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CAPACITY PLANNING AND OPERATION OPTIMIZATION OF HYBRID ELECTRICAL ENERGY SYSTEMS -THE ROLE OF UNCERTAINTY

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ABSTRACT

This article presents a mathematical model to support the decision making process related to the planning of capacity and operation of hybrid electrical energy systems. The model was developed considering case studies of small villages, in rural areas in Angola. Depending of the local conditions, the system considers the following sources: photovoltaic, wind, thermal and mini hydro. The minimum production capacity in each locality was assessed and the load duration curve was also evaluated. For each type of generation, the costs of construction and operation were estimated. The optimisation problem is solved as a multi-stage stochastic programming problem, in which the levels of the load duration curve are treated as random variables. The goal is to select the hybrid combination that minimizes the total cost of energy. The results showed that the consideration of uncertainty in the formulation is crucial for the decision process. The results presented show that the use of natural resources, namely hydropower, has economic advantages even when energy needs are lower than the available potential of generation. We also found that the possibility of energy transfer between villages reinforces the advantages of this solution.

Keywords: energy, management; hybrid, optimisation, uncertainty.

INTRODUCTION

The chronic energy shortage of rural populations, which occurs in many developing countries, has led to the degradation of forests due to the use of their resources as fuel. On the other hand, in recent years, different means of production and energy storage, of flexible use, have been developed using renewable energy systems (Nema et al., 2009).

These systems are called autonomous hybrid systems since they operate without connection to a neighbour network and combine different components of energy production. In their composition we can find production units, such as mini-hydro units, photovoltaic cells, wind turbines and thermal generators. Hybrid systems might include also components of energy storage, such as batteries, and hydrogen production facilities; and elements for the recovery of stored energy, such as fuel cells, generators and fuel gas facilities.

The aim of this work is to contribute to the development of procedures to assist in making decisions, associated with the composition and operation of autonomous hybrid systems. Through the application of the developed model to a case study, we also present results that allow illustrating the economic implications of the solutions that are technically feasible, thus contributing to further clarification of the decision makers involved in this matter.

Mathematical models for the study of autonomous hybrid systems are particularly important under the conditions that characterize the situation of developing countries, particularly the climatic conditions, the low level of technological development, and the lack of basic infrastructures. These systems offer a reasonable compromise between the investment costs and the operating costs, and on many circumstances, particularly in small villages in rural villages, are the most viable alternative to meet the energy needs of the populations (Gutiérrez, 1992; Shaahid and Elhadidy, 2003).

In order to analyse the energy needs of small villages, studies were developed in Angola, in the municipal headquarters of Muxiluando and Ambriz, Bengo province, and in the municipal headquarters of Tombwa, province of Namibe. The objective is to size the components of energy production of autonomous hybrid systems considering photovoltaic, thermal, mini-hydro and wind alternatives, that minimize the investment and operating costs, taking into account the local needs, the technological environment and the weather and water conditions. The choice of the localities accounted for the proximity of small waterfalls that will serve for the construction of mini-hydro and/or localities with average wind speed exceeding 5 m/s.

Currently the studied localities are supplied by electricity locally thermal generated. The solutions studied aim to meet the current demand, and adapt the technologies and their economic, social and environmental impacts for sustainable development. Accordingly, we also consider the possibility of the growing of energy needs, thus resulting in the need of using optimisation tools that take into account uncertainty.

There are several alternatives to formulate and solve the problem used to define the optimum composition of the hybrid system. These problems differ essentially with regard to the consideration, or not, of uncertainty and the consideration of one or more objectives. The simplest models are usually based on the minimization of the cost and do not consider the uncertainty (Patil et al., 2010; Chedid and Rahman, 1997). The multi-objective models (Fadaee and Radzi, 2012) are used when it is desired to complement the economic analysis with the environmental impact. A stochastic approach is generally used to introduce the effect of the uncertainty in the analysis and, if the uncertainty is related to the future demand it is obtained, generally, a stochastic programming problem in two stages with recourse (Cabral et al., 2010).

In this study, the formulated optimisation problem is a multi-stage stochastic programming problem on the generation capacity and the power requirements of the different sources, which objective criterion is the equivalent investment costs plus the operating costs, subjected to constraints on the capacity of the power plants and on the available power, taking into account the uncertainty associated with the temporal evolution of the energy needs. The formulation presented also considers the possibility of energy transfer between nearby localities.

CAPACITY PLANNING

The decisions concerning the capacity planning of electricity generation have to account the current needs and constraints, but also their future developments, generally in a long time horizon with high uncertainty.

There are several types of uncertainty influencing the optimal decisions that must be taken in the first step, as for example the technological development of production systems and their impact on investment and operating costs, changes in demand and economic development, and the capability to promote and implement changes if necessary.

Therefore, in light of future developments, solutions that do not take into account potential risks of transforming decisions initially considered optimal in decisions with unnecessary costs for taxpayers, may have negative implications on the economic development and welfare of the population. These factors amplify the importance of developing strategies to minimize the risk with respect to a variety of future events in which the costs are explicitly considered.

The results presented in this study were supported by three real-world case studies involving the villages of Ambriz, Muxiluando and Tombwa. Yet, in this article, only the case of Ambriz is treated, since it is the one that has conditions for serving other locations, leading to a more general mathematical modelling.

In the first phase of this study were assessed the current minimum production capacity and the local energy needs for the current conditions. Following the guidelines in (Beggs, 2009), the evaluation was performed by considering the consumption of households, and public and social services. A consumption of 13.2 MWh/day and a necessary power of 2.1 MW were estimated. Taking also in account cultural habits, the load duration curve (LDC) was constructed. The daily consumption was divided in three periods of different rates of consumption, leading to the LDC that is represented graphically in Fig. 1

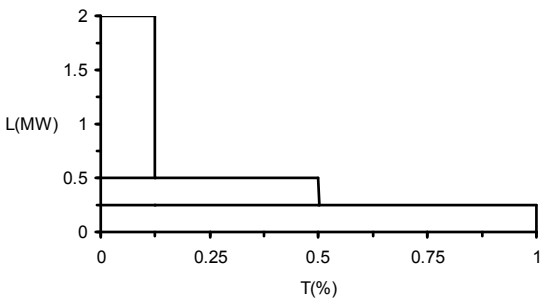

Fig. 1 - Distribution of the energy needs for a daily cycle, for the current levels of consumption in the village of Ambriz

where the vector of average power levels is:

$$L = \{0.25, 0.5, 2.0\} \text{ MW} \tag{1}$$

and the vector of time intervals, given as a percentage of the day in which the loads are exceeded, is:

$$T = \{100, 50, 12.5\} \tag{2}$$

The next step was to decide the main alternatives for energy generation. Due to the proximity of water falls and a weak wind speed, we consider three different alternatives for energy production to be implemented together or separately: thermal, hydro and photovoltaic production.

For each alternative source, costs of construction and operation were estimated. The cost structure was decomposed considering fixed and variable components and, taking into account the service life of the equipment, and the opportunity cost of capital, the equivalent annual cost of each alternative was evaluated.

With regard to thermal production (equipment and installation), generators of 600 kW are available. Each generator has a cost of about 285 kUSD and a useful life of 10 years. In terms

of hydro production, taking into account the flow rate and the height of fall of water, it is considered the construction of a small dam with a capacity of 7 MW. The estimated cost for this project is 23415 kUSD and the lifetime is considered infinite (i.e. more than 100 years). Regarding solar energy, 300 kW modules are available, costing 1338 kUSD and have a service life of 20 years.

The optimal strategy for the satisfaction of energy needs is determined solving an optimization problem, which goal is to select the hybrid combination of energy sources that minimizes the total cost of energy generation. To formulate this problem, equivalent investment costs in the same time base are evaluated. These costs represent an equivalent rent that tax payers would have to pay to an independent investor who decided to build and rent the infrastructure. This rent is equivalent to a fixed cost to be added to the fixed costs related to operating the system.

Relatively to the operating fixed costs, we consider essentially maintenance costs, direct labor costs, and transportation costs. Based on the needs for each type of generation, we obtained the following results: 20.1 kUSD a year per generator for thermal production; 53.03 kUSD a year per module for photovoltaic production; and 1078.7 kUSD a year for hydro production.

To evaluate the equivalent investment annual cost, we assume that the present value of the rents to pay the investor equals the value of the investment (Richard and Stewart, 2000). Accordingly, the discount rate is the opportunity cost of capital for investments of equal risk level. Since the rents would be guaranteed by the Angolan state, the appropriate cost of capital will be approximately 3% per year, which corresponds to the yield on long-term bonds issued in dollars by the Angolan state.

Adding the equivalent investment annual cost, to the fixed operating costs, leads to the annual fixed cost that covers the financial and operational components. Dividing this value by 365 days and the production capacity of each unit, a (daily) equivalent acquisition cost per unit of capacity is obtained.

However, the procedure to evaluate the total fixed cost has to distinguish two situations: the permanent introduction of capacity, with replacement of equipment at the end of the economic life, and the single introduction, without renewal. The permanent introduction of capacity is considered in the penultimate stage in the multi-stage modeling, and in the underlying deterministic LP model.

In the case of permanent introduction of capacity the equivalent annual cost is obtained from the expression:

$$I + \frac{I}{(1+r)^n - 1} + \frac{CF}{r} = \frac{CA}{r} \tag{3}$$

where I is the investment, CF is the annual operating cost, CA is the total annual fixed cost, r is the opportunity cost of capital, and n is the economic life of the investment.

In the case of single introduction of capacity, without renewal, the equivalent annual cost is obtained from the expression:

$$I + \frac{CF}{r}\left(1 - \frac{1}{(1+r)^n}\right) = \frac{CA}{r} \tag{4}$$

Without loss of generality, we assume the possibility of single investments, for all technologies, for the period of 10 years, which also corresponds to the time interval adopted between stages.

In practice, in the case of solar energy production, that assumption corresponds to assume that the rental contract can be cancelled in the middle of the economic life of the units, without cost. In this case, we consider as investment, the value of the equivalent provision if the payment is made in two equal installments, one at the beginning and another half life.

Relatively to hydropower, since it can not be discontinued, that assumption corresponds to assume that we consider the cost of introducing it in the penultimate stage and then we account for the anticipation of its introductions in early stages. In this case, we consider as investment, the value of the equivalent provision if the payment is made in constant installments every 10 years.

Using the indices 1, 2 and 3 to designate respectively the means of production thermal, hydropower and photovoltaic, the vectors of equivalent acquisition cost per unit of capacity are:

$$cr = \{0.2443, 0.6971, 1.3056\} \ \frac{\$}{kW\,day} \tag{5}$$

$$cs = \{0.0625, 0.1784, 0.3341\} \ \frac{\$}{kW\,day} \tag{6}$$

the vector *cr* applies to investments with replacements in the end of the economic life, and *cs* is for single investments.

The variable costs are the opportunity costs of the resources used to produce an additional unit of energy. In our case, we have to consider only the costs of production of thermal energy. To assess these costs we consider the calorific value of the fuel as 42.7 MJ/kg and the efficiency of the generators as 30% (Heywood, 1988). If the opportunity cost of fuel is 1 $/kg, since this cost is zero for the other means of production, we obtain the following vector of variable costs:

$$f = \{0.2812, 0, 0\} \ \frac{\$}{kW\,h} \tag{7}$$

In the underlying LP model the variable costs are considered constant from the first year of operation. In the multi-stage formulation we consider that they are constant between stages and constant forever from the year following the penultimate stage. These costs are also introduced calculating, if appropriate, their equivalent perpetuity.

UNCERTAINTY CHARACTERIZATION

The stochastic parameters considered in the multi-stage model are the three levels of the LDC. Considering that they are perfectly correlated, the LDC can be set from the maximum level of power. Thus, we consider that the future evolution of power needs can be described by a generalized Wiener process. Adopting a normal distribution at every stage, the evolution of power requirements can be described from the following stochastic differential equation:

$$dU(t, W) = \mu\, dt + \sigma\, dW \tag{8}$$

where U represents the level of power needs, μ is the expected rate of change of U, σ^2 is its variance per unit of time, W represents a Brownian motion, and t is the time. The parameters of this differential equation are evaluated imposing the distribution of long-term power requirements (in 30 years). Thus we adopt, as reference, a distribution with an expected value of $2U_0$, and a variance of $2/3\,U_0^2$, being U_0 the current level of power requirements. This distribution is consistent with the forecasts of growing energy needs worldwide, and with the expected increase, above average, in energy needs in Angola, in the coming decades (USEIA, 2011).

Due to the multivariate nature of the optimisation problem, integrations involving the distribution of power requirements are in general impracticable. However this limitation may be overcome adopting a methodology that makes possible to substitute the continuous distribution by a discrete distribution with identical first and second moments.

To obtain the discrete distribution the temporal evolution of U was simulated using trinomial movements (Hull, 1993) in a uniform grid that is obtained by discretizing the time and the power.

To the first node, which represents the current conditions, is assigned a probability of one. From each node, at stage τ and state s (see table 1) with power level U_τ^s, where τ and s are the indexes of the time and power coordinates, respectively, we consider three possibilities of evolution for the next stage: one for the central node $U_m = U_{\tau+1}^s$, one of rising $U_u = U_{\tau+1}^{s+1}$ and another of declining $U_d = U_{\tau+1}^{s-1}$. The evolution probabilities for these states are obtained from the following expressions (Barreiros, 2005):

$$P_u = \frac{\sigma^2 \Delta t + \eta^2}{2\Delta U^2} + \frac{\eta}{2\Delta U} \tag{9}$$

$$P_m = 1 - \frac{\sigma^2 \Delta t + \eta^2}{\Delta U^2} \tag{10}$$

$$P_d = \frac{\sigma^2 \Delta t + \eta^2}{2\Delta U^2} - \frac{\eta}{2\Delta U} \tag{11}$$

where Δt and ΔU are the discretization parameters of time and power, and η represents the deviation from the central node relative to the expected value, that is $\eta = \mu\,\Delta t + U_\tau^s - U_{\tau+1}^s$. The probabilities of occurrence of the nodes (states) of each stage are obtained from the following recurrence relation (Barreiros and Cardoso, 2008):

$$P_\tau^s = P_u\, P_{\tau-1}^{s-1} + P_m\, P_{\tau-1}^{s} + P_d\, P_{\tau-1}^{s+1} \tag{12}$$

Without loss of generality, in this study we consider a network of U with three time steps, with the final central value equal to 4400 kW and the parameters ΔU and Δt equal to 1200 kW and 10 years, respectively. Table 1 shows the trinomial tree, obtained with these assumptions, showing the power values and the probabilities, for the time evolution of the energy needs for the village of Ambriz.

Table 1 - Trinomial tree of the power needs for the village of Ambriz

τ							
0				2000 1			
1			1600 0.3704	2800 0.3703	4000 0.2593		
2		1200 0.1372	2400 0.2743	3600 0.3292	4800 0.1921	6000 0.0672	
3	800 0.0508	2000 0.1524	3200 0.2591	4400 0.2642	5600 0.1814	6800 0.0747	8000 0.0174
s	-3	-2	-1	0	1	2	3

MODELLING APPROACHES

Before the formulation of the optimisation problem, let's consider the assumption of no combination of the production systems. In this case, the most economically advantageous solution can be evaluated directly by comparing the total cost for the different alternatives, with or without the use of excess capacity.

If it is not possible to take advantage of excess capacity, the available alternatives correspond to acquire the minimum required capacity and the best solution depends on the maximum power and the daily energy consumed. A photovoltaic power plant may be the best alternative for the actual scenario, with a daily cost of 2742 USD, against the cost for thermal production of 4382 USD, and the cost of hydro production of 4880 USD. However, if the consumption duplicates, the mini-hydro plant should be the best solution keeping the daily cost equal to 4880 USD.

To account for the benefits of overcapacity, particularly in water production, we have to introduce a price of energy transfer to other locations. In this case, the maximum allowance is the fuel savings induced in neighbouring towns, which only alternative is the thermal production. However, to evaluate a sale price to other localities, we should consider that not all the energy transferred is received by potential customers. Additionally, the means of production themselves are already installed, so it will be convenient to provide incentives to not use them. So, in order to determine the benchmark selling price P_E of the kWh produced to other localities, we introduce two additional parameters: the transfer efficiency and the incentive factor of not using thermal production. Considering both of these factors equal to 0.75, we get $P_E = 0.1582$ \$/kWh.

Now, to evaluate the possibility of combining two or more sources of energy we consider also the possibility of receiving energy from other systems, being P_D the benchmark acquisition unit price of thermal energy from nearby localities. Considering the same transfer efficiency and assuming a profit margin of the producer equal to 0.25 we get $P_D = 0.4686$ \$/kWh.

The underlying deterministic LP model

The LDC of the Fig. 1 shows the distribution of the energy needs for a daily cycle, considering the current levels of consumption. In practice, the consumption and its daily distribution are time varying and are subject to uncertainty. Then, the optimisation problem should be solved as a stochastic programming problem, in which the levels of the load duration curve are treated as random variables.

An approximate solution can be obtained by ignoring the uncertainty and using the expected values of the LDC levels for a distant horizon. In the formulation of this problem are involved the decision variables, the constraints, and the objective function of the stochastic problem.

So, in the optimisation problem, we can identify two types of variables: variables related to the system definition, and variables related to the operational planning. The decision variables of the first type, x_j, are the generation of capacity (in kW) of type *j*. The decision variables of the second type, y_{ij}, represent the power requirements of the segment *i* served by generator *j*. As the capacity of the plants varies only by constant increments, we also introduce auxiliary variables, z_j, as the number of production units of type *j*. To perform the operational plan (i.e. the allocation of powers of generators to the required power levels) the segments of power, w_i, are still defined, as well as the corresponding time intervals, β_i, on the adopted daily basis. Then, the underlying deterministic LP problem can be formulated as:

$$Min\ \ C_T = \sum_j cr_j x_j + \sum_i \sum_j f_j \beta_i y_{ij} + P_D \sum_i \beta_i d_i - \sum_j \left(P_E - f_j\right) E_j \tag{13}$$

$$\begin{aligned}
&s.t.\\
&\sum_i \beta_i y_{ij} + E_j - \beta_1 x_j = 0, \quad \forall j \in J\\
&\sum_i y_{ij} + q_j - x_j = 0, \quad \forall j \in J\\
&\sum_j y_{ij} + d_i = w_i, \quad \forall i \in I\\
&x_j - Q_j z_j = 0, \quad \forall j \in J\\
&\sum_j x_j \le Q_u\\
&x_j, y_{ij}, q_j, d_i, E_j \ge 0,\ z_j \in \square, \quad \forall i \in I, \forall j \in J
\end{aligned}$$

The terms of the objective function represent respectively the total fixed costs, the operating costs, the acquisition costs of energy from other locations, and the contribution margin of sales.

The first set of constraints results from the balance between the energy available from the system, the power supplied to the segments of power, and the energy available to transfer to the exterior, E_j. The second set of constraints limits the available power of the medium *j* at its nominal power. The third set of constraints requires that each segment of power receives the necessary energy using, if necessary, the external source, d_i. The fourth set restricts the capacity to be installed by constant increments, that is: $Q = \left(600, 7000, 300\right)$ kW for the case of Ambriz, being Q_j the power of the unit of source *j*. The parameter Q_u is the upper limit of the total power to install, which was set to 9000 kW.

The multi-stage stochastic model

The linear programming model presented above calculates the optimal investment plan, and the operational strategies, for the deterministic problem of capacity setup. With perfect information, this solution represents a lower bound of the present value of the costs for a given scenario. However, omitting the uncertainty of the random factors can lead to solutions of limited utility. Moreover, the stochastic programming approach provides optimised solutions that take into account the possibilities of realization of the random variables, in particular, considering specific strategies to adapt to these realizations.

The stochastic problem, which is obtained from the deterministic problem by incorporating the uncertainty related with the future evolution of the energy needs, is a decision problem with four stages, where the planning decisions of the system are considered in the first three stages, and the operational decisions are evaluated in the three last stages.

The objective is to minimize the expected value of the total annual cost over the planning horizon:

$$Min \sum_{\tau\geq 1}\gamma_{\tau-1}E\left\{\sum_j c_j^{\tau-1}x_j^{\tau-1}+\alpha_\tau\left[\sum_i\sum_j f_j\beta_i y_{ij}^\tau+P_D\sum_i\beta_i d_i^\tau-\sum_j\left(P_E-f_j\right)E_j^\tau\right]\right\} \tag{14}$$

$$\begin{aligned}
s.t.\quad & \\
& \sum_i \beta_i y_{ij}^\tau + E_j^\tau - \beta_1 x_j^{\tau-1} = 0, \quad \forall j\in J, \forall\tau\geq 1\\
& \sum_i y_{ij}^\tau + q_j^\tau - x_j^{\tau-1} = 0, \quad \forall j\in J, \forall\tau\geq 1\\
& \sum_j y_{ij}^\tau + d_i^\tau = w_i^\tau, \quad \forall i\in I, \forall\tau\geq 1\\
& x_j^{\tau-1} - Q_j z_j^{\tau-1} = 0, \quad \forall j\in J, \forall\tau\geq 1\\
& \sum_j x_j^{\tau-1} \leq Q_u, \quad \forall\tau\geq 1\\
& x_j^\tau, y_{ij}^\tau, q_j^\tau, d_i^\tau, E_j^\tau \geq 0,\ z_j^\tau\in\square, \quad \forall i\in I, \forall j\in J, \forall\tau\geq 0
\end{aligned}$$

Where $\gamma_\tau=\left(1+r\right)^{-n\tau}$ is the discount factor from the time of stage τ (n is the number of years between two consecutive stages), α_τ is a factor that relates the PV of the annuity and the PV of the perpetuity: $\alpha_\tau=1-\left(1+r\right)^{-n}$ if $1\leq\tau<\tau_T$ and $\alpha_\tau=1$ if $\tau=\tau_T$ (τ_T signifies the last stage). The equivalent acquisition cost per unit of capacity is defined as $c_j^\tau=cs_j$ if $0\leq\tau<\tau_T-1$ and $c_j^\tau=cr_j$ if $\tau=\tau_T-1$.

To clarify the significance of this problem, we illustrate below, the sequence of calculations. In what follows we omit the set of restrictions and use the symbol $U_{[\tau]}\equiv\left(U_1,..,U_\tau\right)$ to designate the history of the demand process in periods $1,..,\tau$.

In each period ($0\leq\tau\leq\tau_T-1$) the definition of capacity depends on the available information, i.e. an observed realization $u_{[\tau]}$ of the search process, and not on future observations. Thus, starting from the last stage, we need to evaluate the set of contributions for the total cost $Q_{\tau_T-1}\left(x_{\tau_T-1},u_{\tau_T-1}\right)$, which is defined by:

$$Min\ \gamma_{\tau_T-1}\left\{\sum_j c_j^{\tau_T-1}x_j^{\tau_T-1}+\alpha_{\tau_T}E\left[\sum_i\sum_j f_j\beta_i y_{ij}^{\tau_T}+P_D\sum_i\beta_i d_i^{\tau_T}-\sum_j\left(P_E-f_j\right)E_j^{\tau_T}\right]\right\} \quad (15)$$

Since the random process of U follows the Markov property, the expectation in the above equation is conditional only to $U_{\tau-1}=u_{\tau-1}$. Therefore, considering that we have the discrete representations of $U_{[\tau_T-1]}$, it depends only on the three possibilities of evolution from $u_{\tau-1}$, and it is evaluated considering the three elementary probabilities defined by equations (9), (10) and (11). Additionally, as the discrete representation of U_{τ_T-1} is represented by $2\tau_T-1$ states, the expected value of the contributions for the total cost is evaluated from:

$$\bar{Q}_{\tau_T-1}\left(x_{\tau_T-1}\right)=E\left[Q_{\tau_T-1}\left(x_{\tau_T-1},u_{\tau_T-1}\right)\right]=\sum_{s=-\tau_T+1}^{s=\tau_T-1}P\left(\tau_T-1,s\right)Q_{\tau_T-1}\left(x_{\tau_T-1},u_{\tau_T-1}\right) \quad (16)$$

where $P\left(\tau_T-1,s\right)$ represents the probability of $U_{\tau-1}=u_{\tau-1}$.And, this procedure is applied to all stages, backward to $\tau=1$.

It matters also to mention some details related to the numerical procedure of the optimal search. First, we note defining the availability of hydroelectric generation. The introduction of the dam was tested in the penultimate stage, from each state. That is, if introduced into the state s, it is also introduced in all states with index higher than s. Then, adopting the same procedure, the anticipation of its introduction is tested in previous stages. In these circumstances, in each state of each stage are tested complementarities with different combinations of the other means of production.

RESULTS

The solution of the problem depends on the assumptions that we introduce, which are related with various parameters whose values are not known accurately. In this process, we highlight the parameters related to the evolution of energy demand, and the possibility of utilization of the energy surplus. Indeed there are other uncertain parameters that also influence the best decision, such as the development of technology acquisition costs. However, regarding these, since we have no information to forecast their evolution, we decided to consider them constant. Thus we analyse cases with, and without, uncertainty and with, and without, transference of energy to other locations.

Thus, considering the doubling of the long-term energy needs, if we don't consider the transfer of energy to other locations, and if we use the underlying deterministic LP model, we get, as most advantageous solution, a combination of thermal and photovoltaic generation: $x=\left[3000,0,1200\right]$ kW with $C_T=4662$ USD/day, being 49.3% related with total fixed costs. In this case the first two levels of the LDC would be met by the photovoltaic production, $y_{13}=y_{23}=500$ kW, and the third one by both types of production, thermal $y_{31}=2800$ kW and photovoltaic $y_{33}=200$ kW.

Nevertheless, in these conditions, the total daily cost for the solution corresponding to the mini-hydro only generation is slightly higher (about 5%). But if the exchanges with other locations are feasible, and if it is possible to cede a small part of the energy surplus (e.g. 5%) the decision falls on the dam, with a total fixed cost of 3765 USD/day, where 4880 USD/day

are related to the investment and 1115 USD/day are revenues from concessions to other locations.

This analysis shows that if the decision has to be taken immediately, the definition of the use of the surplus energy has a decisive influence. But, ideally the decisions related to this type of problem should be influenced by the ability to change the configuration of the energy production system according to the developments of the uncertain parameters. The results of the multi-stage model with uncertainty illustrate this possibility and also show that the utilization of part of the surplus energy influences the conditions of introduction of hydropower.

If we don't consider the exchange of energy between locations, the solution of the multi-stage model is characterized by an expected total fixed cost of 4085 USD/day, which corresponds to the investment decisions presented in the following table.

Table 2 - Trinomial tree of the power needs for the village of Ambriz

τ					
0			3000 0 1200		
1		2400 0 1200	3600 0 1200	0 7000 0	
2	2400 0 900	3000 0 1500	0 7000 0	0 7000 0	1200 7000 0
s	-2	-1	0	1	2

This solution stipulates that the best decision should be investing now in the thermal power (3000 kW) and in the photovoltaic (1200 kW). If after 10 years the evolution of the demand is very favourable then we should change for hydropower. Otherwise, and if the evolution is of growth in demand, according to the expected, or to a better evolution, the dam should be introduced in 20 years.

The following table shows the results that allow the identification of the different components of the expected total cost of energy. The first line of each stage presents the component of total fixed costs. The second line concerns the variable operating costs, and the costs related to interactions with other locations.

For example, given the probabilities shown in Table 1, we conclude that the contributions of the expected total fixed costs in the three stages will be 588.42, 565.35 and 2143.25 USD/day, respectively, which represents an expected cost of 3297 USD/day (about 81%). Similarly, the expected values of the operating costs, associated with the investments made in the first three stages, the variable costs, and the costs related to interactions with other locations, represent 316.58, 198.30, and 273.14 USD/day, respectively, representing an expected cost of 788.03 USD/day.

Table 3 - Components of the total cost of the energy (USD/day)

τ					
0			588.52 316.58		
1		410.00 171.32	465.83 364.06	929.21 0	
2	975.28 468.75	1490.18 731.66	2701.86 0	2701.86 0	2864.21 121.08
s	-2	-1	0	1	2

Moreover, the operating costs of 121.08 USD/day, associated with the second state of stage 2, can be explained as follows. The power installed on this node is 8200 kW, being 7000 for the hydro generation and 1200 for the thermal. According to the possibilities of evolution from this node, shown in Table 1, the future power needs can be 5600, 6800 and 8000 kW. In the first two scenarios the thermal energy is not necessary. In the third it will be necessary 1000 kW for 3 hours per day. The estimated cost will be 843.60 USD/day. Considering the probability of occurrence of this scenario, i.e. $P_u = 0.2593$, and that these costs are considered constant from the year following the installation of the means of production, we arrive to that value.

If the exchanges with other locations are feasible, and if it is possible to cede 5% of the energy surplus, the expected total fixed cost is 3587 USD/day, and the investment policies are presented in table 4.

Table 4 - Capacity decisions (kW) if 5% of surplus energy is transferred

τ					
0			1800 0 1200		
1		1200 0 1200	0 7000 0	0 7000 0	
2	1200 0 900	0 7000 0	0 7000 0	0 7000 0	0 7000 0
s	-2	-1	0	1	2

Comparatively to the previous solution, the best decision should be investing now in the thermal power but less (1800 kW), because it would compensate to acquiring energy from the exterior, if it is available and demand grows more than expected, and the same capacity of the photovoltaic (1200 kW).

If after 10 years the evolution of the demand is according to expected, or better, the dam should be introduced.

If it is possible to cede 10% of the energy surplus, the best solution corresponds to the immediate construction of the dam at a cost of 2613 USD/day which 4880 are investment-related and 2267 USD/day are revenue assignments to other locations.

CONCLUSIONS

The motivation for this work arose from the need to find sustainable solutions that contribute to minimizing the cost of energy needs of rural populations in developing countries.

This paper presents a study on the optimisation of hybrid systems for generating electrical energy, including the following energy sources: photovoltaic, wind, hydro and thermal. The optimisation problem is formulated as a multi -stage stochastic programming problem on the generation capacity, and the power requirements of the different sources.

This study was conducted with the support of three real case studies of Angola, which were developed in the villages of Ambriz, Muxiluando and Tombwa. Nevertheless, the conclusions presented are based on the results obtained for the case of the Village of Ambriz. The aim was to find the best solution for electricity generation seeking the exploitation of natural resources.

One should note that the use of natural resources, namely hydropower, has economic advantages even when energy needs are lower than the potential of generation available. An important contribution to support this conclusion comes from the consideration of uncertainty regarding the future energy needs. We also found that the possibility of energy transfer among villages also reinforces the advantages of this solution.

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