epoxy composite system (Scotchply 1002, Tsai 1987) and the four first positions in Tsai number ranking (in ascending order) are considered in the analysis. The correspondent reliability index, $\beta_k = F(\mathrm{R_k})$, is obtained and plotted in Fig. 2.

Figure 2 shows the influence of ply angle on reliability indices for E-glass/epoxy composite system. This dependence comes from the changes in stresses in Eq. (8) when calculated at Gauss point of ply level. The Tsai number ranking changes also in Eq. (12). It must be observed that the changes in reliability indices are higher than Tsai number values. Since Tsai numbers are obtained from the deterministic analysis, they depend on changes in mean values of mechanical properties presented in Table 1. However, the reliability indices being obtained from probabilistic analysis depend on mean values, standard deviations and joint probability density functions of random variables. Furthermore, changes in ply angle modify significantly the constrained optimisation problem defined in Eq. (13) for reliability analysis.

Since only one R-rank position of Tsai number $R_k$ is considered in Fig.2, its location on the composite structure (Gauss point) is not specifically reported. For ply angles equal to 10º and 20º it is observed changes in the $\beta$-rank of reliability index values occurring in points belonging to the same laminate (dashed line circle). For a ply angle equal to 70º the change in $\beta$-rank of reliability index is observed for points belonging to different laminates of the composite structure (solid line circle).

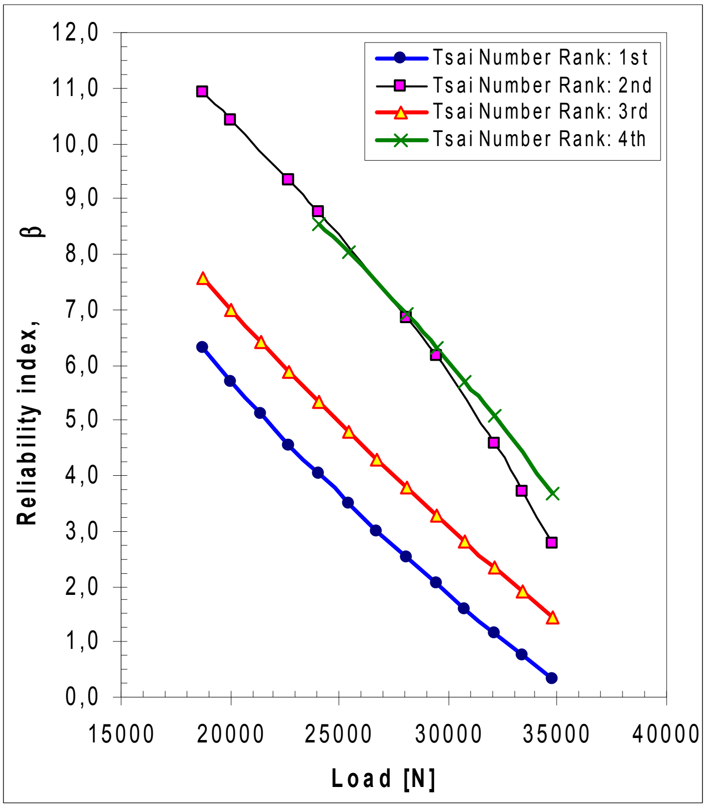


Fig. 3 - Influence of load on reliability index for multiple failure points, ply angle, *a*=70º (Scotchply 1002, 1st case study)

Figure 3 shows the relationship between load and the corresponding reliability index for the specific stacking sequence $\left[-70^{o}/+70^{o}/0^{o}/+70^{o}/-70^{o}\right]$ for all composite laminates built with E-glass/epoxy composite system. In this case the proposed surrogate model for reliability analysis of composite structures defined in Eq. (9) and Eq. (10) kept valid but the changes in the $\beta$-rank of reliability index values (2nd and 3rd R-rank positions) indicates a loss of generality of the surrogate model approach. The reliability index of the failure point ranked in 3rd R-rank position is lower than the reliability index of the 2nd R-rank position. Although the most critical Tsai number is associated to critical reliability index, Fig. 2 and Fig. 3 show that the ranking in terms of Tsai number R (R-rank) is not the same when made in terms of reliability index values ($\beta$-rank).

The second case study is performed for the Carbon/epoxy (T300/N5208, Tsai 1987) composite system (CFRP) and using the previous stacking sequence $\left[-a/+a/0^{o}/+a/-a\right]$ for all laminates. Figure 4 shows that for 60% of the ply angle values there are changes in the positions when the Tsai number-based ranking is compared with the reliability index-based ranking. Changes in the ranking of reliability index values occurring in points belonging to the same laminate (dashed line circle) are visible in Figure 4. For a ply angle $a$=50º the change in reliability index ranking is observed for points belonging to different laminates of the composite structure (solid line circle). Once, there is a loss of accuracy of the surrogate modelling based on the use of reliability index associated to the most critical Tsai number (R-rank) considered as the critical reliability index of the structure. In this scenario, there is an underestimation of the reliability of the composite structure.

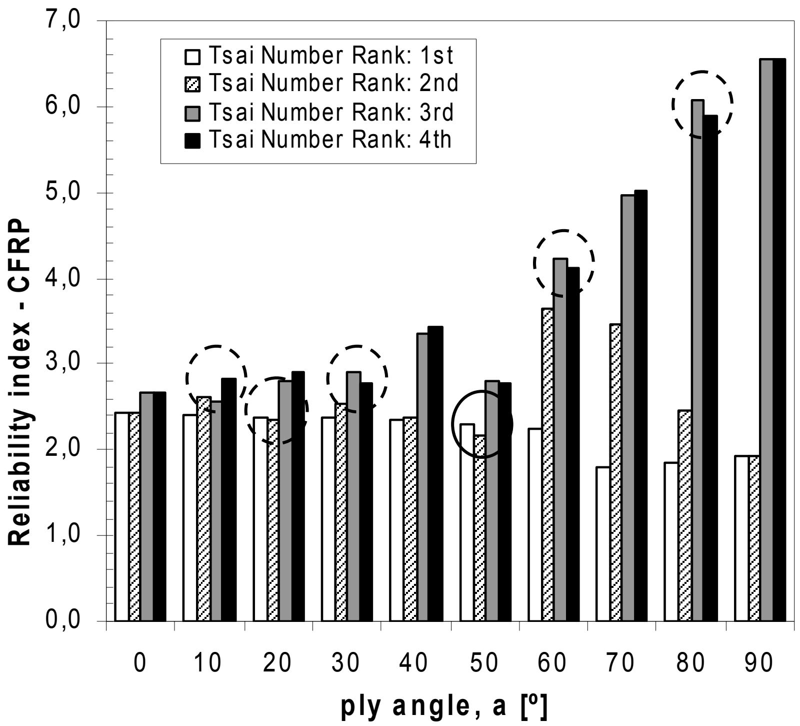


Fig. 4 - Structural reliability assessment: Influence of multiple failure points.
Applied load=1.05 × maximum load for $\beta_a$=3 (T300/N5208, 2rnd case study)

From the previous analysis is interesting to analyse the relationship between load and the corresponding reliability index for the specific stacking sequence $\left[-50^{o}/+50^{o}/0^{o}/+50^{o}/-50^{o}\right]$ for all CFRP composite laminates. Although the influence of load on reliability index is different from first GFRP case there are some important aspects to be pointed. The $\beta$-rank position and R-rank position depends on load value as shown in Fig.5 indicating a loss of generality of the approach.

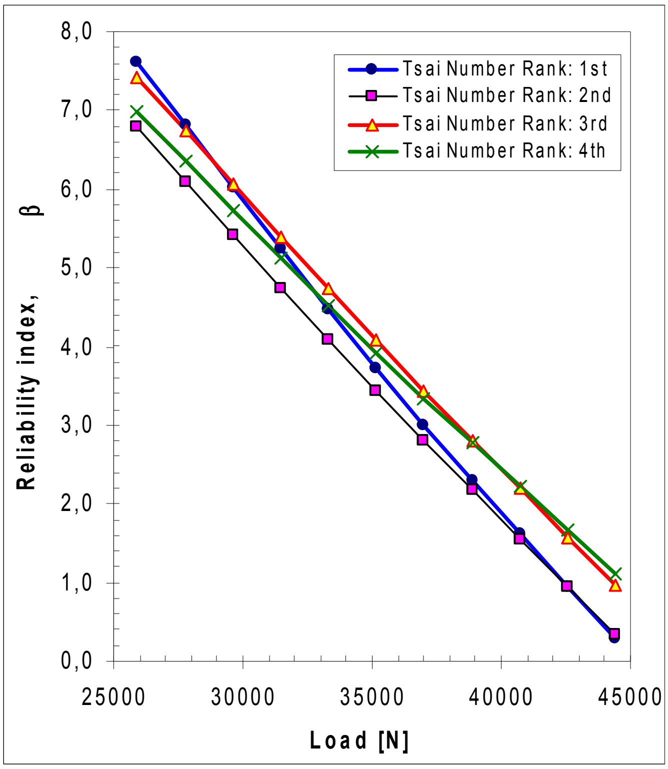


Fig. 5 - Influence of load on reliability index for multiple failure points, ply angle, $a$=50º (T300/N5208, 2nd case study)

## CONCLUSIONS

The influence of multiple failure modes in reliability assessment of composite structures is studied within the context of RBDO. The validity of the approximation used to define the structural reliability $\beta_S$ from critical R is discussed comparing with the $\beta_S$ obtained from the critical $\beta$ on the structure. Although the surrogate model uses the most critical Tsai number to define critical reliability index $\beta_S$ this must be considered as first deign. The previous analysis shows that in many situations the ranking in terms of Tsai number (R - rank, Eq. (9) and Eq. (10)) is not the same when made in terms of reliability index values ($\beta$ - rank, Eq.(12)). So, this can drive for an underestimated reliability assessment of the composite structures. Furthermore, the load level can improve the differences between the values of structural reliability index $\beta_S$ obtained from surrogate model and the ones obtained using the more realistic model defined in Eq. (12).

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## REFERENCES

[1]-Ahmad S. Curved finite elements in the analysis of solid, shell and plate structures. Ph.D. Thesis, University College of Swansea, UK, 1969.

[2]-Boyer C, Béakou A, Lemaire M. Design of a composite structure to achieve a specified reliability level. Reliability Engineering and System Safety, 1997, 56, p.273-283.

[3]-Conceição António CA. Optimization of structures using composite materials made of polymeric matrix (in Portuguese). Ph.D. Thesis, Faculty of Engineering, University of Porto, Portugal, 1995.

[4]-Conceição António CA, Torres Marques A. Gonçalves JF. Reliability based design with a degradation model of laminated composite structures. Structural Optimization, 1996, 12, p.16-28.

[5]-Conceição António C, Hoffbauer LN. An approach for reliability-based robust design optimisation of angle-ply composites. Composite Structures, 2009, 90, p.53-59.

[6]-Carbillet S, Richard F, Boubakar L. Reliability indicator for layered composites with strongly non-linear behaviour. Composites Science and Technology, 2009, 69, p.81-87.

[7]-Hasofer AM, Lind NC. Exact and Invariant Second-Moment Code Format. Journal of the Engineering Mechanics Div., ASCE, 1974, 100, p.11-121.

[8]-Melchers RE. Structural Reliability Analysis and Prediction. Chichester: Wiley; 1999.

[9]-Tsai S.W. Composites Design. Dayton, USA, Think Composites, 1987.