MEASUREMENTS OF MASS MOMENTS OF INERTIA OF MULTI-BODY SYSTEMS

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ABSTRACT
This paper summarizes the evidence and results of the research focused on determining the inertia matrix (ellipsoid of inertia) of an aggregate and a passenger car. The work describes a proposed computational algorithm and further presents the results of measurements. The measured moment of inertia matrices may be further used in practice as one of the input parameters for the simulation and computation.

Keywords: measurement, inertia matrix, ellipsoid of inertia, vehicle, aggregate, engine.

INTRODUCTION
For determining moments of inertia, various methods are used. Generally, they are based on a principle of dependence between the moment of inertia of a body and the natural oscillation frequency. The basic methods for determining moments of inertia are based on the principle of a physical pendulum, a torsion suspension or a bifilar suspension, eventually a trifilar or a quadrafilar one (Aachen, 2006) (Best, 2004). Was created an experimental stand for determining the mass, center-of-gravity position and moments of a powertrain’s inertia (aggregate - combustion engine and gearbox) where necessary measurements (with the greatest degree of possible accuracy) were later carried out. For determining the moments of inertia, the indirect method of measuring the time of oscillation of the body hung on the unifilar (torsion) suspension was used. For providing the moments of inertia of bodies of more complex shapes, special preparations were used. These preparations enabled to hung the body in various positions onto the torsion suspension. Therefore, a subframe for gripping the aggregate was used. This aggregate was gripped into the subframe in a defined way so that the axes of the chosen system of coordinates of the aggregate were parallel with the axes of the system of coordinates of the subframe. Because the moments of inertia of the subframe are not insignificant, it was necessary to perform the measuring twice. The first one was performed for the assembly of the sub-frame with the aggregate. In the second case only the subframe itself was measured by reason of separate parameters evaluation and determination of elements of inertia matrix of the aggregate and the subframe. For determining the inertia matrix, at least six measurements are needed. In our case, at least ten measurements towards an arbitrary axis passing through the beginning were performed so that we would reach the highest accuracy and subsequent calculation of the size and position of the ellipsoid of inertia. In general, for n measurements we will get the matrix of n equations with six unknowns. In this case, these unknowns can be determined from the set of measured data by using the method of least squares (Brabec, 2009). These unknowns are 3 moments of inertia towards the relevant axes: \( J_x \), \( J_y \) and \( J_z \) and similarly 3 deviation moments: \( D_{xy} \), \( D_{xz} \) and \( D_{yz} \).
PRINCIPLE OF CREATING AN INERTIA ELLIPSOID OF AGGREGATE AND MEASUREMENT ACCURACY

If we plot the values of $\sqrt{\frac{1}{J}}$ from the origin using a specific scale as vectors along the axis of the suspension apparatus, the end points will form an ellipsoid of inertia in a space. The following simplified picture shows a process for plotting the inertia ellipsoid in the plane passing through the axis of the crankshaft and axes of the cylinders. Naturally, to enhance the accuracy, more measurements should be taken, ideally a few measurements using the suspension apparatuses which determine the points of the inertia ellipsoid in the proximity of ends of its half-axes (i.e. around the vertices of ellipse).

![Fig. 1 - Principle of plotting an inertia ellipsoid in a plane of the crankshaft.](image)

To verify the measurement method the following procedure was selected. Firstly, a geometrically simple body was chosen and the body passed through the complete measurement process as a separate powertrain - i.e. firstly, the frame-body assembly and then the frame itself were measured. A simple form of the body was selected due to a simple calculation of the body’s moment of inertia matrix which was further compared with the measured matrix. Considering the semi-finished products being available in our laboratory, an assembly consisting of two semi-finished products (Ø100 and Ø130 mm) was chosen. The position of the semi-finished products was chosen in such a way so that one of the deviation moments was not zero. The principal moments of inertia showed very good conformity, the most significant deviation occurred along the x-axis and it was less than 1.5%. The position of the semi-finished products can be considered asymmetrical. However, the asymmetry is chosen in such a way that two deviation moments are zero. For these two deviation moments the relative error cannot be determined, we are able to determine only the absolute one (i.e. -0.0895 kg.m² and -0.0198 kg.m²). The last deviation moment was measured approximately with the relevant error of 2.5%.
By further measurement to verify the method, a semi-finished product having the dimensions of Ø60-1500 mm was located into the position of the auxiliary frame diagonal. A comparison of the computed and measured moments of inertia values resulted in an average error of 1.10 %, the max. deviation occurred along the y-axis (2.54 %). The absolute error along the principal axis of inertia (identical with the semi-finished product axis) was less than 1° for all solid angles.

![Fig. 2 - Illustration of the measured inertia ellipsoid.](image)

**MEASUREMENT OF MOMENT OF INERTIA FOR WHOLE A CAR**

At our workplace, we also measured the moment of inertia of the whole vehicle and that his transverse and longitudinal axis through the centre of gravity. The experimental measuring device was created with the help of the student and his diploma work (Bárta, 2015). Student worked on an assignment with the company AZOS CZ, s.r.o. Measurement method is based on the principle of the physical pendulum. It was created by the optimized design proposal for the implementation of measurement device. Transverse, longitudinal and height position of the center of gravity of the vehicle (input parameters for measuring) was measured by weighing the vehicle in a horizontal position using four scales under each wheel and by tilting the vehicle on the front axle. Further was using an optical measuring system TRITOP for better results of measurement. Created methodology was verified by measuring inertia of passenger car ŠKODA Fabia III.
Furthermore, was carried out verification of the device and the determination of measurement error. As a suitable etalon for device verification options were chosen (because of the possibility of test room and limited financial resources) the same four barrels filled completely with liquid - that did not change the positions of centre of gravity of the barrel during the pendulum motion. With the position of the barrels on the platform relative to the axis of swing of the device has been verified, so that the measured values of the moments of inertia drums approximately cover a range of values of moments of inertia of the passenger vehicles. The resulting values were compared with the results obtained using the model of the situation in the CAD program. The entire method of measuring the moment of inertia exhibited total relative error smaller than 8%. This error is total and includes any influences on the measurement.

The advantage of the device lies in its simplicity, the minimum used sensors, possibilities of dismantling and the possibility of its further development. The disadvantage of this measurement device is more time consuming whole methodology and the impossibility to measure the moment of inertia of vehicles to their vertical axis. Use of measured results as the center of gravity and moment of inertia of the vehicle are particularly useful for dynamic simulation of driving a vehicle.

Fig. 3 - Examples of existing devices for measuring the moment of inertia of the vehicle: a) product InTenso from SMARTMechanical Company, b) TU-Dresden, c) device VIMM (source, www).
Fig. 4 - Measuring the inertia moment of the vehicle - modification of the platform for longitudinal and transverse axis of the car (Bárta, 2015).
RESULTS

The methodology was verified when measuring several aggregates or motors, in some cases for more variants of motor accessories layout (thirteen aggregates and one motor designed for passenger vehicles, one motor designed for a commercial vehicle). The measurement was always performed without any filling, which means without (engine) oil and coolant. The proposed measurement methodology was able to reveal even a relatively small difference in measurement depending on the powertrain’s accessories. Consequently, the change resulting from removal of the engine’s intake filter (1.4, MPI engine with mechanical gearbox) which formed 0.9 % of the total powertrain’s mass, could be observed.

Fig. 5 - Values of axial moments of inertia for axle Y of passenger vehicles aggregate, the values are presented in relation to a referential aggregate which was an ordinary light three-cylinder spark ignition engine with the volume of 1.2 dm³ with a manual five-gear gearbox.

From the graph in the Figure is obvious that the values sizes of axial moments of inertia of the aggregate are not linear dependent on the aggregate weight. The resulting values of moments of inertia and the centre-of-gravity position of the aggregate influence both the weight and also the position of additional devices and motor accessories. Neither relatively low physical parts which (regarding relatively big remoteness) can also influence the size of the overall moment of inertia of the aggregate cannot be neglected. The influence of the accessories was examined with careful attention; therefore, variant measuring of aggregates in different configuration with different accessories was also performed.
Fig. 6 - Arrangement of engine accessories for the impact assessment - e.g. the measurement carried out on the truck engine (Cummins 3.8 l Avalon) resulted in nearly 20 % change in the axial moment of inertia when approximately a 10 % change in mass occurred.

In the workplace were also carried out design work for the realization of devices (platforms) for measuring the moment of inertia of passenger vehicles at its lateral and longitudinal axis. For the measurement method was chosen based on principle of a physical pendulum. The methodology was verified and has made a measurement Skoda Fabia third generation (Bárá, 2015). The measurement results can be used for simulation of the vehicle dynamic behaviour.

CONCLUSION

Was created an experimental stand for determining the mass, center-of-gravity position and moments of a powertrain’s inertia where necessary measurements (with the greatest degree of possible accuracy) were later carried out.

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