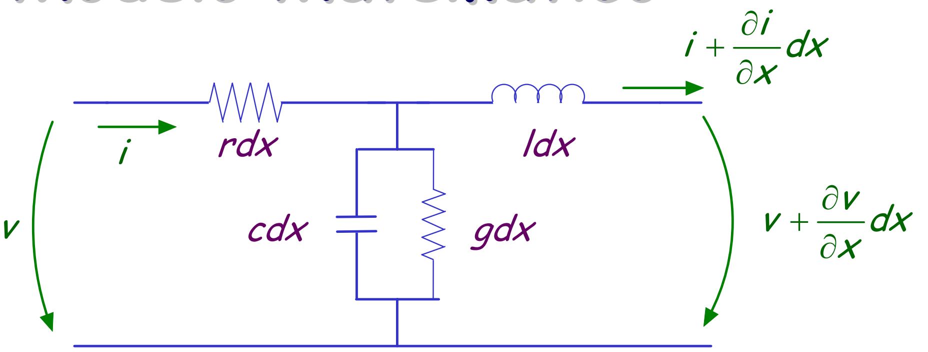
SISTEMAS ELÉCTRICOS DE ENERGIA I

Funcionamento das linhas aéreas

Modelo Matemático



$$\underline{Z} = r + j\omega l \quad \Omega m^{-1}$$

$$\underline{Y} = g + j\omega c \quad Sm^{-1}$$

Equações gerais

$$\begin{cases} -\dfrac{\partial v}{\partial x} = ri + l\dfrac{\partial i}{\partial t} \\ -\dfrac{\partial i}{\partial x} = gv + c\dfrac{\partial v}{\partial t} \end{cases}$$

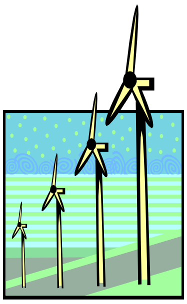
Em regime permanente e sinusoidal (T. Steinmetz)

$$\begin{cases} -\dfrac{\partial \underline{V}}{\partial x} = (r + j\omega l)\underline{I} \\ -\dfrac{\partial \underline{I}}{\partial x} = (g + j\omega c)\underline{V} \end{cases}$$

Separando as variáveis $\underline{I}$ e $\underline{V}$

Equações diferenciais lineares de coeficientes constantes acopladas ou correlacionadas

$$\begin{cases} -\dfrac{\partial^2 \underline{V}}{\partial x^2} = (r + j\omega l)(g + j\omega c)\underline{V} = \underline{Z}\underline{Y}\underline{V} \\ -\dfrac{\partial^2 \underline{I}}{\partial x^2} = (g + j\omega c)(r + j\omega l)\underline{I} = \underline{Y}\underline{Z}\underline{I} \end{cases}$$



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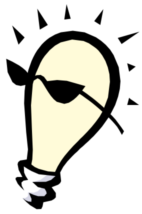


Funcionamento das linhas aéreas

considerando $\underline{\gamma} = \sqrt{(r + j\omega l)(g + j\omega c)} = \sqrt{\underline{Z}\underline{Y}} \quad (m^{-1})$

Constante de propagação

e $\underline{Z}_c = \sqrt{\dfrac{(r + j\omega l)}{(g + j\omega c)}} = \sqrt{\dfrac{\underline{Z}}{\underline{Y}}} \quad \Omega$

Impedância característica

obtém-se

$$\begin{cases} -\dfrac{\partial^2 \underline{V}}{\partial x^2} = \underline{\gamma}^2 \underline{V}(x) \\ -\dfrac{\partial^2 \underline{I}}{\partial x^2} = \underline{\gamma}^2 \underline{I}(x) \end{cases}$$

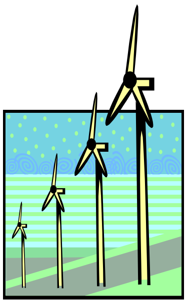
resolvendo

$$\begin{cases} \underline{V}(x) = \underline{V}_i.e^{-\underline{\gamma}x} + \underline{V}_r.e^{\underline{\gamma}x} \\ \underline{I}(x) = \dfrac{1}{\underline{Z}_c}\left(\underline{V}_i.e^{-\underline{\gamma}x} - \underline{V}_r.e^{\underline{\gamma}x}\right) \end{cases}$$

Equações de onda

Considerando a origem no início da linha (x=0)

$$\begin{cases} \underline{V}_i = \dfrac{\underline{V}_0 + \underline{Z}_c.\underline{I}_0}{2} \\ \underline{V}_r = \dfrac{\underline{V}_0 - \underline{Z}_c.\underline{I}_0}{2} \end{cases}$$



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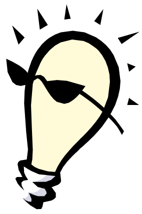


Funcionamento das linhas aéreas

E finalmente, as equações de onda

$$\begin{cases} \underline{V}(x) = \dfrac{\underline{V}_0+\underline{Z}_c.\underline{I}_0}{2}e^{-\underline{\gamma}x} + \dfrac{\underline{V}_0-\underline{Z}_c.\underline{I}_0}{2}e^{\underline{\gamma}x} \\ \underline{I}(x) = \dfrac{1}{\underline{Z}_c}\left(\dfrac{\underline{V}_0+\underline{Z}_c.\underline{I}_0}{2}e^{-\underline{\gamma}x} - \dfrac{\underline{V}_0-\underline{Z}_c.\underline{I}_0}{2}e^{\underline{\gamma}x}\right) \end{cases}$$

Vou dar um jeito a estas equações e ficar na história

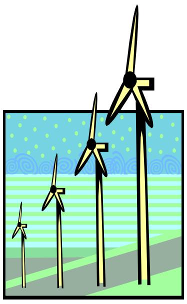
$$\begin{cases} \underline{V}(x) = \underline{V}_0\, ch\underline{\gamma}x - \underline{Z}_c\underline{I}_0\, sh\underline{\gamma}x \\ \underline{I}(x) = \underline{I}_0\, ch\underline{\gamma}x - \dfrac{\underline{V}_0}{\underline{Z}_c} sh\underline{\gamma}x \end{cases}$$

Equações dos telegrafistas

Valores típicos para as linhas de energia

$Z_c = 400\Omega$

$\beta = \pi/3000$ rad/km



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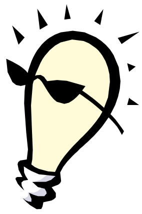


Funcionamento das linhas aéreas

Considerando x = L (comprimento da linha) define-se:

$$\theta = \gamma . L$$

Ângulo característico

0,1 — 0,5 — θ

Linhas curtas | Linhas médias | Linhas longas

Matricialmente

$$\begin{bmatrix} \underline{U}_L \\ \underline{I}_L \end{bmatrix} = \begin{bmatrix} ch\theta & -\underline{Z}_c sh\theta \\ -\dfrac{sh\theta}{\underline{Z}_c} & ch\theta \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_0 \\ \underline{I}_0 \end{bmatrix}$$

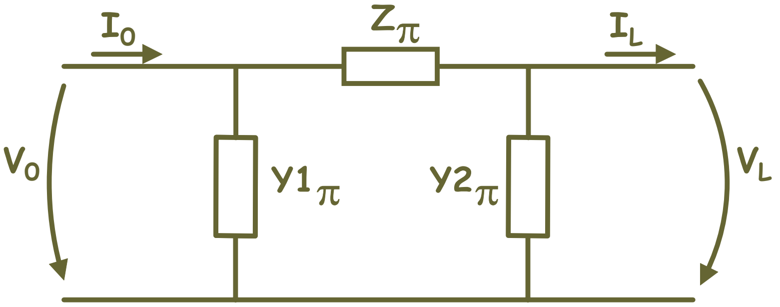
por inversão

$$\begin{bmatrix} \underline{U}_0 \\ \underline{I}_0 \end{bmatrix} = \begin{bmatrix} ch\theta & \underline{Z}_c sh\theta \\ \dfrac{sh\theta}{\underline{Z}_c} & ch\theta \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_L \\ \underline{I}_L \end{bmatrix}$$

A linha é um quadripolo passivo, simétrico

Funcionamento das linhas aéreas

Esquemas equivalentes



Esquema em π

Para este circuito

$$\begin{cases} \underline{V}_L = \underline{V}_0 - \underline{Z}_\pi \left(\underline{I}_0 - \underline{Y1}_\pi . \underline{V}_0 \right) \\ \underline{I}_L = \underline{I}_0 - \underline{Y1}_\pi . \underline{V}_0 - \underline{Y2}_\pi . \underline{V}_L \end{cases}$$

De outra forma

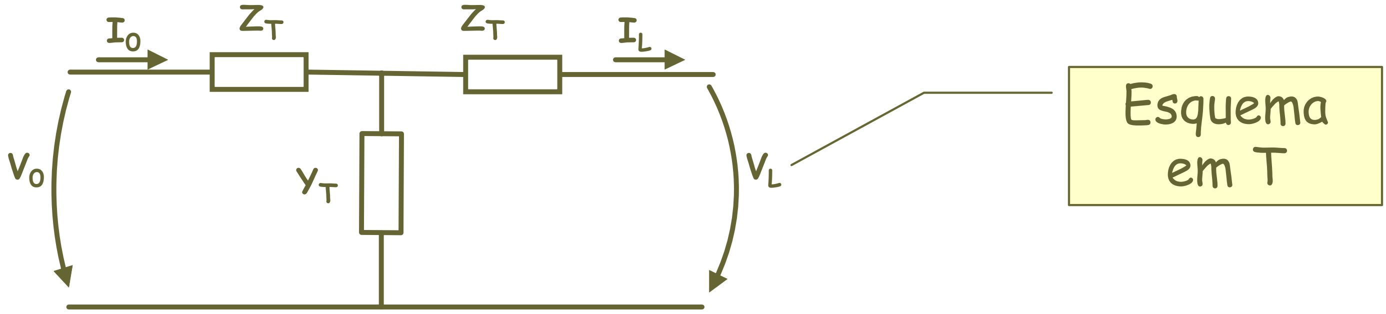
$$\begin{cases} \underline{V}_L = \underline{V}_0 \left(1 + \underline{Z}_\pi . \underline{Y1}_\pi \right) - \underline{Z}_\pi \underline{I}_0 \\ \underline{I}_L = \underline{I}_0 \left(1 + \underline{Z}_\pi . \underline{Y2}_\pi \right) - \underline{V}_0 \left(\underline{Y1}_\pi + \underline{Y2}_\pi + \underline{Z}_\pi . \underline{Y1}_\pi . \underline{Y2}_\pi \right) \end{cases}$$

Comparando com as equações dos telegrafistas

$$\begin{cases} \underline{Z}_\pi = \underline{Z}_c . sh\gamma x \\ \underline{Y1}_\pi = \underline{Y2}_\pi = \dfrac{1}{\underline{Z}_c} \dfrac{ch\gamma x - 1}{sh\gamma x} \end{cases}$$

Funcionamento das linhas aéreas

Esquemas equivalentes



Da mesma forma para este circuito

$$\begin{cases} \underline{Z}_T = \underline{Z}_c . \dfrac{ch\gamma x - 1}{sh\gamma x} \\ \underline{Y}_T = \dfrac{sh\gamma x}{\underline{Z}_c} \end{cases}$$

Linhas curtas

$$x = L \Rightarrow \quad \gamma x = \gamma L \quad \Rightarrow \quad \gamma L = \theta$$

Para linhas curtas são válidas as aproximações

$$sh\underline{\theta} \simeq \underline{\theta} \qquad ch\underline{\theta} \simeq 1 + \frac{\underline{\theta}^2}{2}$$

assim

$$\underline{Z}_c . sh\underline{\theta} \simeq \underline{Z}_c . \underline{\theta} = \underline{Z} . L$$

$$\frac{sh\underline{\theta}}{\underline{Z}_c} \simeq \frac{\underline{\theta}}{\underline{Z}_c} = \underline{Y} . L$$

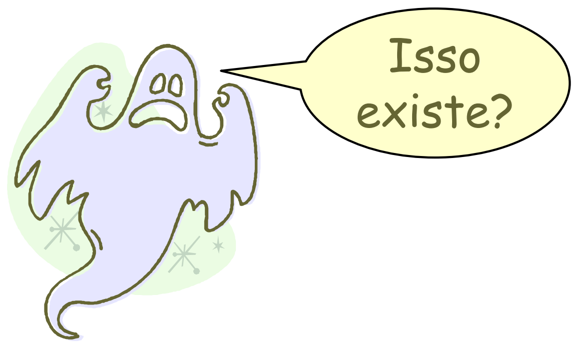
SISTEMAS ELÉCTRICOS DE ENERGIA I

Funcionamento das linhas aéreas

Esquemas equivalentes (Quadro resumo)

	π	π (L. Curta)	T	T (L. Curta)
Z	$\underline{Z}_c.sh\theta$	$\underline{Z}.L$	$\frac{ch\theta-1}{sh\theta}.\underline{Z}_c$	$\frac{\underline{Z}.L}{2}$
Y	$\frac{ch\theta-1}{\underline{Z}_c.sh\theta}$	$\frac{\underline{Y}.L}{2}$	$\frac{sh\theta}{\underline{Z}_c}$	$\underline{Y}.L$

Linha ideal (sem perdas)



r=0

g=0

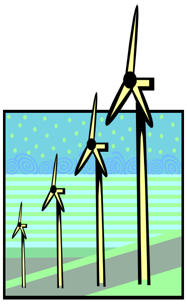
$$\underline{Z}_c = R_c = \sqrt{\frac{l}{c}}$$

$$\gamma = j\omega\sqrt{lc} = j\beta$$

$$ch\gamma x = \cos\beta x$$

$$sh\gamma x = j\,sen\beta x$$

$$\begin{bmatrix}\underline{U}_{(x)}\\ \underline{I}_{(x)}\end{bmatrix} = \begin{bmatrix}\cos\beta x & -jR_c sen\beta x\\ -j\dfrac{sen\beta x}{R_c} & \cos\beta x\end{bmatrix}.\begin{bmatrix}\underline{U}_0\\ \underline{I}_0\end{bmatrix}$$



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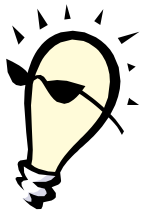
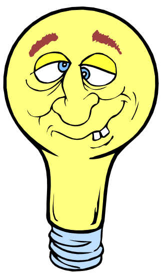


Funcionamento das linhas aéreas

POTÊNCIA NATURAL (OU CARACTERÍSTICA)

É a potência activa transmitida por uma linha sem perdas quando fornece potência reactiva nula na recepção, sendo as tensões iguais nas extremidades



$$\underline{U}_0 = \cos(\beta L).\underline{U}_L + jR_c \,\mathrm{sen}(\beta L).\underline{I}_L$$

conjugando

$$\underline{U}_0^* = \cos(\beta L).\underline{U}_L^* - jR_c \,\mathrm{sen}(\beta L).\underline{I}_L^*$$

$(\times \underline{U}_L)$

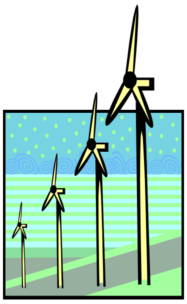
$$\underline{U}_0^*\underline{U}_L = \cos(\beta L).U_L^2 - jR_c \,\mathrm{sen}(\beta L).\underline{U}_L\underline{I}_L^*$$

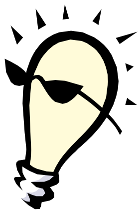
Considerando $\underline{U}_L = U_L e^{j0}$ e $\underline{U}_0 = U_0 e^{j\delta}$

$$U_0 U_L e^{-j\delta} = \cos(\beta L).U_L^2 - jR_c \,\mathrm{sen}(\beta L).(P_L + jQ_L)$$

Da definição $U_L = U_0 = U$ e $Q_L = 0$ $\Rightarrow$ $P_L = P_N$

$$U^2\cos(-\delta) + jU^2\mathrm{sen}(-\delta) = \cos(\beta L).U^2 - jR_c \,\mathrm{sen}(\beta L).P_N$$





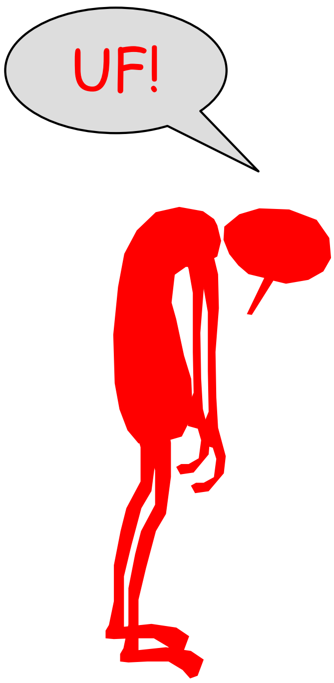
Funcionamento das linhas aéreas

Separando em parte real e imaginária

$$\begin{cases} U^2\cos\delta = U^2\cos(\beta L) \\ -U^2\operatorname{sen}\delta = -R_c\operatorname{sen}(\beta L).P_N \end{cases}$$

resolvendo

$$\begin{cases} \delta = \beta L \\ P_N = \dfrac{U^2}{R_c}\cdot\dfrac{\operatorname{sen}\delta}{\operatorname{sen}(\beta L)} \end{cases}$$



$$P_N = \frac{U^2}{R_c}$$