Multicriteria Decision-Aid

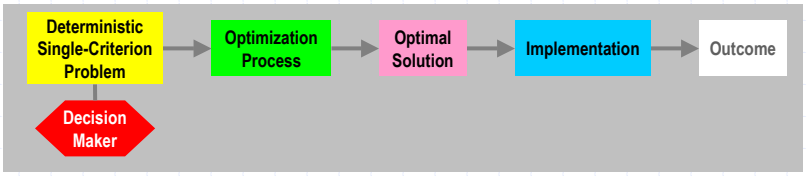
basic concepts and definitions

Manuel Matos
INESC Porto & FEUP
Portugal

INESC PORTO FEUP

The role of the decision maker

- Deterministic, single-criterion problems
  - The DM participates only in the problem formulation
  - The rest of the process is mainly technical, leading (hopefully) to the optimal solution
  - The decision is embedded in the problem formulation



Trivial decision problems

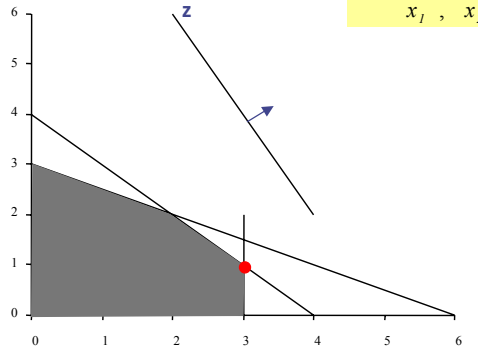
- Minimize Cost

n	Cost
1	65
2	58
3	72
4	72
5	60
6	65
7	71
8	51
9	67
10	90
11	67
12	86
13	66
14	52
15	76

- Maximize profit z

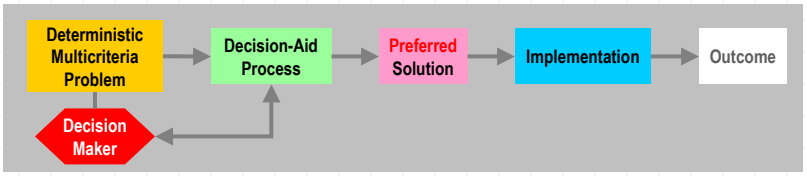
$$\max\ z = 2x_1 + x_2$$

$$\begin{aligned} suj:\quad & x_1 + x_2 \le 4 \\ & x_1 + 2x_2 \le 6 \\ & x_1 \le 3 \\ & x_1,\ x_2 \ge 0 \end{aligned}$$



The role of the decision maker

- Deterministic, multicriteria problems
  - The DM participates in the problem formulation
  - The structure of preferences of the DM must be incorporated in the problem
  - The process leads to the preferred solution



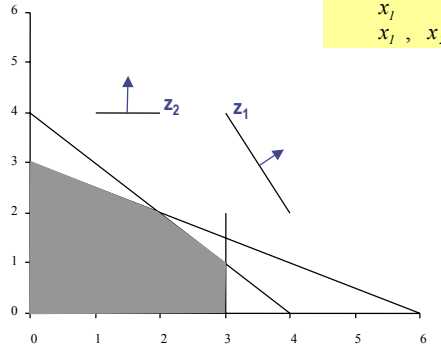
Multicriteria problems

- Minimize Cost
- Maximize Reliability

n	Cost	Reliability
1	65	0.994586
2	58	0.993677
3	72	0.995333
4	72	0.995531
5	60	0.994064
6	65	0.994641
7	71	0.995954
8	51	0.992906
9	67	0.995111
10	90	0.998551
11	67	0.995425
12	86	0.997641
13	66	0.994653
14	52	0.992848
15	76	0.995913

- Maximize profit z_1
- Maximize export z_2

$$\max\ z_1 = 2x_1 + x_2$$
$$\max\ z_2 = x_2$$
$$suj:\quad x_1 + x_2 \le 4$$
$$x_1 + 2x_2 \le 6$$
$$x_1 \le 3$$
$$x_1,\ x_2 \ge 0$$

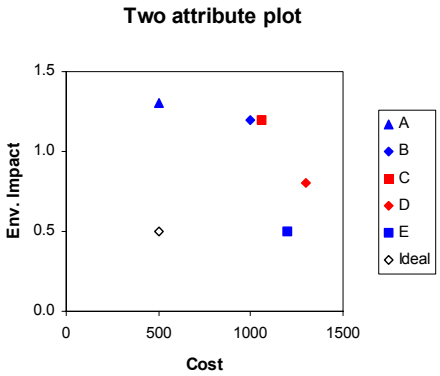


Some definitions

- Dominated (inferior) alternative
  - A solution is dominated iff there exists another one that is better in at least one criterion, without being worse in any of the remaining criteria
- Efficient (nondominated, noninferior, Pareto optimal) alternative
  - A solution is efficient iff it is not dominated by any other feasible alternative
- Ideal
  - (Non feasible) solution that joins up the individual optima
  - Defined only in the attributes' space

Example

- E dominates D
  - **E** is strictly better than **D** in both criteria
- B dominates C
  - **B** is strictly better than **C** in the **Cost** criterion
  - **B** is not worse than **C** in any criterion
- C and D are dominated
- A, B and E are efficient
  - They are not dominated by any other alternative



NB:
A possible rank: B, C, E, D, A

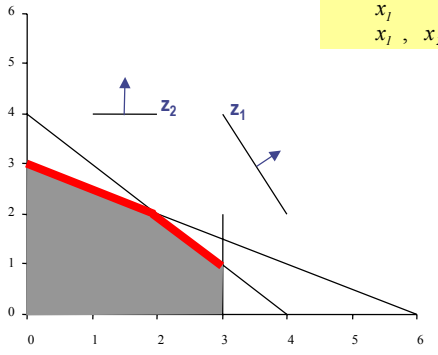
Examples

- Minimize Cost
- Maximize Reliability

n	Cost	Reliability
1	65	0.994586
2	58	0.993677
3	72	0.995333
4	72	0.995531
5	60	0.994064
6	65	0.994641
7	71	0.995954
8	51	0.992906
9	67	0.995111
10	90	0.998551
11	67	0.995425
12	86	0.997641
13	66	0.994653
14	52	0.992848
15	76	0.995913

- Maximize profit z_1
- Maximize export z_2

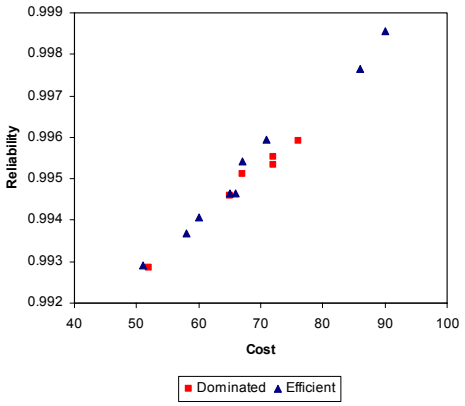
$$\max\ z_1 = 2x_1 + x_2$$
$$\max\ z_2 = x_2$$
$$suj:\quad x_1 + x_2 \le 4$$
$$x_1 + 2x_2 \le 6$$
$$x_1 \le 3$$
$$x_1\ ,\ x_2 \ge 0$$



Examples

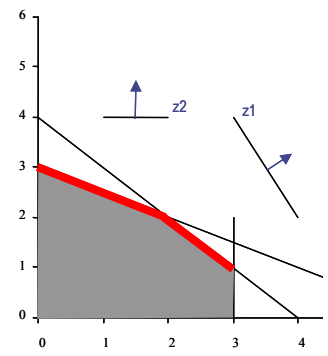
- Minimize Cost
- Maximize Reliability

n	Cost	Reliability
1	65	0.994586
2	58	0.993677
3	72	0.995333
4	72	0.995531
5	60	0.994064
6	65	0.994641
7	71	0.995954
8	51	0.992906
9	67	0.995111
10	90	0.998551
11	67	0.995425
12	86	0.997641
13	66	0.994653
14	52	0.992848
15	76	0.995913



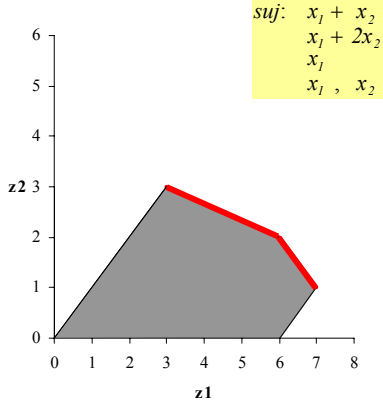
Decision space vs attribute's space

- Decision space



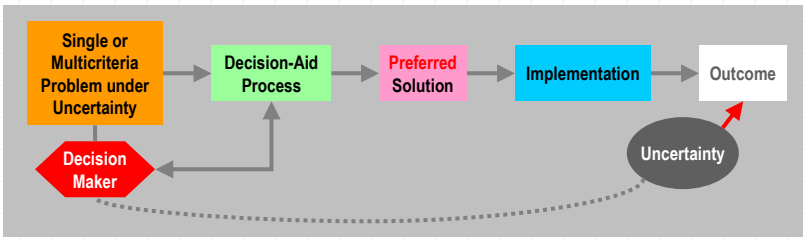
- Attribute's space

$$\max\ z_1 = 2x_1 + x_2$$
$$\max\ z_2 = x_2$$
$$suj:\quad x_1 + x_2 \le 4$$
$$x_1 + 2x_2 \le 6$$
$$x_1 \le 3$$
$$x_1,\ x_2 \ge 0$$



The role of the decision maker

- Single or multicriteria problems under uncertainty
  - The DM participates in the problem formulation and in the uncertainty characterization
  - The preferred solution results from the incorporation in the problem of the structure of preferences of the DM, including its risk attitude



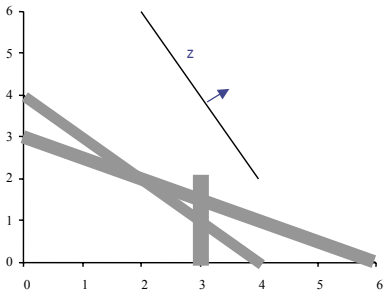
Different types of uncertainty

- Probabilistic - Different scenarios with probabilities

n	Cost		
	C1 (0.1)	C2 (0.6)	C3 (0.3)
1	59	65	75
2	**50**	58	71
3	68	72	**60**
4	69	72	62
5	53	60	63
6	51	59	65
7	68	71	77
8	56	57	75
9	62	58	80
10	62	**55**	70

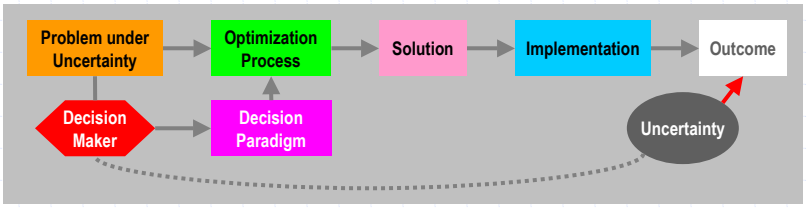
- Fuzzy - Vague or imprecise constraints

$$\max\ z = 2x_1 + x_2$$
$$suj:\quad x_1 + x_2 \tilde{\leq} 4$$
$$x_1 + 2x_2 \tilde{\leq} 6$$
$$x_1 \tilde{\leq} 3$$
$$x_1,\ x_2 \geq 0$$



The role of the decision maker

- Problems under uncertainty
  - Sometimes, the risk attitude of the DM is incorporated in the form of a pre-defined decision paradigm (expected value, regret, etc.)
  - This leads generally to an optimization process



Use of decision paradigms (or rules)

- Original problem
  - Dominated solutions shown

n	Cost		
	C1 (0.3)	C2 (0.6)	C3 (0.1)
1	59	65	75
2	**50**	58	71
3	68	72	**60**
4	69	72	62
5	53	60	63
6	51	59	65
7	68	71	77
8	56	57	75
9	62	58	80
10	62	**55**	70

- Min E(Cost)

n	Expected Cost
1	64.2
2	**56.9**
3	69.6
4	70.1
5	58.2
6	57.2
7	70.7
8	58.5
9	61.4
10	58.6

- Minimax Cost

n	Minimax Cost
1	75
2	71
3	72
4	72
5	**63**
6	65
7	77
8	75
9	80
10	70

Modeling

- Identification of
  - Agents (DM, regulators, competitors, consumers, etc)
  - Relevant criteria (how to compare the outcomes of two alternatives)
  - Main uncertainties
  - Alternatives
    - in the case of multiattribute problems
- Formulation of
  - Decision variables
  - External variables and parameters
  - Coherent family of **criteria**
  - Attributes
    - How to measure the satisfaction in each criterion
    - (e.g. **Criterion** – Minimize environmental impact. **Attribute** - $\%CO_2$

Modeling

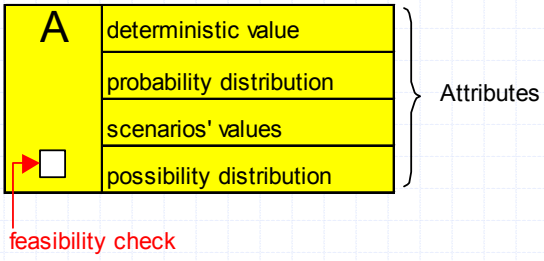
- A coherent family of criteria must be:
  - Exhaustive – All important points of view must be included
  - Consistent – If two alternatives A and B are equivalent except in criterion k, and A_k is better than B_k, then A must be at least as good as B
  - Non-redundant - Eliminating a criterion leads to the violation of one of the preceding axioms
- Other desirable proprieties
  - Legibility - The number of criteria used must be relatively low
  - Operationality - The family of criteria must be accepted by the stakeholders and the decision makers

Modeling

- Impact
  - Outcome of each particular decision (e.g. objective functions)
- Physical model
  - How to evaluate feasibility (e.g. mathematical constraints)
- Forecasting and estimation
  - Traditional (expected consumptions, wind power, etc)
  - Agents' behavior (demand curves, offer curves, criteria, etc)
- Uncertainty
  - Probability distributions
  - Scenarios (with or without probabilities)
  - Possibility distributions (fuzzy sets)

Alternatives

- Alternatives may be explicit (MA) or implicit (MO)
- To be a candidate, an alternative must be feasible
  - or *almost* feasible
- Decisions are made based on the attributes of each alternative



Multiattribute problems

	Criteria			
Alternatives	C_1	C_2		C_m
A_1	a_{11}	a_{12}	$\cdots$	a_{1m}
A_2	a_{21}	a_{22}	$\cdots$	a_{2m}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_n	a_{n1}	a_{n2}	$\cdots$	a_{nm}

Attributes
may be
real numbers, intervals, probability distributions, possibility distributions, qualitative labels

- Main characteristics
  - The alternatives are completely defined and assumed feasible
  - Attributes may be determinist, probabilistic, fuzzy (or mixed)
  - The problem may be: choice, ranking or sorting

Multiobjective problems

$$\begin{aligned} min \quad & \mathbf{f(x)} \\ st: \quad & \mathbf{g(x)} = \mathbf{0} \\ & \mathbf{h(x)} \le \mathbf{0} \\ & \mathbf{x} \ge \mathbf{0} \end{aligned}$$

$\mathbf{x}$	vector of decision variables *(may include integer or binary variables)*
$\mathbf{f(x)}$	vector of objective functions
$\mathbf{g(x)}$	set of equality constraints
$\mathbf{h(x)}$	set of inequality constraints

- Main characteristics
  - Alternatives are not known in advance
  - Optimization procedures are always needed
  - May have a big number of constraints and decision variables
  - May not be completely described by the mathematical formulation
  - Planning problems are generally combinatory

Multicriteria analysis - main approaches

- Ensure that the DM follows a "rational" behavior (Normative option)
  - *Value functions, Utility theory, distance to the Ideal*
- Give some advice based on reasonable (but not indisputable) rules
  - *The French School*
- Find the preferred solution from partial decisions about decision hypothesis
  - *Interactive methods*
- Prepare decision sets
  - *Generation methods*
    Filtering of efficient solutions

from Benjamin Franklin to the President

Dear Sir,

In the affair of so much importance to you, wherein you ask my advice, I cannot, for want of sufficient premises, advise you what to determine, but if you please I will tell you how. When those difficult cases occur, they are difficult, chiefly because while we have them under consideration, all the reasons pro and con are not present to the mind at the same time; but sometimes one set present themselves, and at other times another, the first being out of sight. Hence the various purposes or informations that alternatively prevail, and the uncertainty that perplexes us. To get over this, my way is to divide half a sheet of paper by a line into two columns; writing over the one Pro, and over the other Con. Then, during three or four days consideration, I put down under the different heads short hints of the different motives, that at different times occur to me, for or against the measure. When I have thus got them all together in one view, I endeavor to estimate their respective weights; and where I find two one on each side, that seem equal. I strike them both out. If I find a reason pro equal to some two reasons con, I strike out the three. If I judge some two reasons con, equal to three reasons pro, I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of further consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly. And, though the weight of the reasons cannot be taken with the precision of algebraic quantities, yet when each is thus considered, separately and comparatively, and the whole lies before me, I think I can judge better, and am less liable to make a rash step, and in fact I have found great advantage from this kind of equation, and what might be called moral or prudential algebra.

Wishing sincerely that you may determine for the best, I am ever, my dear friend, yours most affectionately.

B. Franklin