Class Notes
MAD – Decision Aid Methodologies – FEUP 2005

Decisions under uncertainty

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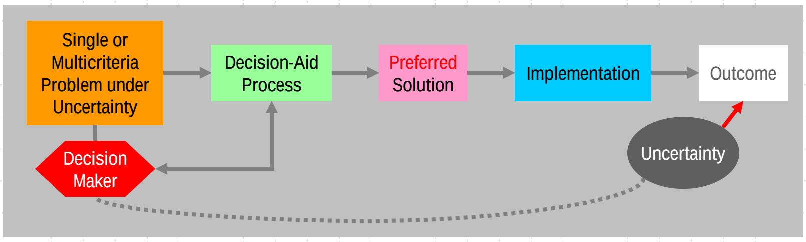
Uncertainty issues

- Uncertainty about the data
  - Loads, costs, wind power, hydro inflows, economical parameters
  - Reliability parameters
- Uncertainty about the outcome of random variables
  - Quality of service indices
- Uncertainty about the behavior of other agents
  - Regulatory decisions
  - Environmental pressure
  - Competitors decisions
- Uncertainty about the model

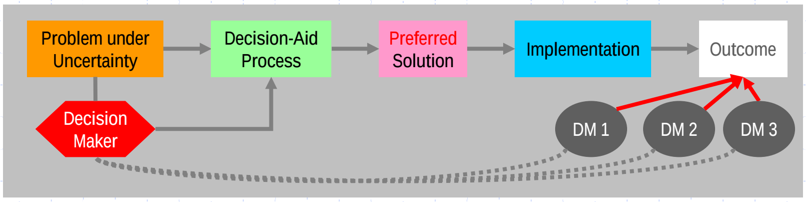
The role of the decision maker

- Single or multicriteria problems under uncertainty
  - The DM participates in the problem formulation and in the uncertainty characterization
  - The preferred solution results from the incorporation in the problem of the structure of preferences of the DM, including its risk attitude



The role of the decision maker

- Problems under uncertainty
  - Sometimes, part of the uncertainty comes from the decisions of other decision makers
  - Models of their behavior must also be incorporated in the problem



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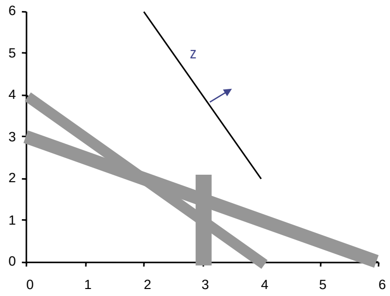
Different types of uncertainty

- Probabilistic - Different scenarios with probabilities

	Cost		
n	C1 (0.3)	C2 (0.6)	C3 (0.1)
1	59	65	75
2	50	58	71
3	68	72	60
4	69	72	62
5	53	60	63
6	51	59	65
7	68	71	77
8	56	57	75
9	62	58	80
10	62	55	70

- Fuzzy - Vague or imprecise constraints

$$\max\ z = 2x_1 + x_2$$
$$suj:\quad x_1 + x_2 \precsim 4$$
$$x_1 + 2x_2 \precsim 6$$
$$x_1 \precsim 3$$
$$x_1\ ,\ x_2 \geq 0$$



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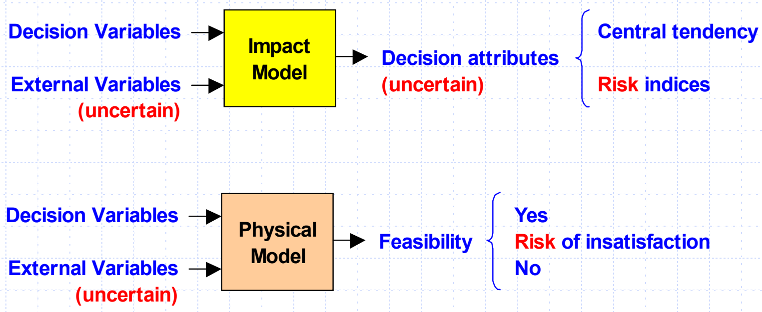
Modeling

- Uncertain environment



Uncertainty models

- Scenarios
  - with or without probabilities
- Probabilistic models
  - continuous, discrete, *subjective*
- Fuzzy models
  - (intervals are fuzzy sets)

Scenarios

- Scenarios are coherent estimates of the uncertain environment
  - Taking into account the relations and dependencies between variables
  - Although quantitatively characterized, they correspond to qualitatively different realizations of the uncertainties
    - e.g. “Moderate economic development”, “Economic stagnation”
  - Probabilities may be assigned to each scenario
    - Sometimes, *subjective probabilities*
    - But also *interval* or *fuzzy* probabilities
- Output
  - Impact of the decisions in each scenario

Probabilistic models

- Input
  - Continuous variables with known distributions
  - Discrete independent variables
  - Scenarios
- Output
  - Probability distributions of the attributes
  - Expected values of the attributes
  - Other moments of the distributions of the attributes
- Methods
  - Analytic
  - Simulation (e.g. Monte-Carlo)

Fuzzy models

- Basic concept
  - The degree of membership of an element of the universe of discourse to the concept associated to the fuzzy set may take any value in (0,1)
    - e.g. **u(7, "near 9")=0.3**
    - e.g. **u(17 min, "a quarter of hour")=0.9**
- Typical applications
  - "This load will be around 800 kW"
  - "The consumption will grow from 3 to 5%"
  - "The deficit should not exceed too much 3%"
- Output
  - Possibility distributions of the attributes
  - Robustness regarding constraint violations

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Why fuzzy sets?

- Fuzzy sets incorporate implicitly an infinite number of scenarios
  - But we can conjugate the two concepts
- Experts' knowledge can be "translated" in a simple way
- In most cases, uncertainty is better captured than with probabilities
  - Most of the times, no significant statistical data exists
  - Fuzzy Sets do not substitute for probabilistic models when these are adequate
- It is possible to develop physical and impact models that propagate this kind of uncertainty (extension principle)
- We can even construct fuzzy-probabilistic methodologies

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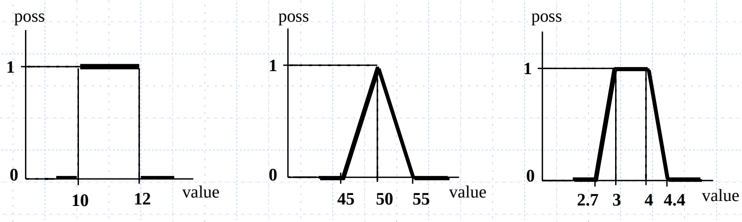
Basic fuzzy models for data

- A "dictionary" for qualitative declarations:



"Between 10 and 12"

"About 50"

" More or less between 3 and 4"

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A global view on alternatives

- In each criterion, each alternative has an outcome:
  - a real number, when no uncertainty exists
  - a list of real numbers, when a finite number of scenarios exists
  - a list of pairs (attribute value, probability), when a finite number of scenarios exists with assigned probabilities
  - a discrete probability distribution or a probability density function, when the uncertainty is probabilistic
    - Dependencies may exist between random variables
  - a possibility distribution, when a fuzzy model is used
    - e.g. to describe mathematically qualitative labels
    - Intervals are a particular case of fuzzy sets
- An alternative may be a stream of decisions over time
  - Including conditional decisions in the future
- Hedging policies can “generate” additional alternatives

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Uncertain environment (probabilistic)

- Single criterion

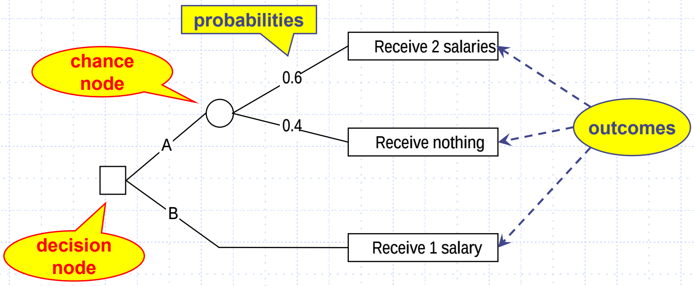
	Cost	
Alternatives	p=0.9	p=0.1
A	100	1000
B	150	550

- Multicriteria

	scenarios			
	C1		C2	
	cost	env	cost	env
X	1000	0.9	1000	0.8
Y	800	1.6	900	1.9
Z	500	1.7	1300	2
prob	0.7		0.3	

Decision trees - basics

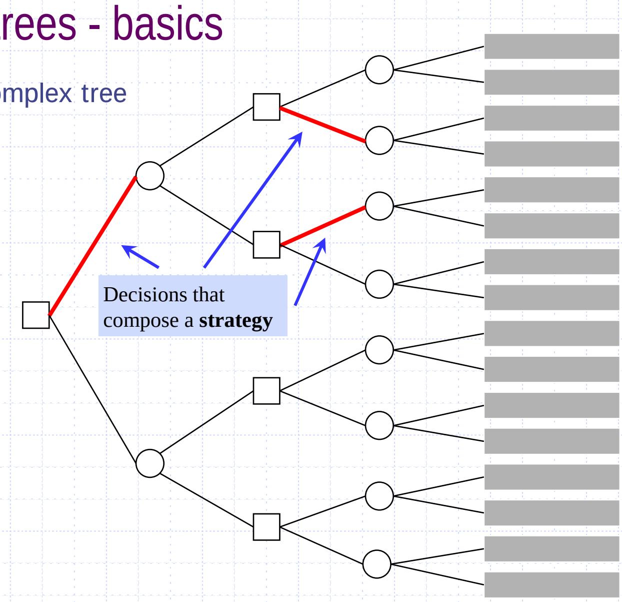
- A systematic way to represent a sequence of decisions and uncertain events through time, along with their outcomes
  - Complemented with procedures that identify the best strategy, according to some **decision paradigm**



Decision	1st Scen. (0.6)	2nd Scen.(0.4)
A	2 salaries	0
B	1 salary	1 salary

Decision trees - basics

- A more complex tree



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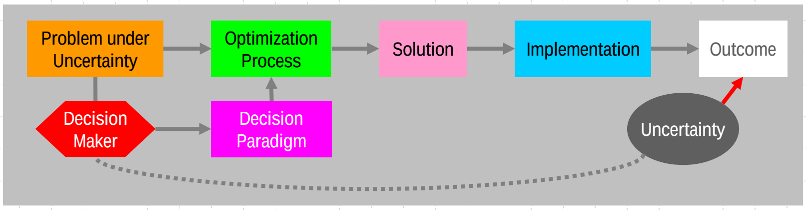
The role of the decision maker

- Problems under uncertainty
  - Sometimes, the risk attitude of the DM is incorporated in the form of a pre-defined decision paradigm (expected value, regret, etc.)
  - This leads generally to an optimization process



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Use of decision paradigms (or rules)

- Original problem
  - Dominated solutions shown

n	Cost		
	C1 (0.3)	C2 (0.6)	C3 (0.1)
1	59	65	75
2	**50**	58	71
3	68	72	**60**
4	69	72	62
5	53	60	63
6	51	59	65
7	68	71	77
8	56	57	75
9	62	58	80
10	62	**55**	70

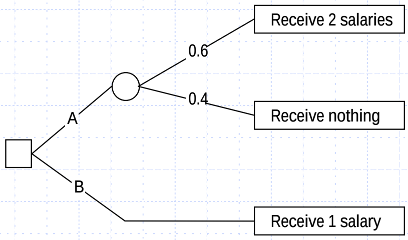
- Min E(Cost)

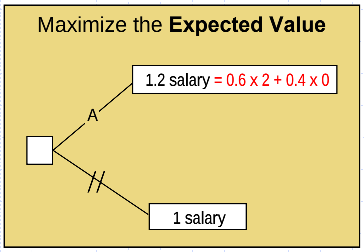
n	Expected Cost
1	64.2
2	**56.9**
3	69.6
4	70.1
5	58.2
6	57.2
7	70.7
8	58.5
9	61.4
10	58.6

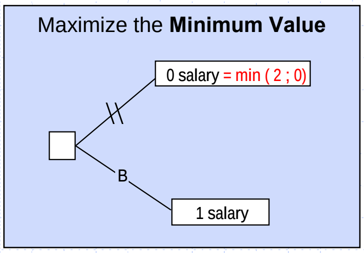
- Minimax Cost

n	Max Cost
1	75
2	71
3	72
4	72
5	**63**
6	65
7	77
8	75
9	80
10	70

Applying decision rules to a decision tree







- Procedure
  - (from the leaves to the root)
  - Chance nodes are collapsed according to the paradigm
  - Decision nodes are collapsed according to Max ou Min
  - Conditional decisions are saved

Decision paradigms for uncertainty

- Expected value paradigm
- E-V analysis
- Utility theory
- Robust optimization
- Bellman and Zadeh fuzzy decision

Expected value paradigm

- Basic paradigm
  - *Choose the alternative with the best expected value of the attribute*
  - *(implies risk indifference of the DM – linear utility)*

$$A_k = \{(z_{ik}, p_{ik}), i = 1..s\} \xrightarrow{EVP} A_k = \sum_{i=1}^{s} p_{ik} . z_{ik}$$

$$A_k = f_k(z) \xrightarrow{EVP} A_k = \int_{-\infty}^{\infty} z . f_k(z)\, dz$$

E-V analysis

- Basic paradigm
  - *Choose the alternative that simultaneously has the best expected value of the attribute and the smaller variance*
    - *Greater variance for risk seeking DM!*
  - multicriteria approach (E-V diagrams are common)
- Value functions
  - Risk equivalents added to the expected value

$$\max E[z] - \alpha . E\left[(z - \bar{z})^2\right]$$

$$\max E[z] - \alpha . E\left[(z - \bar{z})^2\right] + \beta . E\left[(z - \bar{z})^3\right]$$

α for risk, $\beta > 0$ for skewness

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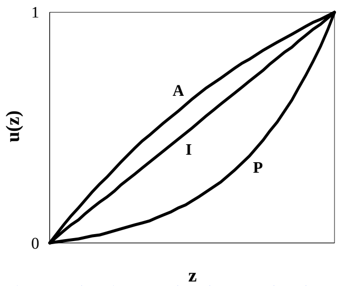
Utility theory

- A formal way to include risk in the evaluation of alternatives
- Basic paradigm
  - Choose the alternative with the greatest expected utility
  - (incorporates the risk attitude of the DM through an utility function)
- Risk attitude:
  - Risk Aversion: concave function
  - Risk Proneness: convex function
  - Risk Indifference: linear function
    - Equivalent to the Expected Value P.



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Utility theory - fundamentals

- Axiomatic (Von Newman, Morgenstern)
  - Transitivity:
    A≿B and B≿C implies A≿C
  - Continuity
    if A≿B≿C, there exists a probability p such that: B ~ {(p, A); (1-p, C)}
  - Completeness:
    either A≿B or B≿A

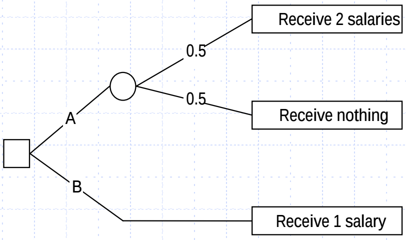
Lottery between A and C with probability *p* for A

- Theorem of existence
  - If the axioms hold, then there exists U(.) such that:
    A≿B ⇔U(A)≥U(B)
- Decision rule → maximize the expected utility
  - With c scenarios: $U(A)=\sum_{k=1}^{c} p_k U_k(A_k)$

NOT the expected value of the attribute

Utility theory - single attribute

- Check the risk profile of the DM
  - Prefers A? (Risk prone)
  - Prefers B? (Risk averse)
  - Indifferent? (Risk neutral)



- Build the utility function
  - Using a predetermined form
    or
  - Point by point, using lotteries to interrogate the DM

Utility theory – single attribute

- Decision rule: maximize the expected utility: $U(A)=\sum_{k=1}^{c} p_k U_k(A_k)$

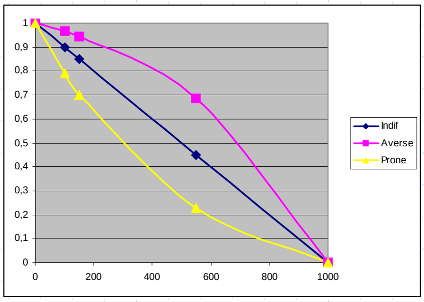
Cost			
Alternatives	p=0.9	p=0.1	E
A	100	1000	190
B	150	550	190

Utility Functions					
Cost	*0*	*100*	*150*	*550*	*1000*
Indif	1	0,90	0,85	0,45	0
Averse	1	0,97	0,95	0,69	0
Prone	1	0,79	0,70	0,23	0

Risk Indifferent			
Alternatives	p=0.9	p=0.1	U
A	0,90	0	**0,81**
B	0,85	0,45	**0,81**

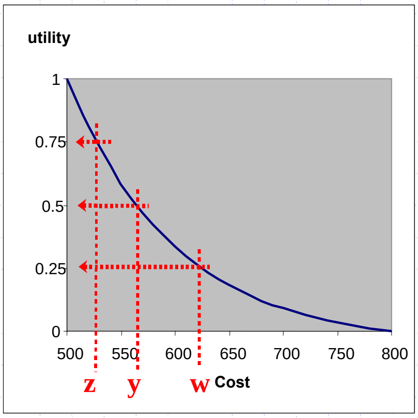
Risk Averse			
Alternatives	p=0.9	p=0.1	U
A	0,97	0	0,87
B	0,95	0,69	**0,92**

Risk Prone			
Alternatives	p=0.9	p=0.1	U
A	0,79	0	**0,71**
B	0,70	0,23	0,65



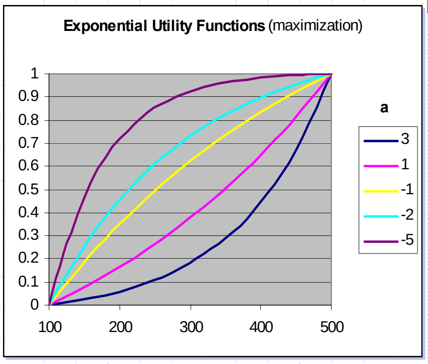
Building an utility function

- Attribute: cost (minimization)
- U(500)=1
  - (min cost)
- U(800)=0
  - (max cost)
- Find a cost y such that
  - y ~ {500; 800}
    - a 50% lottery between 500 and 800
- Then
  - U(y) = 0.5 U(500)+0.5 U(800) = 0.5
- …
  - z ~ {500;y}, w ~ {y; 800}
  - checking: y ~ {z;w}



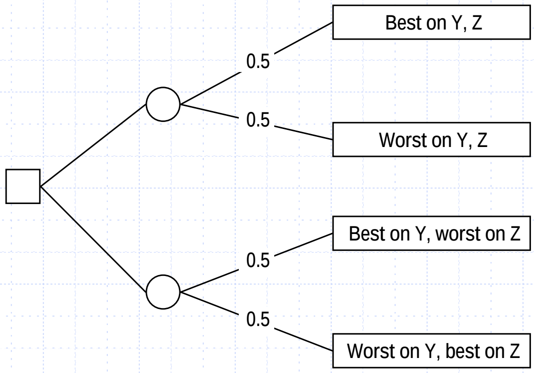
Some useful functions...

- You may want to use a predefined function to ease your work
  - linear $U(X) = x = \frac{X - X^{worst}}{X^{best} - X^{worst}}$
  - polynomial $U(X) = x^k$
  - exponential $U(X) = \frac{e^{ax} - 1}{e^{a} - 1}$
    - a<0 means constant aversion
    - a>0 means constant proneness



Utility theory - multiattribute

- Verify additive independence:
  - the DM must be indifferent between these two lotteries



Utility theory - multiattribute

- Typical forms
  - additive $U(X)=\sum_{i=1}^{m} k_i U_i(X_i)$
  - multiplicative $1+kU(X)=\prod_{i=1}^{m}(k k_i U_i(X_i)+1)$
  - multilinear
    - 2 attributes $U(X)=k_1 U_1(X_1)+k_2 U_2(X_2)+k_{12} U_1(X_1) U_2(X_2)$

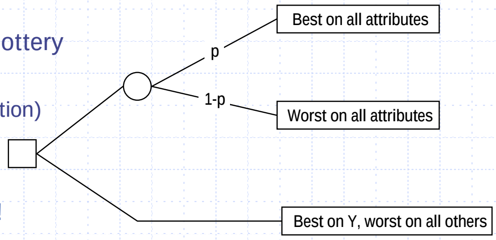
Utility theory - parameters

- Build the multiattribute utility function
  - parameters result from judgments
    - eg two attributes

$$A \sim B \Rightarrow U(A)=U(B)$$

$$\begin{cases} k_1.U_1(a_1)+k_2.U_2(a_2)=k_1.U_1(b_1)+k_2.U_2(b_2) \\ k_1+k_2=1 \end{cases}$$

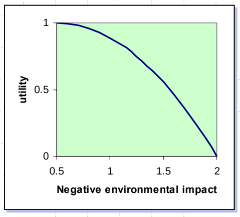
  - or decisions between a lottery and a sure value
    - eg k_Y=p (additive function)



  - but never from guesses!

Utility theory - parameters (example)

- X=(cost, negative environmental impact)
  - 500<cost<1500, 0.5<env<2.0



$$U_{cost}(cost)=\frac{1500-cost}{1500-500} \qquad U_{env}(env)=1-\frac{(env-0.5)^2}{(2-0.5)^2}$$

  - A=(1000, 0.5) ~ B=(700, 1.7)

$$U_{cost}(A)=0.5 \qquad U_{env}(A)=1$$

$$U_{cost}(B)=0.8 \qquad U_{env}(B)=0.36$$

$$U(A)=U(B)$$

$$k_1.U_{cost}(A)+k_2.U_{env}(A)=k_1.U_{cost}(B)+k_2.U_{env}(B)$$

$$0.5k_1+k_2=0.8k_1+0.36k_2$$

$$0.3k_1-0.64k_2=0$$

$$0.3k_1-0.64(1-k_1)=0$$

$$k_1=0.68 \qquad k_2=0.32$$

$$U(X)=0.68\,U_{cost}(X)+0.32\,U_{env}(X)$$

Utility theory - parameters (example)

- 3 alternatives X,Y,Z
  2 scenarios

	scenarios			
	C1		C2	
	cost	env	cost	env
X	1000	0.9	1000	0.8
Y	800	1.6	900	1.9
Z	500	1.7	1300	2
prob	0.7		0.3	

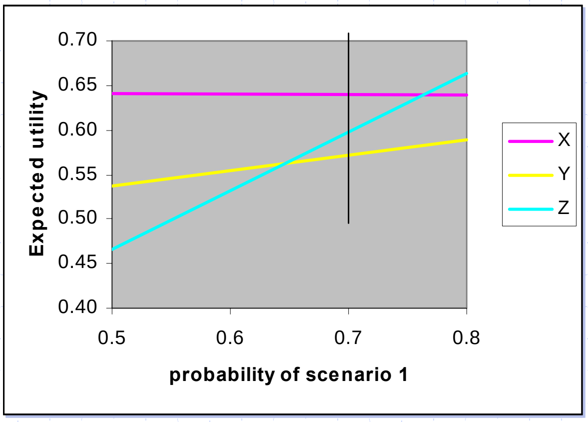
- Calculate the utilities in each scenario and the expected utility

$$U(X) = 0.68\, U_{cost}(X) + 0.32\, U_{env}(X)$$
$$E(U(X)) = 0.7\, U_{sc1}(X) + 0.3\, U_{sc2}(X)$$

	scenarios						
	C1			C2			
	cost	env	**U1**	cost	env	**U2**	**E(U)**
X	0.5	0.93	**0.64**	0.5	0.96	**0.65**	**0.64**
Y	0.7	0.46	**0.62**	0.6	0.13	**0.45**	**0.57**
Z	1	0.36	**0.80**	0.2	0.00	**0.14**	**0.60**
prob	0.7			0.3			

Sensitivity analysis

- If your are not sure about the scenarios' probabilities you may study the sensitivity of the final order to them
  - example: p(C1) from 0.8 to 0.5



Robust optimization

- Idea
  - This approach deals with uncertainty by trying to avoid unpleasant outcomes in adverse scenarios
- Basic paradigm
  - *Choose the alternative that, in the worst case, has the best value (minimax paradigm)*
- Basic concepts
  - Robustness, disappointment, regret
- Single attribute approach
  - In multicriteria problems, you must first aggregate

Absolute robust approach

- When
  - goal satisfaction
  - situations where the uncertainty comes from competitors' decisions
- Rule
  - Choose the alternative corresponding to: $\min_{z \in Z} \max_{s \in S} Cost(z,c)$

Z - set of alternatives
S - set of scenarios

	Cost			
alternative	Sc 1	Sc 2	Sc 3	order
A	850	**1100**	900	2
B	500	**1650**	1600	4
C	**1200**	1100	1150	3
D	900	**1000**	950	1
E	500	800	**1700**	5

Minimax regret approach

- when the quality of the decision is evaluated *ex post facto*
- when your losses are automatically gains of your competitors
- Rule: $\min_{z \in Z} \max_{s \in S} Regret(z,s) = \min_{z \in Z} \max_{s \in S} \left(Cost(z,s) - Cost^*(s)\right)$

Best in scenario s

alternative	Cost Sc 1	Cost Sc 2	Cost Sc 3
A	850	**1100**	900
B	500	**1650**	1600
C	**1200**	1100	1150
D	900	**1000**	950
E	500	800	**1700**
Best in Sc	500	800	900

alternative	Regret Sc 1	Regret Sc 2	Regret Sc 3	order
A	**350**	300	0	1
B	0	**850**	700	5
C	**700**	300	250	3
D	**400**	200	50	2
E	0	0	**800**	4

Minimax *weighted* regret approach

- when scenarios have very different probabilities
- Rule: $\min_{z \in Z} \max_{s \in S} \{prob(s).Regret(z,s)\}$

alternative	Regret Sc 1	Regret Sc 2	Regret Sc 3
A	**350**	300	0
B	0	**850**	700
C	**700**	300	250
D	**400**	200	50
E	0	0	**800**
prob	0.3	0.6	0.1

alternative	Weighted Regret Sc 1	Weighted Regret Sc 2	Weighted Regret Sc 3	order
A	105	**180**	0	3
B	0	**510**	70	5
C	**210**	180	25	4
D	**120**	**120**	5	2
E	0	0	**80**	1

- We may have also uncertainty on the probabilities
  - e.g. modeled with intervals or fuzzy sets

A related concept: exposure

- *Exposure* is the percentage of scenarios where an alternative *z* leads to unacceptable regret (as defined by a threshold)

$$S_E(z) = \{ s \in S \mid Regret(z,s) > threshold \}$$

$$Exposure(z) = \frac{\# S_E(z)}{\# S}$$

- or, if you have scenarios' probabilities

$$Exposure(z) = \sum_{s \in S_E(z)} prob(s)$$

Exposure: an example

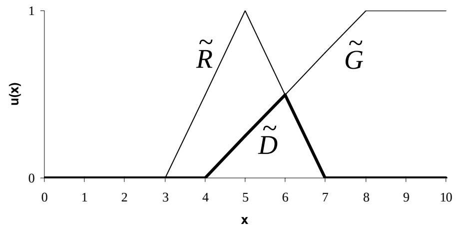
	Regret		
alternative	Sc 1	Sc 2	Sc 3
A	350	300	0
B	0	850	700
C	700	300	250
D	400	200	50
E	0	0	800
prob	0.3	0.6	0.1

- Threshold: 230

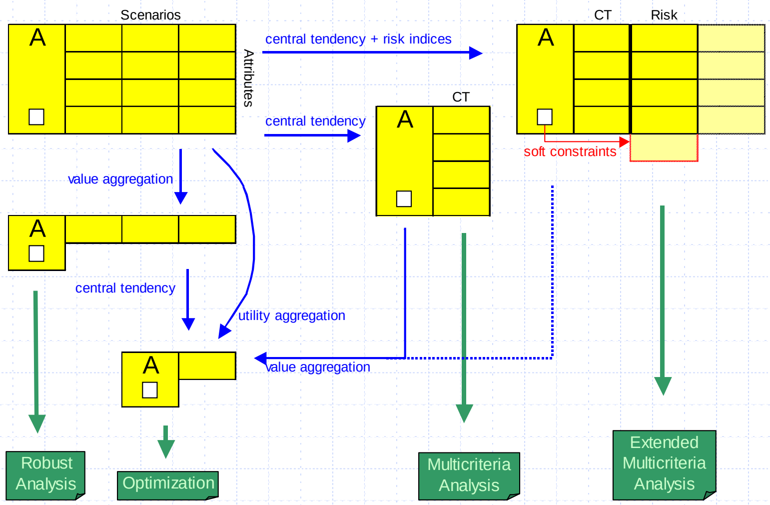
alternative	Sc 1	Sc 2	Sc 3	exposure1	exposure2
A	yes	yes		67%	0.9
B		yes	yes	67%	0.7
C	yes	yes	yes	100%	1
D	yes			33%	0.3
E			yes	33%	0.1

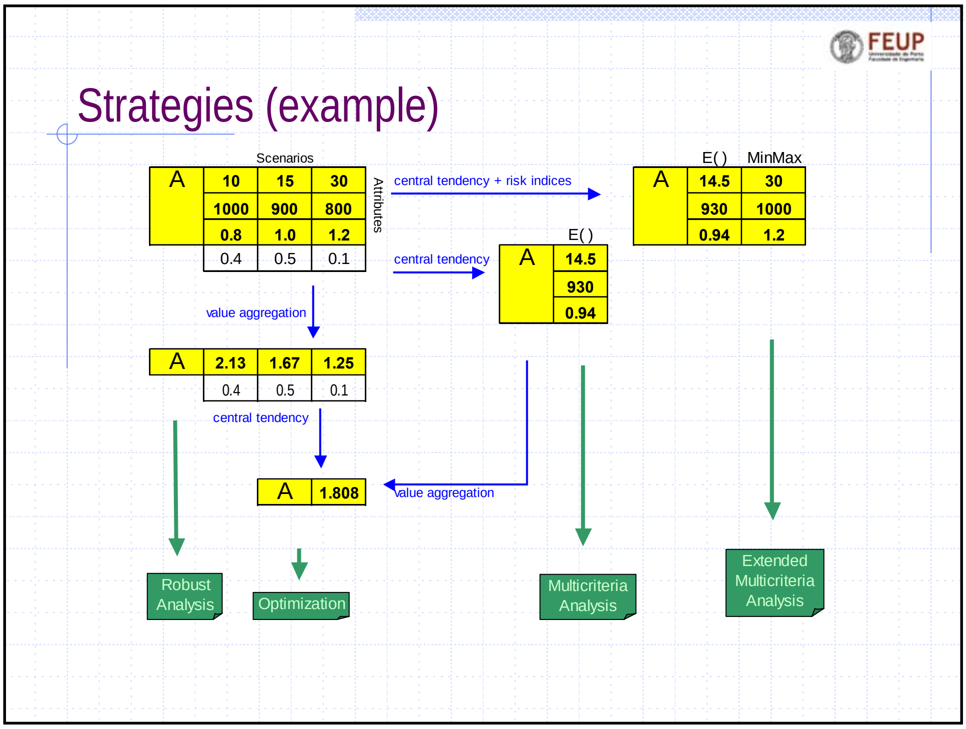
Fuzzy decision (Bellman and Zadeh)

- Basic paradigm:
  - *Choose the alternative that has the greatest degree of membership to the fuzzy set Decision, defined as intersection of the sets Goal and Restriction*



Strategies for MC problems with uncertainty

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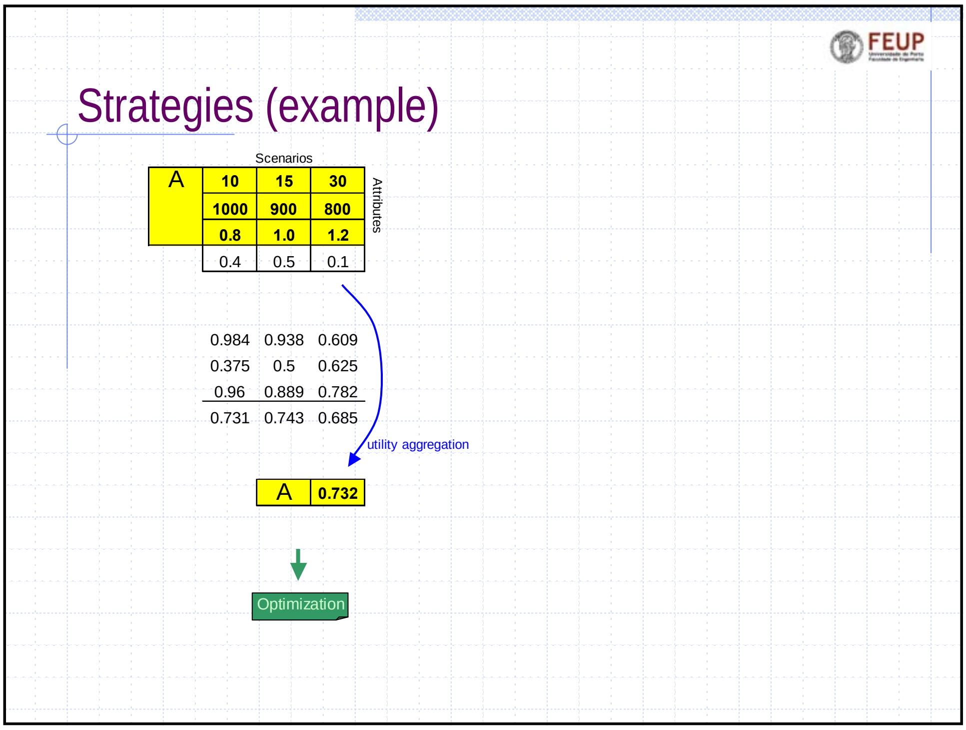
Strategies (example)
Scenarios
A
10
15
30
1000
900
800
0.8
1.0
1.2
0.4
0.5
0.1
Attributes
central tendency + risk indices
E( )
MinMax
A
14.5
30
930
1000
0.94
1.2
central tendency
E( )
A
14.5
930
0.94
value aggregation
A
2.13
1.67
1.25
0.4
0.5
0.1
central tendency
A
1.808
value aggregation
Robust Analysis
Optimization
Multicriteria Analysis
Extended Multicriteria Analysis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Strategies (example)
Scenarios
A
10
15
30
1000
900
800
0.8
1.0
1.2
0.4
0.5
0.1
Attributes
0.984 0.938 0.609
0.375 0.5 0.625
0.96 0.889 0.782
0.731 0.743 0.685
utility aggregation
A
0.732
Optimization

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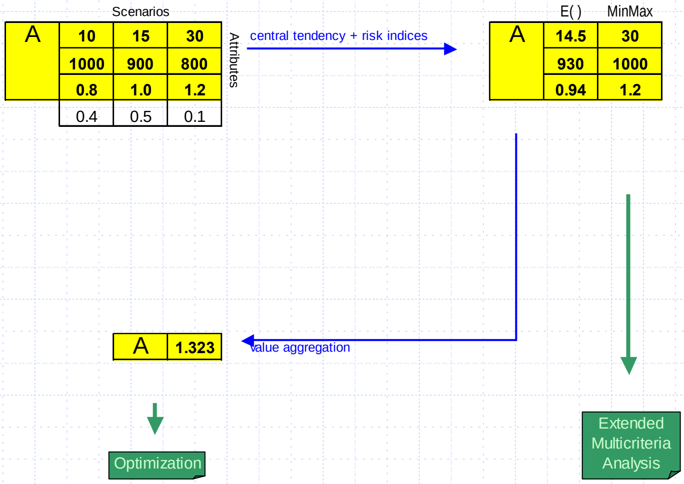
Strategies (example)



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Comparing different paradigms

maximizing

	0.9	0.1
A	20	-10a
B	10	-a

Exercise: For every paradigm, find the minimum value of a that makes B≻A

Expected value (a=10)

	0.9	0.1	EV
A	20	-100	8
B	10	-10	8

Absolute Robust (all values of a)

			minimax
A	20	-10a	-10a
B	10	-a	-a

Minimax Regret (a=10/9)

			Regret		minimax
A	20	-11.11	0	-10	-10
B	10	-1.11	-10	0	-10

Minimax Weighted Regret (a=10)

			0.9	0.1	W. Regret		minimax
A	20	-100	0	-90	0	-9	-9
B	10	-10	-10	0	-9	0	-9

Utility (u(x)=exp, par=-1) (a=7.678)

			0.9	0.1	E(U)
A	20	-76.78	1	0.6784	0.9678
B	10	-7.68	0.9729	0.922	0.9678

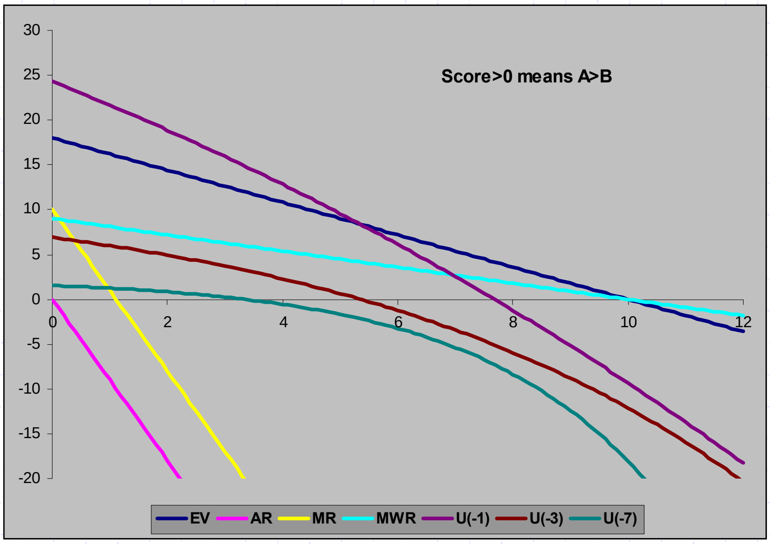
Utility (u(x)=exp, par=-3) (a=5.36)

			0.9	0.1	E(U)
A	20	-53.6	1	0.9095	0.9909
B	10	-5.36	0.9923	0.9784	0.9909

Utility (u(x)=exp, par=-7) (a=3.347)

			0.9	0.1	E(U)
A	20	-33.47	1	0.9959	0.9996
B	10	-3.35	0.9997	0.999	0.9996

Comparing different paradigms



Multicriteria approach

- Decision-aid in uncertain environments can be performed by constructing multicriteria models
  - Mathematically deterministic
- Prescriptive paradigms turn to be points of view
  - Traditional decision paradigms may be interpreted as possible points of view, leading to different (deterministic) attributes
  - The **Uncertainty model** and the **Decision methodology** are separated
- Other risk related attributes can be built and used
  - Must be **meaningful** for the Decision Maker
  - Problem dependent!

Indices for Pure scenarios

- Direct use of the scenarios' outcomes
- Regret
- Exposure
- Minimax value
- Maximax value
  - (outcome of the alternative in the most favorable scenario)

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Indices for Probability models

- Central measure attribute: $z_0 = E[z]$
- Risk related indices
  - Variance, skewness, regret, etc.
- Percentiles

$\{z_1, z_2 \mid p(z \le z_1) = 0.9,\ p(z \le z_2) = 0.1\}$ $\{p_1 = p(z \le 0),\ p_2 = p(z \le 100),\ p_3 = p(z \ge 1000)\}$

- “Optimistic” indices:
  - Probability of a positive outcome
  - Variance of positive outcomes
  - Expected value of gains
  - Best-case value
- Constraint related indices $p = p\left\{ \bigcup_{k \in C} \left(b_k(z) > b_k \right) \right\}$

Indices for Fuzzy models

- Central measure attribute:
  - Removal
  - Center of gravity
  - Maximum value
- (specific) Risk related indices
  - Largest and smallest of maximum
  - Largest and smallest possible values
  - Divergence
  - Set of possible results
  - Measures of fuzziness
    - a scalar index to measure the degree of fuzziness of a fuzzy set
- Constraint related indices

Example1 – generate deterministic attributes

	Cost			Expected	Max	Max Cost	Std	Prob	Skewness
n	C1 (0.3)	C2 (0.6)	C3 (0.1)	Cost	Cost	p>0.7	Dev	C≥60	
1	59	65	75	64.2	75	65	4.49	0.9	0.93
2	50	58	71	**56.9**	**71**	**58**	**5.91**	**0.3**	0.89
3	68	72	60	69.6	72	68	3.67	1.0	**-1.65**
4	69	72	62	70.1	72	69	**3.01**	1.0	**-1.80**
5	53	60	63	**58.2**	**63**	60	**3.52**	0.9	**-0.64**
6	51	59	65	**57.2**	**65**	59	**4.42**	0.3	**-0.24**
7	68	71	77	70.7	77	71	**2.49**	1.0	1.24
8	56	57	75	58.5	75	**57**	5.52	0.3	2.63
9	62	58	80	61.4	80	58	6.45	0.4	2.31
10	62	55	70	58.6	70	**55**	4.92	0.4	1.11

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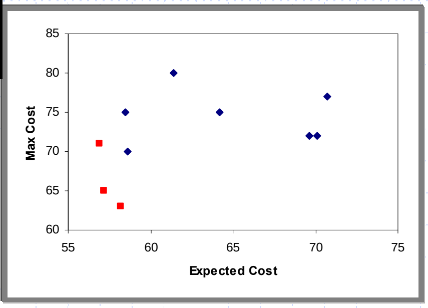
Example1 – two attributes (E, Max)

n	Cost C1 (0.3)	Cost C2 (0.6)	Cost C3 (0.1)	Expected Cost	Max Cost
1	59	65	75	64.2	75
2	50	58	71	**56.9**	**71**
3	68	72	60	69.6	72
4	69	72	62	70.1	72
5	53	60	63	**58.2**	**63**
6	51	59	65	**57.2**	**65**
7	68	71	77	70.7	77
8	56	57	75	58.5	75
9	62	58	80	61.4	80
10	62	55	70	58.6	70



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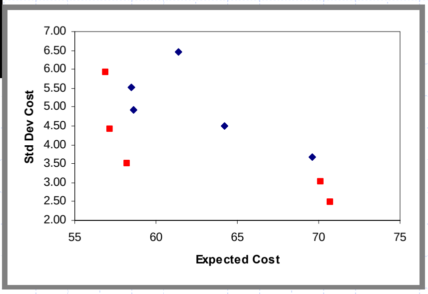
Example1 – two attributes (E, V)

n	Cost C1 (0.3)	Cost C2 (0.6)	Cost C3 (0.1)	Expected Cost	Std Dev
1	59	65	75	64.2	4.49
2	50	58	71	**56.9**	**5.91**
3	68	72	60	69.6	3.67
4	69	72	62	70.1	**3.01**
5	53	60	63	**58.2**	**3.52**
6	51	59	65	**57.2**	**4.42**
7	68	71	77	70.7	**2.49**
8	56	57	75	58.5	5.52
9	62	58	80	61.4	6.45
10	62	55	70	58.6	4.92



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Example2 – generate deterministic attributes

n	Cost C1 (0.85)	Cost C2 (0.10)	Cost C3 (0.05)	Expected Cost	Max Cost	Prob C≥300	Prob C≥350	Std Dev
1	59	325	375	101.4	375	15	5	101
2	50	290	355	89.3	355	5	5	94
3	68	360	300	108.8	360	15	10	98
4	69	360	310	110.2	360	15	10	98
5	53	300	315	**90.8**	**315**	15	0	**90**
6	51	295	325	**89.1**	**325**	**5**	**0**	**91**
7	68	355	385	112.6	385	15	15	106
8	56	285	375	94.9	375	5	5	94
9	62	290	400	101.7	400	5	5	97
10	62	275	350	97.7	350	5	5	**86**

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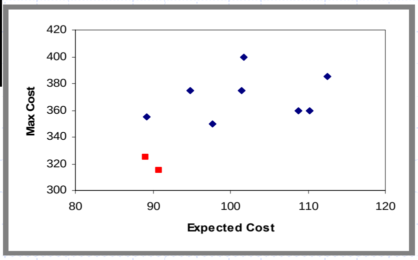
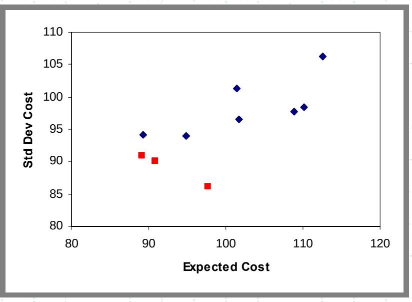
Example2 – two attributes (E, Max)

n	Cost C1 (0.85)	Cost C2 (0.10)	Cost C3 (0.05)	Expected Cost	Max Cost
1	59	325	375	101.4	375
2	50	290	355	89.3	355
3	68	360	300	108.8	360
4	69	360	310	110.2	360
5	53	300	315	**90.8**	**315**
6	51	295	325	**89.1**	**325**
7	68	355	385	112.6	385
8	56	285	375	94.9	375
9	62	290	400	101.7	400
10	62	275	350	97.7	350



Example2 – two attributes (E, V)

n	Cost			Expected Cost	Std Dev
	C1 (0.85)	C2 (0.10)	C3 (0.05)		
1	59	325	375	101.4	101
2	50	290	355	89.3	94
3	68	360	300	108.8	98
4	69	360	310	110.2	98
5	53	300	315	90.8	90
6	51	295	325	89.1	91
7	68	355	385	112.6	106
8	56	285	375	94.9	94
9	62	290	400	101.7	97
10	62	275	350	97.7	86



Final remarks

- Uncertainty can be interpreted as an additional dimension of decision problems
- Methodologies must incorporate the DM preferences, generally known as his risk attitude
  - But risk is a vague concept, with many operational translations
- There is no decision paradigm that prevails
  - Choosing one of them is a kind of meta-decision
  - Once chosen one d.p., the remaining process is essentially technical
- We also may build multicriteria approaches
  - Using deterministic equivalents