Class Notes
MAD – Decision Aid Methodologies – FEUP 2005

Multiattribute problems

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Multiattribute problems

	Criteria			
Alternatives	C_1	C_2		C_m
A_1	a_{11}	a_{12}	$\cdots$	a_{1m}
A_2	a_{21}	a_{22}	$\cdots$	a_{2m}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_n	a_{n1}	a_{n2}	$\cdots$	a_{nm}

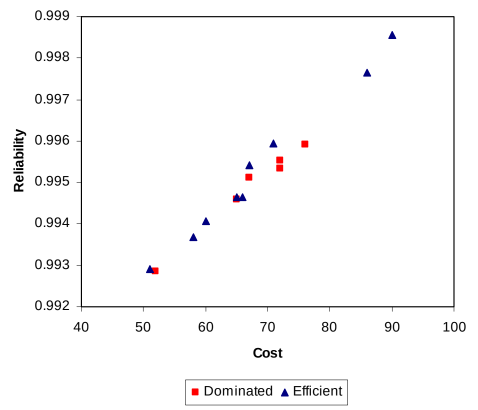
Attributes
may be
real numbers, intervals, probability distributions, possibility distributions, qualitative labels

- Main characteristics
  - The alternatives are completely defined and assumed feasible
  - Attributes may be determinist, probabilistic, fuzzy (or mixed)
  - The problem may be:
    - Choice – Select the best alternative
    - Ranking – Draw a complete order of the alternatives
    - Sorting – Select the best k alternatives from a list of n>k

Example

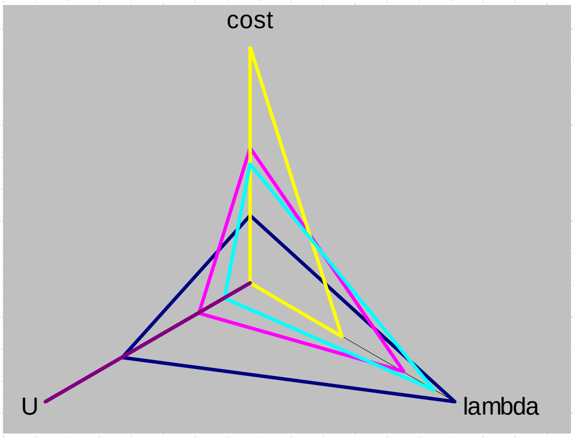
- Minimize Cost
- Maximize Reliability

n	Cost	Reliability
1	65	0.994586
2	58	0.993677
3	72	0.995333
4	72	0.995531
5	60	0.994064
6	65	0.994641
7	71	0.995954
8	51	0.992906
9	67	0.995111
10	90	0.998551
11	67	0.995425
12	86	0.997641
13	66	0.994653
14	52	0.992848
15	76	0.995913



Example

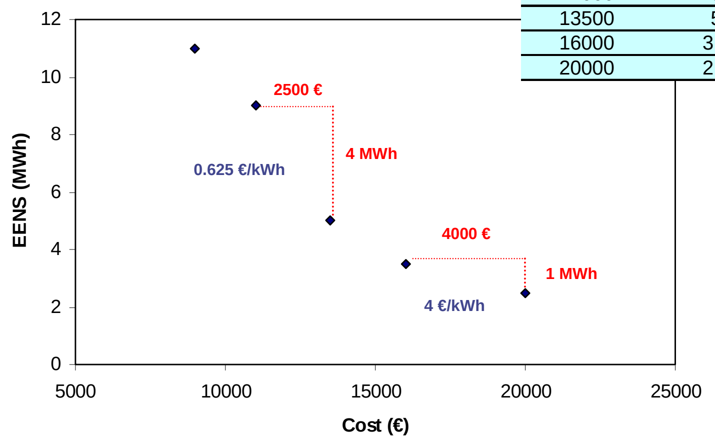
- 5 alternatives in 3 criteria



Trade-off analysis

- 5 possible investment plans

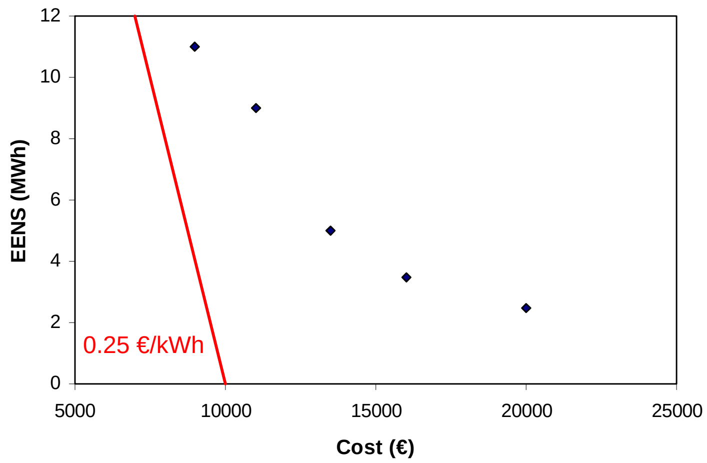
Cost (€)	EENS (MWh)
9000	11
11000	9
13500	5
16000	3.5
20000	2.5



Trade-off analysis

- When comparing B to A (two efficient alternatives)
  - We gain something in one criterion
  - We loose something in another criterion
- If we have a reference value for the trade-off
  - We know immediately if we prefer A or B
  - It's easy to select the preferred alternative

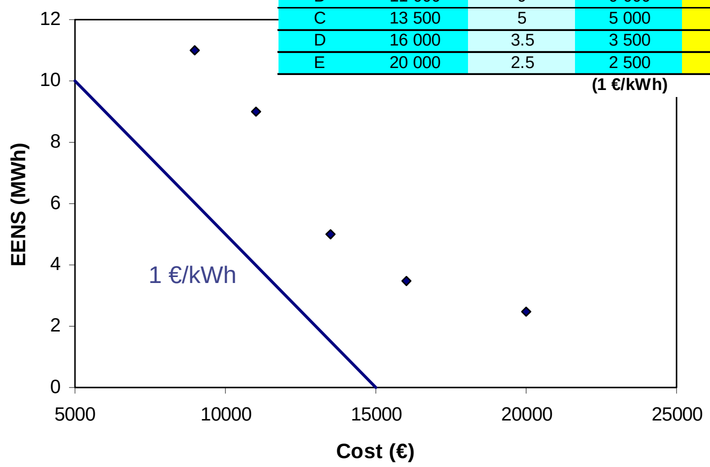
Trade-off analysis



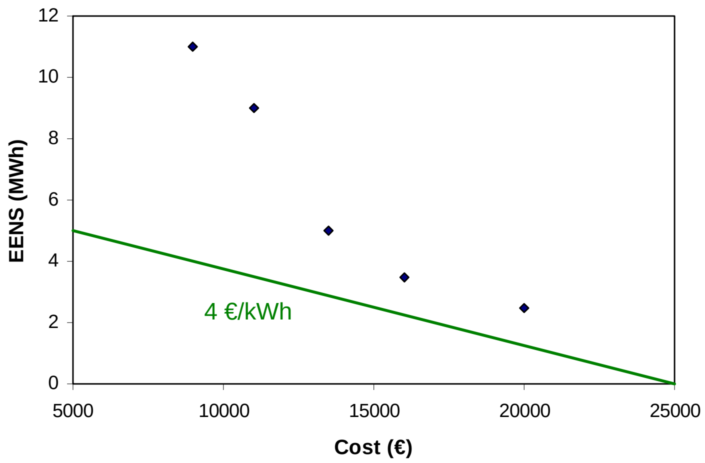
Trade-off analysis

alternative	Cost (€)	EENS (MWh)	EENS (€)	Total
A	9 000	11	11 000	20 000
B	11 000	9	9 000	20 000
C	13 500	5	5 000	**18 500**
D	16 000	3.5	3 500	19 500
E	20 000	2.5	2 500	22 500

(1 €/kWh)

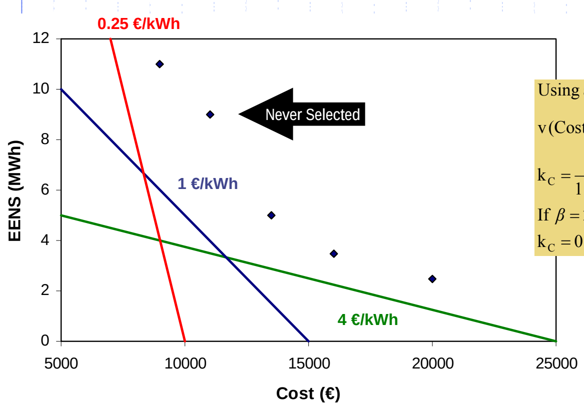


Trade-off analysis



Trade-off analysis

- Each trade-off β defines a family of indifference lines
  - f(Cost, EENS) = Cost + β. EENS   β in €/MWh



Using a normalized **value function**:

$$v(\text{Cost}, \text{EENS}) = k_C \frac{20000 - \text{Cost}}{20000 - 9000} + k_E \frac{11 - \text{EENS}}{11 - 2.5}$$

$$k_C = \frac{11000}{11000 + 8.5\beta} \qquad k_E = \frac{8.5\beta}{11000 + 8.5\beta}$$

If $\beta = \mathbf{1000}$ **€/MWh**

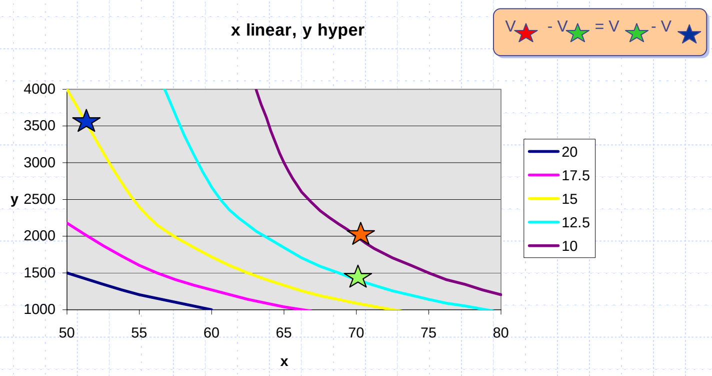
$k_C = 0.564 \quad k_E = 0.436$

Trade-off analysis

- Conclusions:
  - Constant trade-offs lead to linear indifference curves
  - ... and to linear value functions
  - ... with constant weights
    - that have no special meaning as indicators of the relative importance of the criteria in general
- Important issues
  - The process may be extended to more than two criteria
  - Trade-offs are not always constant
    - e.g. beyond a certain level, your willingness to pay for extra reliability decreases
  - ... leading to non-linear indifference curves
  - ... and non-linear value functions
    - but generally still additive, with constant parameters

Indifference curves

- Indifference curves join all the points with the same *global value*
- The DM is indifferent between two points in the same curve



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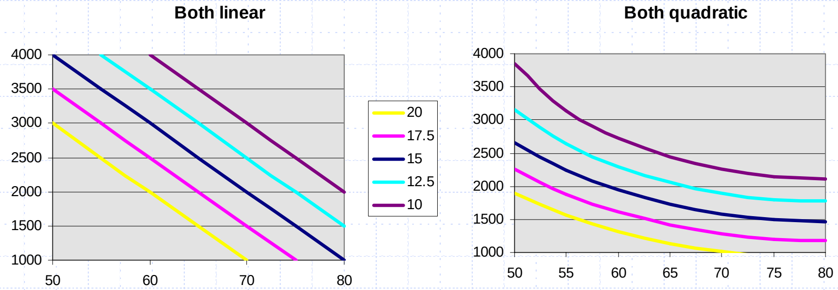
Indifference curves

- Other (additive) value functions...



Minimization in both criteria
Value scale (20≻17.5, etc)

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Summarizing

- Indifference curve (attribute space)
  - Set of the alternatives that are valued the same way by the Decision Maker
  - The indifference curves completely describe the structure of preferences of the Decision Maker
- Trade-off between two attributes X and Y
  - What you must lose in X to increase one unit in Y, without leaving the indifference curve (slope of the curve)
- Weights
  - If and only if the trade-offs are constant, weights are constant

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Multiattribute analysis - main approaches

- Ensure that the DM follows a "rational" behavior (Normative option)
- *Value functions, Utility theory, distance to the Ideal*
- Give some advice based on reasonable (but not indisputable) rules
- *The French School*
- Find the preferred solution from partial decisions about decision hypothesis
- *Interactive methods*

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Value functions

- A formal way to address multiattribute problems
  - Sometimes also called deterministic utility functions
- Requires
  - Verifying assumptions
  - Construction of the individual value functions
  - Indifference judgments to build the multiattribute value function
- Difficulties
  - Building individual value functions
- Problems
  - Tendency to use naïve weights asked directly to the DM

BIG MISTAKE!

Value functions - existence

- If Z is a subset of R_m
  - i.e. if each alternative A is described by m attributes (A_1, A_2, …, A_m)
- and
  - ($A \geq B$ and $A \neq B$) $\Rightarrow A \succ B$ for all A, B $\in$ Z
  - For all A, B, C $\in$ Z such that $A \succ B \succ C$, it exists exactly one $\lambda \in (0, 1)$ such that $B \sim [\lambda.A + (1-\lambda).C]$

    (Archimedean Condition)
- Then, it exists a real value function v() such that:
  - $A \succ B \Leftrightarrow v(A) > v(B)$
  - $A \sim B \Leftrightarrow v(A) = v(B)$

Independence and additivity (m>2)

- Given a set of attributes K, a subset X of K is said to be preferentially independent (p.i.) from its complement Y=K-X iff, for a particular value P_Y

$$(A_X, P_Y) \succeq (B_X, P_Y) \Rightarrow (A_X, Q_Y) \succeq (B_X, Q_Y)$$

  - stands for all Q_Y, A and B being arbitrary.
- A set K is mutually preferentially independent (m.p.i.) if every subset X of K is p.i. from its complement K-X
- For three or more criteria (m>2), this is a sufficient condition to additivity: $A \succeq B \Rightarrow v_1(A_1) + \ldots + v_m(A_m) \geq v_1(B_1) + \ldots + v_m(B_m)$

Additivity (m=2)

- For two criteria, an additional condition is necessary for additivity
  - For instance, the Thomsen condition:

$$\text{For all } P, Q, A$$
$$(P_1, A_2) \sim (A_1, Q_2) \text{ and } (A_1, P_2) \sim (Q_1, A_2) \Rightarrow (P_1, P_2) \sim (Q_1, Q_2)$$

  - or the cancellation condition
    - also guaranties that K is m.p.i.

$$\text{For all } P, Q, A$$
$$(P_1, A_2) \succeq (A_1, Q_2) \text{ and } (A_1, P_2) \succeq (Q_1, A_2) \Rightarrow (P_1, P_2) \succeq (Q_1, Q_2)$$

- More weak conditions exist for difficult cases

Building value functions

- Direct construction
  - Too complicated
- Verify preferential independence conditions
  - Then: $v(A) = \Psi(v_1(A_1), \ldots, v_m(A_m))$
- Check for additivity conditions...
  - If they hold: $v(A) = k_1 v_1(A_1) + k_2 v_2(A_2) + \ldots + k_m v_m(A_m)$
- ...or less restrictive conditions
  - That let you use (eg two normalized individual value functions)

$$v(A) = k_1 v_1(A_1) + k_2 v_2(A_2) + k_{12} v_1(A_1) v_2(A_2)$$

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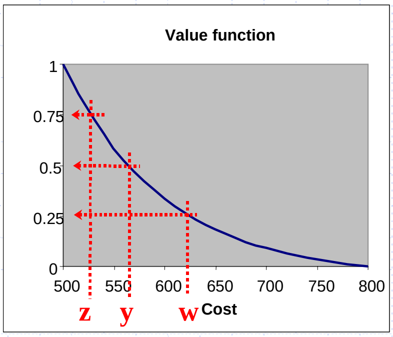
Building individual value functions

- Fix $v(x_{min})=1$, $v(x_{max})=0$
- Find *y* such that
  - $x_{max} \rightarrow y$ or $y \rightarrow x_{min}$
  - is indifferent to the DM
- Then, $v(y)=0.5$
- Repeat to find w (interval $[y, x_{max}]$)
  - *v(w)=0.25*
- and z (interval $[x_{min}, y]$)
  - *v(z)=0.75*
- ... (trace the curve)



- Verify! *The DM should be indifferent between $w \rightarrow y$ and $y \rightarrow z$*

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Individual value functions

- Individual (or conditional) value function
  - Measures the satisfaction in one criterion, regardless of the values of the other criteria
- Typical value functions (minimization):
  - Linear $v(x) = x_N = \dfrac{x_{max} - x}{x_{max} - x_{min}}$
  - Quadratic 1 $v(x) = (x_N)^2$
  - Quadratic 2 $v(x) = 2.x_N - (x_N)^2$
  - Exponential $v(x) = \dfrac{e^{a.x_N} - 1}{e^a - 1}$

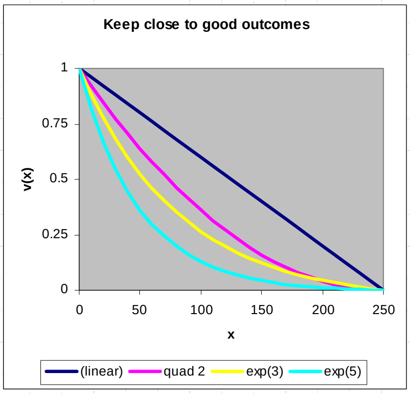
Generally **v(x)** is normalized, with:

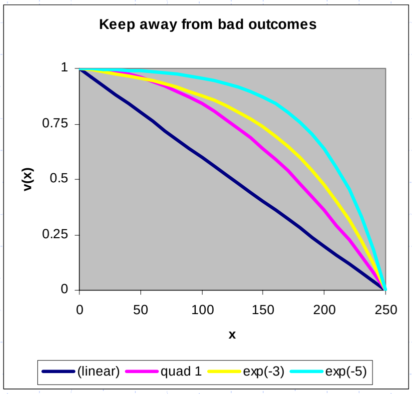
v(best x) = 1

v(worst x) = 0

Individual value functions

- The convexity of the value function reflects the variation of the DM's satisfaction in the range of the attribute
  - The same difference in the attribute (e.g. 50-100 and 200-250) does not correspond to the same increase in satisfaction (exception: linear v.f.)





MA value functions - parameters

- Assess the parameters k_1 and k_2 $\quad v(A)=k_1v_1(A_1)+k_2v_2(A_2)$
  - Build "extreme" alternatives:

$Ideal: best\ A_1, best\ A_2$	$P: best\ A_1, worst\ A_2$	$Q: worst\ A_1, best\ A_2$
$v=1, v_1=1, v_2=1$	$v_1=1, v_2=0$	$v_1=0, v_2=1$
$k_1+k_2=1$	$v(P)=k_1$	$v(Q)=k_2$

  - Ask for a judgment (eg: P $\succeq$ Q, that implies $k_1 \geq k_2$)
  - Find M = (z, worst A_2) ~ Q
    - Then:

$$v(M)=v(Q) \Rightarrow k_1v_1(z)=k_2$$

$$k_1=\frac{1}{1+v_1(z)} \qquad k_2=1-k_1$$

    - **This is very different from asking directly for weights!**

Example

- Build "extreme" alternatives:
  - P=(9000, 11), Q=(20000, 2.5)
- Search for an indifference
  - *P or Q?*
    - The DM says P $\succeq$ Q
  - *P'=(11000, 11) or Q?*
    - P' $\succeq$ Q
  - *P''=(12000, 11) or Q?*
    - Q $\succeq$ P''
  - M=(11500, 11) ~ Q=(20000, 2.5)
  - $v_C(11500)=0.773$
  - $k_C=0.564$ $k_E=0.436$

Cost (€)	EENS (MWh)
9000	11
11000	9
13500	5
16000	3.5
20000	2.5

$$v(\text{Cost}, \text{EENS}) = k_C \frac{20000 - \text{Cost}}{20000 - 9000} + k_E \frac{11 - \text{EENS}}{11 - 2.5}$$

NB:
8 500 € compensates 8.5 MWh
Trade-off = 1 €/kWh

$$v(M) = v(Q) \Rightarrow k_C v_C(z) = k_E$$

$$k_C = \frac{1}{1 + v_C(z)} \qquad k_E = 1 - k_C$$

Minimum distance to the Ideal

- A possible decision paradigm for deterministic multiattribute problems
  - Induces an order in the set of the alternatives
  - May also be used in multiobjective problems
- Ideal (Zeleny)
  - (Non feasible) solution, defined only in the attributes' space, that joins up the individual optima
- Limitations
  - If scales are very different, some kind of normalization is mandatory
  - We are implicitly accepting equal compensation between attributes
    - e.g. 30% loss in Attribute X are compensated by 30% gain in Attribute Y

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Minimum distance to the Ideal

- Distance measures
  - w_k are scale factors
  - Choice of p is a decision problem!

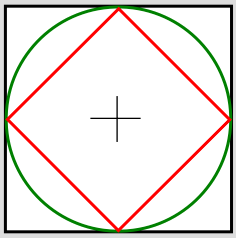
$$d_p(A, Ideal) = \left(\sum_{k=1}^{m} w_k . \left(\left| A_k - Ideal_k \right| \right)^p \right)^{1/p}$$

$$d_1(A, Ideal) = \sum_{k=1}^{m} w_k . \left| A_k - Ideal_k \right| \quad \textit{Manhatan distance}$$

$$d_2(A, Ideal) = \sqrt{\sum_{k=1}^{m} w_k . \left(\left| A_k - Ideal_k \right| \right)^2} \quad \textit{Euclidian distance}$$

$$d_\infty(A, Ideal) = \max_k \left\{ w_k . \left| A_k - Ideal_k \right| \right\} \quad \textit{Chebyshev distance}$$

(implicit)
indifference curves



Points equidistant from the center:
- **Manhatan**
- **Euclidian**
- **Chebyshev**

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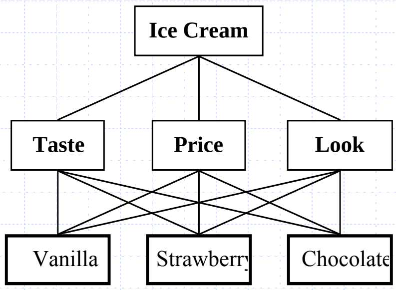
AHP

- Analytic Hierarchy Process
  - Thomas Saaty
- Hierarchical organization of the criteria
- Comparison matrices
  - Between sub-criteria, regarding the parent criterion
  - Between alternatives, regarding a level 1 criterion
- Calculation of a final order of priorities



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AHP: input and calculations (1)

- Input - Judgments about the relative preference of the alternatives, regarding each attribute
  - May be expressed by linguistic labels
  - Converted then to numbers (the Saaty scale)
  - Form a matrix of comparisons
  - Inconsistencies are allowed (to a certain degree)
- Calculations – The priorities on an attribute correspond to the greatest eigen-vector of its matrix
  - May be approximated by the average of the normalized columns

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AHP: example

Taste

	Vanilla	Strawberry	Chocolate	
Vanilla	**1**	**3/2**	**5**	0.540
Strawberry	**2/3**	**1**	**3**	0.348
Chocolate	**1/5**	**1/3**	**1**	0.112

Price

	Vanilla	Strawberry	Chocolate	
Vanilla	**1**	**1/3**	**1**	0.185
Strawberry	**3**	**1**	**5**	0.659
Chocolate	**1**	**1/5**	**1**	0.156

Look

	Vanilla	Strawberry	Chocolate	
Vanilla	**1**	**1/5**	**1**	0.149
Strawberry	**5**	**1**	**4**	0.691
Chocolate	**1**	**1/4**	**1**	0.160

AHP: input and calculations (2)

- The process is repeated with the relative importance of the attributes
  - Or the relative importance of sub-attribute of an attribute

	Taste	Price	Look	
Taste	**1**	**5**	**7**	0.731
Price	**1/5**	**1**	**3**	0.188
Look	**1/7**	**1/3**	**1**	0.081

- Conclusion - global priorities of the alternatives

	Taste	Price	Look				
Vanilla	0.540	0.185	0.149		0.731		**0.442**
Strawberry	0.348	0.659	0.691	X	0.188	=	**0.434**
Chocolate	0.112	0.156	0.160		0.081		**0.124**

AHP: a surprise...

- Eliminating CHOCOLATE, but keeping the remaining judgments...

	Taste	Price	Look
Vanilla	3/5	1/4	1/6
Strawberry	2/5	3/4	5/6

- ... The following new global priorities are obtained !

Vanilla	**0.499**
Strawberry	**0.501**

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AHP - comments

- Strong points
  - Easy to use and understand
    - Accepts linguistic labels
  - Flexible - allow small inconsistencies
  - Judgments substitute unavailable information
    - The attributes' values are not used in the calculations
- Weak points
  - Uses value ratio evaluations instead of value difference evaluations
    - *"How many times is alternative A preferred to B?"*
  - Rank reversal problems
  - Most of the work and conclusions are specific of the problem in hand

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Decision-aid methodologies

- The French School of decision-aid proposes a number of methods that try to better model the structure of preferences of the DM, without prescribing a total order
- The methodologies include
  - indifference thresholds
  - hesitations between strict preference and indifference (weak preference)
  - veto thresholds
  - incomparability situations
  - the complementary concepts of concordance and discordance
- Aggregation of preferences mainly by rules
  - as opposed to formulas
- Members of the family
  - ELECTRE I, IS, II, III, IV, Tri, PROMETHEE, GAIA

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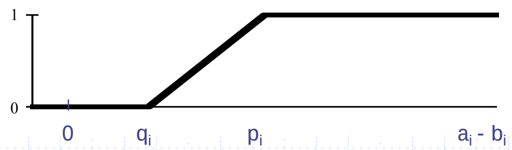
The French School

- Extension of the classic paradigm (P, I) by considering two additional situations:
  - Q - weak preference   R – incomparability
- Definition, in each criterion i, of indifference limits q(i) and preference limits p(i), used to define intervals of indifference, weak preference and strict preference



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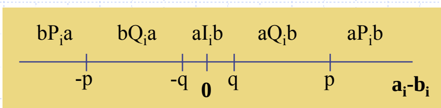
Electre IV - basic ideas

- The method is based on pairwise comparisons between alternatives
- In each criterion i, some thresholds are defined:
  - q - indifference threshold
  - p - strict preference threshold
  - v - veto threshold
- We may have (alternatives **a** and **b**, maximization):



I - indifference
P - strict preference
Q - weak preference

Electre IV - procedure

- Aggregation rules
  - Comparison between alternatives **a** and **b** may lead to different types of dominance (quasi, canonic, pseudo, sub, veto) of **a** over **b** (or vice-versa), or to no dominance
    - Each alternative has a **qualification** (# situations where it dominates - # situations where it is dominated) for each type of dominance
- Distillation
  - Descending: begins with the alternatives with greater qualification
  - Ascending: begins with the alternatives with lesser qualification
    - In both cases, the effect of the selected alternatives is annulled on the remaining ones
- Final preoder
  - Combination of the two distillations

Electre IV – binary relations

- Quasi-dominance - The couple (b, a) verifies the relation of quasi-dominance if and only if:
  - for every criterion, b is either preferred or indifferent to a,
  - and if the number of criterion for which the performance of a is better than the one of b (a staying indifferent to b) is strictly inferior to the number of criteria for which the performance of b is better than the one of a.
- Canonic-dominance - The couple (b, a) verifies the relation of canonic-dominance if and only if:
  - for no criterion, a is strictly preferred to b,
  - and if the number of criteria for which a is weakly preferred to b is inferior or equal to the number of criteria for which b is strictly preferred to a,
  - and if the number of criteria for which the performance of a is better than the one of b is strictly inferior to the number of criteria for which the performance of b is better than the one of a.

Electre IV – binary relations

- Pseudo-dominance - The couple (b, a) verifies the relation of pseudo-dominance if and only if:
  - for no criterion, a is strictly preferred to b,
  - and if the number of criteria for which a is weakly preferred to b is inferior or equal to the number of criteria for which b is strictly or weakly preferred to a.
- Sub-dominance - The couple (b, a) verifies the relation of sub-dominance if and only if:
  - for no criterion, a is strictly preferred to b.
- Veto-dominance - The couple (b, a) verifies the relation of veto-dominance if and only if:
  - either for no criterion, a is strictly preferred to b,
  - or a is strictly preferred to b for only one criterion but this criterion not vetoing the outranking of a by b and furthermore, b is strictly preferred to a for at least half of the criteria.

Electre IV - illustration

- A small distribution planning problem

alternative	cost	lambda	U
A	1000	0.10	7
B	800	0.15	10
C	500	0.21	12
D	850	0.12	11
E	1200	0.30	4

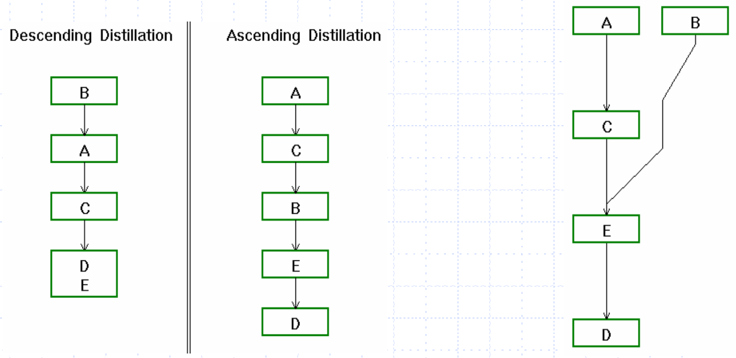
threshold	cost	lambda	U
q	50	0.05	0
p	150	0.1	2
v	500		6

1.0 - quasi
0.8 - canonic
0.6 - pseudo
0.4 - sub
0.2 - veto

	A	B	C	D	E
A	1	0	0.2	0.8	0.8
B	0	1	0	1	0
C	0	0.4	1	0.4	0
D	0	0.4	0	1	0
E	0	0	0	0	1

Electre IV - illustration

- Distillations and final preorder



2nd example: original data

	Algorithm		no rot	with rot	blazewic	shirt	milenk
1	MAX_OVERLAP	DISTANCE	69.00	72.00	32.57	66.44	308.37
2	MAX_OVERLAP	WASTE+OVERLAP+DISTANCE	73.00	71.00	33.96	77.96	314.95
3	MAX_OVERLAP	WASTE	74.50	67.50	30.00	70.28	275.55
4	MAX_OVERLAP	WASTE+DISTANCE	74.50	67.50	30.73	69.93	273.45
5	MAX_OVERLAP	OVERLAP+DISTANCE	75.50	65.00	31.83	69.95	275.89
6	MAX_OVERLAP	WASTE+OVERLAP	76.00	66.00	32.07	84.86	333.71
7	MAX_OVERLAP	OVERLAP	78.00	67.75	31.83	70.00	314.45
8	MIN_AREA	OVERLAP	67.00	67.00	33.26	68.12	315.82
9	MIN_AREA	DISTANCE	68.00	77.50	30.90	67.80	298.87
10	MIN_AREA	OVERLAP+DISTANCE	69.50	71.00	32.75	67.43	291.37
11	MIN_AREA	WASTE	73.00	67.00	31.09	73.83	281.99
12	MIN_AREA	WASTE+DISTANCE	73.00	67.50	31.75	76.07	282.80
13	MIN_AREA	WASTE+OVERLAP	77.00	66.50	32.65	76.15	305.45
14	MIN_AREA	WASTE+OVERLAP+DISTANCE	77.00	73.00	32.03	71.61	339.50
15	MIN_LENGTH	OVERLAP	66.75	67.00	29.48	68.12	276.11
16	MIN_LENGTH	DISTANCE	71.00	73.00	31.91	67.72	282.00
17	MIN_LENGTH	WASTE	73.00	67.00	30.09	70.00	286.35
18	MIN_LENGTH	WASTE+DISTANCE	73.00	67.50	30.42	74.42	280.14
19	MIN_LENGTH	WASTE+OVERLAP	74.00	71.00	29.92	76.13	313.98
20	MIN_LENGTH	WASTE+OVERLAP+DISTANCE	74.50	72.00	32.35	73.10	300.19
21	MIN_LENGTH	OVERLAP+DISTANCE	83.50	67.00	28.90	67.30	281.99

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Example: efficient solutions

	Algorithm		no rot	with rot	blazewic	shirt	milenk
1	MAX_OVERLAP	DISTANCE	69.00	72.00	32.57	66.44	308.37
3	MAX_OVERLAP	WASTE	74.50	67.50	30.00	70.28	275.55
4	MAX_OVERLAP	WASTE+DISTANCE	74.50	67.50	30.73	69.93	273.45
5	MAX_OVERLAP	OVERLAP+DISTANCE	75.50	65.00	31.83	69.95	275.89
9	MIN_AREA	DISTANCE	68.00	77.50	30.90	67.80	298.87
10	MIN_AREA	OVERLAP+DISTANCE	69.50	71.00	32.75	67.43	291.37
13	MIN_AREA	WASTE+OVERLAP	77.00	66.50	32.65	76.15	305.45
15	MIN_LENGTH	OVERLAP	66.75	67.00	29.48	68.12	276.11
16	MIN_LENGTH	DISTANCE	71.00	73.00	31.91	67.72	282.00
21	MIN_LENGTH	OVERLAP+DISTANCE	83.50	67.00	28.90	67.30	281.99

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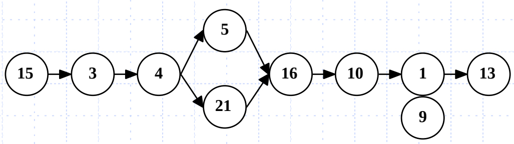
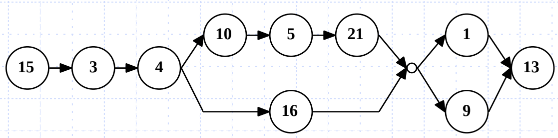
Example: Results

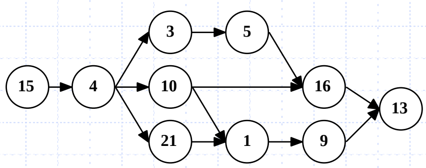
q=5% p=20%



q=5% p=5%



q=0% p=5%



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SAM – Successive Amplification Method

- Designed to deal with a list of many alternatives
- Zooms from global picture...
  - a limited set of pseudo-alternatives (PA), obtained by clustering
- ... to the preferred region of the attribute space...
  - by successive decisions of the DM (chooses one or more of the PA)
- ... until the preferred alternative is identified
- No weights or any kind of parameters

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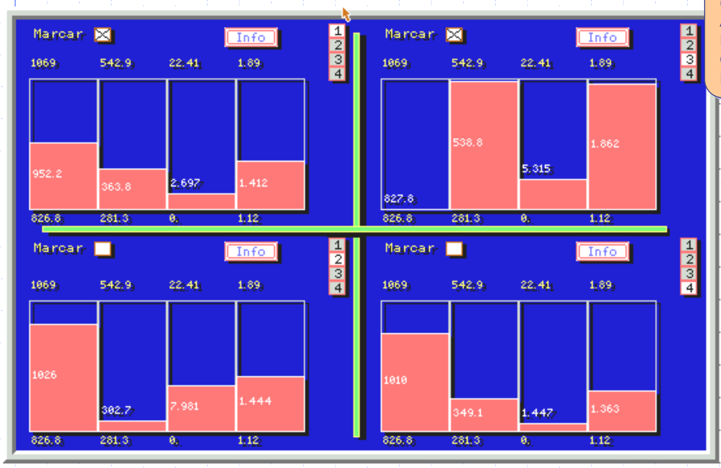
SAM - an example in distribution planning



(minimize)
A - Investment Cost
B - EPNS
C - Voltage Quality
D - Power Losses

53 original alternatives

4 clusters

The DM chooses clusters 1 and 3

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SAM - first reduction

- Global list, showing selected alternatives

1	958.	314.8	0.45	1.47
2	943.5	344.3	0.37	1.45
3	995.6	308.	3.08	1.53
4	995.6	306.	3.08	1.56
5	995.6	309.5	2.71	1.43
6	1017	299.9	2.71	1.59
7	1016	302.4	2.71	1.58
8	1014	307.6	2.71	1.58
9	1062	302.9	14.26	1.37
10	1061	293.2	14.33	1.42
11	1049	304.2	11.19	1.42
12	1025	299.4	14.7	1.46
13	1021	292.3	22.41	1.58
14	1026	288.	11.64	1.45
15	1010	305.2	14.7	1.48
16	1011	300.6	14.7	1.45
17	1000	306.6	11.24	1.41
18	992.2	336.8	11.24	1.43
19	828.5	537.6	3.29	1.89
20	827.	537.2	6.32	1.83
21	826.8	542.9	6.32	1.87
22	1069	281.3	3.1	1.46
23	1040	314.4	0.68	1.21
24	1041	314.5	0.3	1.25
25	1034	315.6	0.68	1.22
26	1030	336.5	0.68	1.19
27	1023	352.6	0.68	1.19
28	1013	358.2	1.81	1.34
29	1007	359.4	1.81	1.34
30	1005	363.5	1.16	1.32
31	925.5	374.8	1.08	1.31
32	924.	380.5	1.08	1.34
33	925.8	375.9	3.e-002	1.41
34	924.9	405.7	3.e-002	1.39
35	912.7	420.7	0.	1.31
36	1017	332.3	4.e-002	1.46
37	1016	336.	4.e-002	1.45
38	1017	334.	1.e-002	1.42
39	1006	353.5	1.e-002	1.36
40	1002	364.6	4.e-002	1.4
41	1002	366.4	4.e-002	1.38
42	993.3	369.9	4.e-002	1.46
43	909.6	392.	0.33	1.63
44	924.	380.	6.79	1.35
45	930.9	371.	0.	1.12
46	1056	318.	0.	1.55
47	984.7	357.3	0.	1.55
48	980.1	358.9	0.	1.53
49	973.2	361.1	0.	1.54
50	957.1	370.9	0.	1.52
51	964.5	327.7	13.17	1.39
52	964.5	331.	13.17	1.28
53	957.3	343.3	13.17	1.29

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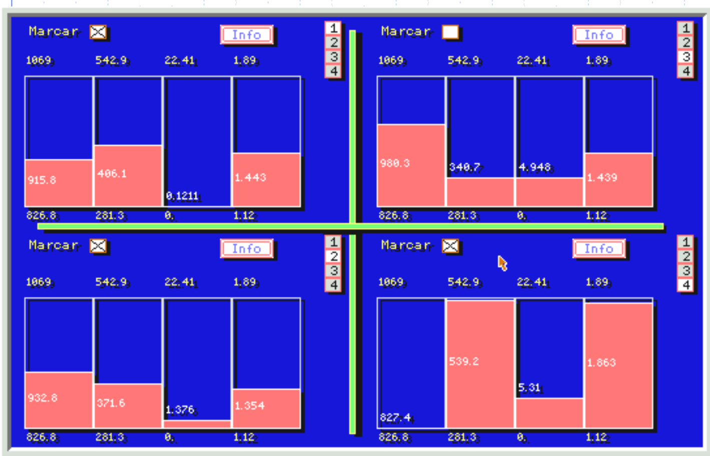
SAM - second clustering

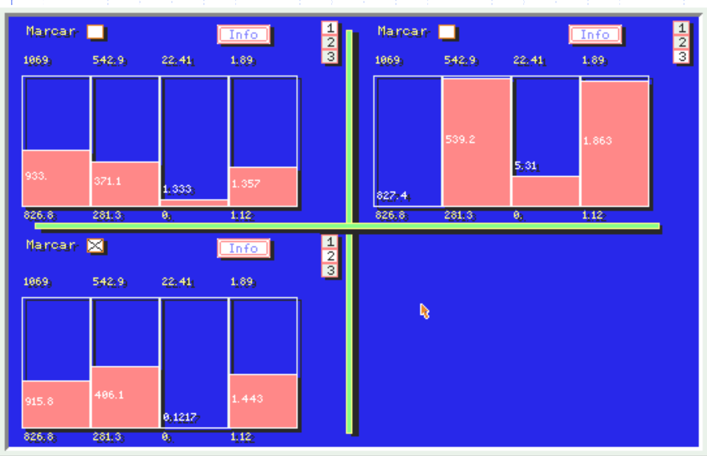


21 alternatives

4 clusters

The DM chooses clusters 1, 2 and 4

SAM - final clustering

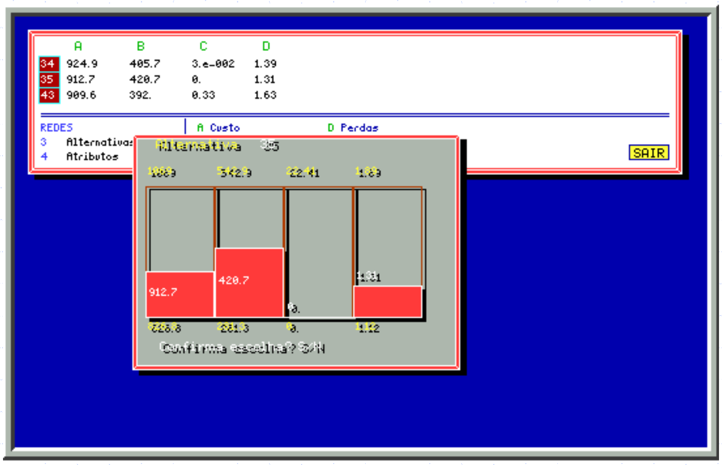


13 alternatives

3 clusters

The DM chooses cluster 2

SAM - final decision



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Final remarks

- In deterministic multiattribute problems, the main issue is preference modeling
- Building correctly a value function may be a good approach, namely if automatic decisions are needed
  - Trade-off analysis is just a particular case
- Decision-aid methods are an interesting alternative when the DM desires a more detailed representation of his preferences
  - Very adequate when a large number of criteria exist
- Filtering procedures and non-parametric approaches help the DM gaining more insight into the problem