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EFFICIENT SIMULATION OF STICK-SLIP DYNAMICS IN MATLAB

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ABSTRACT

The understanding of the stick-slip dynamics is of both fundamental and practical importance. However, the numerical investigation of such dynamical systems can be difficult and time consuming. In this work, we outline the necessary modification in the scientific package MATLAB to achieve an accurate and fast simulation result. The result is demonstrated using a 3-mass system under friction and harmonic excitation.

Keywords: stick-slip dynamics, chaos, MATLAB.

INTRODUCTION

The presence of dry friction is known to create complex stick-slip dynamics in many natural and engineering systems. The numerical simulation of the stick-slip dynamics often requires transitioning to a different set of governing equations based on the force balance when the system is temporarily at rest. However, implementing this relatively simple check of the force balance in the numerical integration is not straightforward. For example, numerically precisely hitting a zero velocity is difficult. Also, the convergence of the integration can change across the transition, which could potentially result in significantly longer integration time. Such issues continue to motivate various alternative, and sometimes *ad hoc*, numerical schemes to study the stick-slip dynamics [1]. In this work, we show a simple solution using the scientific computational package MATLAB [2]. While MATLAB is quite effective to solve various types of differential equations, it is not efficient to solve stick-slip dynamical systems. By modifying the main integration routine, we could reach the numerical result significantly faster with the similar degree of accuracy. This will be demonstrated in the stick-slip dynamics of a 3-mass mechanical system.

RESULTS AND CONCLUSIONS

To simulate the stick-slip dynamics, one must (a) identify the potential transition point, (b) stop the integration, and (c) switch to a different set of dynamical laws when necessary. While these steps can be directly entered from the right hand side of the integration routine (direct method), exceedingly long computational time could result for above mentioned reasons. Alternatively, MATLAB provides the option to terminate the integration at a user-defined event. However, it is strictly defined to be the zero-crossing of the event function, rather than when the function attains a zero value. This can exclude the dynamic scenario given by a sequence of slip-

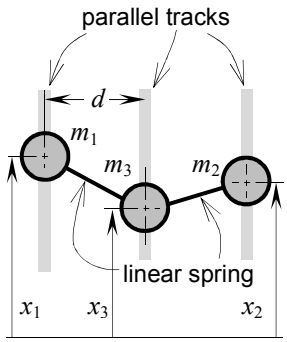

Fig. 1

stick-slip motion in the same direction. Hence, the default setting of MATLAB cannot be directly applied to study the stick-slip dynamics. By adding the codes in the main integration routine, the potential transition can be effectively determined and the integration can be forced to stop using the existing event function facility (modified MATLAB). The time saving is quite remarkable as demonstrated by a 3-mass mechanical system. The system consists of masses, $m_{1,2,3}$, in a horizontal plane moving on parallel tracks separated by a fixed distance d. The masses are connected via linear springs of spring constant $k_{1,2}$ with undeformed length l_0 (Figure 1). The middle mass m_3 is further placed on a moving ground of constant velocity v_0 and forced by the harmonic function $G_0\cos(\omega t)$. Letting the mass position be $x_{1,2,3}$, the magnitude of the spring force exerted on the (side) masses m_i is given by $k_i\left(1 - l_0/\sqrt{(x_3 - x_i)^2 + d^2}\right)(x_3 - x_i)$, $i = 1, 2$, respectively. Now, consider $k = k_{1,2}$ and, for $i = 1, 2, 3$, introduce $T = t\sqrt{k/m_3}, y_i = \frac{x_i}{d}, \mu_i = m_i/m_3, \Lambda_i = F_i/kd$, the friction force, $F_i = F_{is}\mathrm{sgn}(\dot{x}_i)/(1 + 3\,|\dot{x}_i|)$ [3,4], and $\Gamma = G_0/kd, \Omega = \omega/\sqrt{k/m_3}$. The equations of motion based on the 3rd order expansion of $\sqrt{(x_3 - x_i)^2 + d^2}$ can be given by $\mu_i\ddot{y}_i - \alpha(y_3 - y_i) + \beta(y_3 - y_i)^3 + \Lambda_i = 0$, $i = 1, 2$, and $\ddot{y}_3 - \alpha\sum_{i=1}^{2}(y_3 - y_i) + \beta\sum_{i=1}^{2}(y_3 - y_i)^3 + \Lambda_3 = \Gamma\cos(\Omega T)$. Qualitatively similar chaotic *transient* stick-slip dynamics can be obtained using direct method (Fig. 2a) and modified MATLAB method (Fig. 2b). But the modified MATLAB method significantly reduces the computation time (Figure 3).

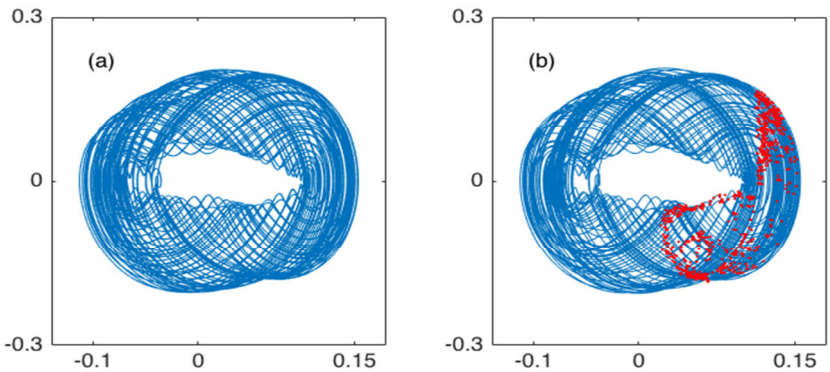

Fig. 2 - Transient stick-slip dynamics

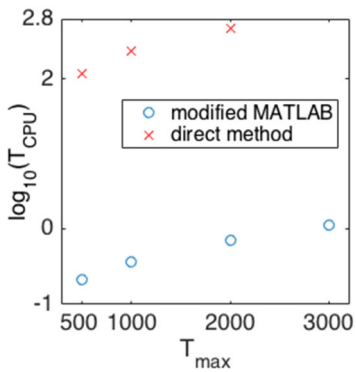

Fig. 3 - CPU time (min) vs max simulation time

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