# Interactions between orthogonally polarized optical beams in photorefractive media 

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#### Abstract

A theory describing the propagation of orthogonally polarized spatial bright beams in a photorefractive crystal (SBN:60) is developed. The interaction between the two beams is further investigated numerically. We show that such a coupling can give rise to interesting effects like beam steering and beam compression.


Photorefractive (PR) media hold great promise for optical data processing [1]. Light incident on such materials is known to induce a nonlinear change in their refractive index. Recently, a class of spatial solitons has been discovered in PR media. These PR solitons are possible when the index confining effect exactly compensates for the process of diffraction [2-5]. Recent theoretical work has shown that a particular kind of soliton domains, the so-called screening solitons, can exist under steady-state conditions provided the PR material is externally biased [4,5]. In these previous investigations, bright, dark and gray steady-state domains have been predicted. Thus far, the above theoretical formulations have implicitly assumed that such beams share the same polarization.

In this paper, we study the interactions between orthogonally polarized steadystate planar bright beams in biased PR media. The evolution equations obeyed by the vector components are derived and the interactions are then investigated numerically, by assuming that the PR crystal is of strontium barium niobate (SBN:60) [6] type.

We first consider planar optical beams that propagate along the $z$-coordinate and are allowed to diffract only along the $x$-direction. Thus, in essence, we develop a one-dimensional diffraction theory. Let the PR crystal be SBN with its optical c-axis oriented in the $x$-direction. The incident optical wavefront $(\overrightarrow{\mathcal{E}})$ is composed of two beams, one polarized along the $x$-axis $\left(\overrightarrow{\mathcal{E}_{x}}\right)$ and other in the $y\left(\overrightarrow{\mathcal{E}_{y}}\right)$. The external bias electric field is parallel to the $c$-axis. Under these conditions, the perturbed refractive index along $x$ is given by $n_{x}^{2}=n_{e}^{2}-n_{e}^{4} r_{33} E_{x}$ and in the $y$-direction by $n_{y}^{2}=n_{o}^{2}-n_{o}^{4} r_{13} E_{x}$ where $n_{e}$ and $n_{o}$ are the extraordinary and ordinary unperturbed indices of refraction, $r_{33}$ and $r_{13}$ are the electrooptic coefficients involved and $\vec{E}_{x}=E_{x} \hat{x}$ is the total static field, a sum of applied dc electric field and that induced from the space charge in this material. In this case where the incident beams are bright like i.e. well confined within the $x$-width W of the PR crystal, the steady-state electric field $E_{x}$ is approximately given by [4,5]

$$
\begin{equation*}
E_{x}(x, z)=E_{0} \frac{I_{d}}{\left[I(x, z)+I_{d}\right]} . \tag{1}
\end{equation*}
$$

In Eq. (1), $\mathrm{I}(\mathrm{x}, \mathrm{z})$ is the total power density of the optical beams involved and $I_{d}$ is the so-called "dark irradiance" which phenomenologically accounts for the rate of thermally generated electrons. $E_{0}$ is the background electric field and is approximately given by $V / W$ where $V$ is the applied voltage. The total incident optical field $\overrightarrow{\mathcal{E}}=$ $\mathcal{E}_{x} \vec{x}+\mathcal{E}_{y} \vec{y}$ can then be expressed in terms of slowly varying envelopes $(X, Y)$ such that $\overrightarrow{\mathcal{E}}=\left(2 \eta_{0} I_{d} / n_{\mathrm{av}}\right)^{1 / 2}\left[\exp \left(i \gamma_{1} z\right) X \vec{x}+\exp \left(i \gamma_{2} z\right) Y \vec{y}\right] \exp (i k z)$ where $\eta_{0}=\left(\mu_{0} / \epsilon_{0}\right)^{1 / 2}$ is the vacuum intrinsic impedance. The average propagation constant, $k$, is given by $k_{0} n_{\mathrm{av}}=\left(2 \pi / \lambda_{0}\right) n_{\mathrm{av}}$ where $n_{\mathrm{av}}=\left(n_{e}+n_{o}\right) / 2 . \lambda_{0}$ is the free-space wavelength of the incident beam and the constants $\gamma_{1,2}=k_{0}^{2}\left(3 n_{e, o}^{2}-n_{o, e}^{2}-2 n_{o} n_{e}\right) / 8 k$. Furthermore for simplicity the PR material is assumed to be lossless. In this case, the normalized
envelopes $X$ and $Y$ satisfy:

$$
\begin{align*}
& i X_{\xi}+\frac{X_{s s}}{2}-\frac{\beta_{1} X}{1+|X|^{2}+|Y|^{2}}=0  \tag{2}\\
& i Y_{\xi}+\frac{Y_{s s}}{2}-\frac{\beta_{2} Y}{1+|X|^{2}+|Y|^{2}}=0 \tag{3}
\end{align*}
$$

where the following transformations have been adopted: $\xi=z /\left(k x_{0}^{2}\right), s=x / x_{0}$ and $x_{0}$ is an arbitrary spatial width. In the above equations $X_{\xi}=\partial X / \partial \xi$ etc. and the dimensionless quantity $\beta_{1}=\left(k_{0} x_{0}\right)^{2} n_{e}^{4} r_{33} E_{0} / 2$ whereas $\beta_{2}=\left(k_{0} x_{0}\right)^{2} n_{o}^{4} r_{13} E_{0} / 2$. If the incident wavefront exhibits only one polarization, the above coupled set of equations reduce to one equation which is known to allow soliton solutions [4,5]. It is of interest to study the interactions among cross-polarized spatial solitons in PR materials. To consider such interactions, Eqs. (2) and (3) are solved numerically using a split-step Fourier method [7]. It can be readily shown that the two beams do not in fact exchange power. Nevertheless, they can greatly influence each other through cross-phase modulation effects.

As an example the $P R$ material is taken to be SBN: 60 with the following parameters: $n_{e}=2.33, n_{o}=2.36, r_{33}=237 \mathrm{pm} / \mathrm{V}$ and $r_{13}=37 \mathrm{pm} / \mathrm{V}$ at a wavelength $\lambda_{0}=0.5 \mu \mathrm{~m}$ [6]. If we let the arbitrary spatial scale to be $x_{0}=40 \mu \mathrm{~m}$ and the applied electric field $E_{0}=40 \times 10^{3} \mathrm{~V} / \mathrm{m}$, we find that $\beta_{1}=35.3$ and $\beta_{2}=5.8$. The initial field distribution at $\xi=0$ are assumed to be the bright soliton solutions of each polarization and are identified by the parameters $r_{1}=|X(\xi=0)|_{\max }^{2}$ and $r_{2}=|Y(\xi=0)|_{\max }^{2}$. Let us first consider a pump-probe configuration with the $x$-polarized envelope $X$ to be much more intense than $Y$. The two beams are taken to be completely overlapping at the input. Numerical solutions of Eqs. (2) and (3) show that the pump propagates without experiencing appreciable change in its form. On the other hand the probe beam exhibits a complex evolution pattern. Fig. 1 shows the evolution of a $r_{2}=0.1$ probe when it co-propagates with a $r_{1}=40$ pump beam. The probe undergoes com-


Figure 1: Evolution of a $y$-polarized probe beam with $|Y(\xi=0)|_{\max }^{2}=r_{2}=0.1$ co-propagating with a $x$-polarized $r_{1}=40$ pump.
pression before it begins to expand. The compression at $\xi=0.6(z \approx 3 \mathrm{~cm})$ is found to be $70 \%$, with its intensity full width half max (FWHM) decreasing from $98 \mu \mathrm{~m}$ to $30 \mu \mathrm{~m}$. The amount of compression is found to increase as the ratio of the input probe beam width to that of the pump increases. If the two beams are allowed to be spatially separated (they overlap partially at the input), the probe experiences an asymmetric compression and develops a side wing. Apart from compression, we find that the cross-polarized beams are initially attracted to each other.

To observe the interaction forces between the $X$ and $Y$ beams in more detail, let the input configuration be equal amplitude $r_{1}=r_{2}=1$ solitons. The evolution of the two waveforms is depicted in Fig. 2 where they are initially separated by $\Delta s=0.5$ which corresponds to $20 \mu \mathrm{~m}$. The $y$-polarized beam which exhibits a small $\beta\left(\beta_{2}=5.8\right)$ does not experience any appreciable change in its form. This is not the case for the $x$-polarized beam where $\beta_{1}=35.3$, which undergoes significant evolution in its width and peak intensity. Moreover, the center of the $x$-polarized beam initially gets steered towards the other beam's center and thereafter oscillates around that


Figure 2: Interaction between a $r_{1,2}=1 x$ and $y$ polarized beams when separation between them at the input is $\Delta s=0.5 .|X|^{2}$ and $|Y|^{2}$ are represented by solid and dashed lines respectively.
point. $X$, which has an FWHM of $18 \mu \mathrm{~m}$ at the input, is found to shift by $7 \mu \mathrm{~m}$ at $\xi=0.2$ that is $z=1 \mathrm{~cm}$.

Other interesting interactions are also possible. We plan to report them in a future publication.

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