Compression, self-bending, and collapse of Gaussian beams in photorefractive crystals

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By means of an exact solution, we demonstrate that a Gaussian beam can undergo spatial compression when it traverses a photorefractive medium. This is possible provided that the external bias field exceeds the critical value necessary to establish photorefractive spatial solitons. In this regime and under paraxial assumptions our analysis indicates that a Gaussian beam tends to exhibit self-focusing collapse. Beam self-deflection effects that arise from the $\pi/2$-phase-shifted component of the photorefractive grating are also considered in our study.

It has been suggested[1,2] that spatial soliton formation may be feasible in photorefractive (PR) media. This phenomenon arises through the process of PR phase coupling (owing to two-wave mixing) that occurs among the spatial plane-wave components of an optical beam. This leads to a self-lensing effect, which in turn is capable of counteracting that of diffraction. As a result, a nondiffracting beam, or what is better known as a PR spatial soliton, can form. As was previously noted,[1,2] this class of solitons can be observed provided that the PR material has been appropriately biased (externally) so appreciable phase coupling effects can take place. Moreover, the PR crystal needs to be oriented properly to preserve symmetry requirements.[1,2] Two recent experimental studies[3,4] have successfully confirmed such spatial soliton behavior. Unlike their $\chi^{(3)}$ counterparts,[5] PR spatial solitons can be observed at very low power densities. Clearly, this opens new exciting opportunities toward all-optical beam switching at microwatt power levels.[3]

In this Letter we solve exactly the evolution problem of one-dimensional Gaussian beams propagating in PR crystals. In the case in which the external bias field is above the threshold value necessary to establish PR spatial solitons, our results indicate that a Gaussian beam can experience spatial compression. It is also found that in this regime Gaussian beams tend to exhibit self-focusing collapse. Even though we recognize that a beam collapse of this sort is physically unrealistic (it is an artifact of our paraxial arguments), we may nevertheless anticipate spatial compression. Beam self-bending effects that arise from the $\pi/2$-phase-shifted component of the PR grating[6,7] are also considered in our analysis.

Let us consider an optical beam that propagates in a PR material along the z axis and is allowed to diffract only along the x direction. In essence, we will be dealing with a one-dimensional diffraction theory. Moreover, let us assume that the PR crystal is oriented in a fashion similar to that suggested in Ref. 2. For illustration purposes let the crystal be, say, strontium barium niobate (SBN), with its optical c axis oriented along the x coordinate. The optical beam is linearly polarized along x, and the external-bias electric field is also applied in the same direction. Any noise fanning effects have been neglected in our analysis. In this case, the spatial envelope U of the optical beam obeys the following equation[1,2]:

$$iU_z + i\alpha U + \frac{1}{2k}U_{xx} + \frac{k}{n_1}\frac{1}{U^*}\iint U(x - x', z)U^*(x + x'', z)g(x', x'')\,\mathrm{d}x'\mathrm{d}x'' = 0, \tag{1}$$

where $U_z = \partial U/\partial z$, etc. α is the bulk loss coefficient, $k = k_0 n_1$, and n_1 is the crystal's index of refraction. The integral in Eq. (1) implicitly accounts for any phase–power-coupling effects that occur among spectral components of the optical beam as a result of two-wave mixing. Moreover, $g(x', x'')$ is related to the complex factor $\delta\hat{n}(k_{x'}, k_{x''})$ through a Fourier transform, i.e.,

$$\delta\hat{n}(k_{x'}, k_{x''}) = \iint g(x', x'') \times \exp\left[-i(k_{x'}x' + k_{x''}x'')\right]\mathrm{d}x'\mathrm{d}x'', \tag{2}$$

where $\delta\hat{n}(k_{x'}, k_{x''}) = (n_1^3/2)r_{\mathrm{eff}}E_m(k_{x'}, k_{x''})$, r_{eff} is an effective electro-optic coefficient, and E_m is the complex coefficient of the induced PR space-charge field, which can be obtained from[8]

$$E_m(k_{x'}, k_{x''}) = \frac{E_p(iE_0 - E_d)}{E_0 + i(E_d + E_p)}, \tag{3}$$

where E_0 is the externally applied electric field, $E_p = eP_d/(\varepsilon_0\varepsilon_r K_g)$ is the saturation space-charge field, and $E_d = K_B T K_g/e$ is the so-called diffusion field.[8] P_d is the trap density, ε_r is the static relative permittivity of the crystal, e is the electronic charge, K_B is Boltzmann's constant, and T is the absolute temperature. $K_g = k_{x'} - k_{x''}$ is the PR grating vector, which in our case is along the x direction.

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If we now assume that the Fourier transform of the intensity profile of this optical beam is well confined around the vicinity of $K_g = 0$, (i.e., by using paraxial arguments), then $E_m(k_{x'}, k_{x''})$ can be expressed as a truncated Taylor series. More specifically, a quadratic expansion of the real part of E_m and a linear expansion of the imaginary part give $\delta\hat{n}(k_{x'}, k_{x''}) \approx B(1 - d^2K_g^2) + i\Gamma K_g$, where $B = (n_1^3/2)r_{\text{eff}}E_0$, $\Gamma = [n_1^3 r_{\text{eff}}\varepsilon_0\varepsilon_r/(2eP_d)][E_0^2 + K_BTP_d/(\varepsilon_0\varepsilon_r)]$, and $d^2 = [\varepsilon_0\varepsilon_r E_0/(eP_d)]^2 + [2K_BT\varepsilon_0\varepsilon_r/(e^2P_d)]$. In obtaining $\delta\hat{n}(K_g)$ we have also assumed that r_{eff} is relatively insensitive to $k_{x'}$ or to $k_{x''}$, which is true for the configuration considered here. B is a dimensionless quantity and can be positive or negative depending on the direction of the externally applied field. Γ and d are measured in units of meters. Using the Taylor expansion of $\delta\hat{n}(K_g)$ and by taking an inverse Fourier transform, one can obtain $g(x', x'')$ explicitly from Eq. (2); it is given by $g(x', x'') = B\delta(x')\delta(x'') + Bd^2\delta(x' + x'')\delta''(x') + \Gamma\delta(x' + x'')\delta'(x')$, where $\delta(x)$, $\delta'(x)$, and $\delta''(x)$ are delta and derivative delta functions of the first and second order. By substituting this form of $g(x', x'')$ back into Eq. (1), one obtains the following envelope evolution equation:

$$iU_z + i\alpha U + \frac{1}{2k}U_{xx} + \frac{k}{n_1}\frac{1}{U^*}\Big[B|U|^2 + Bd^2(|U|^2)_{xx} + \Gamma(|U|^2)_x\Big] = 0. \quad (4)$$

One can further simplify Eq. (4) by introducing dimensionless variables and quantities; that is, let $\xi = z/(kx_0^2)$, $s = x/x_0$, $\beta = (k^2/n_1)Bd^2$, and $\gamma = \Gamma x_0(k^2/n_1)$, where x_0 is associated with the spatial width of the optical beam. Moreover, by adopting the transformation $U = u\exp[i(k^2/n_1)x_0^2B\xi]\exp(-\alpha kx_0^2\xi)$ one can readily show that the auxiliary envelope u satisfies

$$iu_\xi + \frac{1}{2}u_{ss} + \left[\beta\frac{(|u|^2)_{ss}}{|u|^2} + \gamma\frac{(|u|^2)_s}{|u|^2}\right]u = 0. \quad (5)$$

The $U-u$ transformation just employed clearly indicates that bulk losses play no role whatsoever in the dynamics of optical beams in PR media. Previous investigations[1,2] reached similar conclusions. In Eq. (5) the nonlinear term associated with β accounts for the PR phase-coupling process, and it is responsible for beam self-focusing or defocusing. The other nonlinear term, which is proportional to γ, describes the effect that arises from the $\pi/2$-phase-shifted component of the PR hologram,[6,7] which tends to deflect the optical beam. β was derived from the real part of E_m, whereas γ was derived from the imaginary part. The spatial PR solitary (stationary) wave solutions of Eq. (5) were obtained in earlier studies[1,2] under the assumption that γ is negligible ($\gamma = 0$). These waves exist provided that β lies in the range $-1/4 < \beta \le -1/8$. More specifically, for $\beta = -1/8$ the PR solitary wave is given by $u = u_0\exp(-s^2/2)\exp(-i\xi/4)$. If, on the other hand, $-1/4 < \beta < -1/8$, then the soliton waves of Eq. (5) can be obtained from $u = u_0\,\text{sech}^D(s)\exp(ip\xi)$, where $D = -(1 + 4\beta)/(1 + 8\beta)$ and p is a constant to be determined.

It can be directly shown that Eq. (5) exhibits the following evolving Gaussian solution:

$$u(\xi, s) = A(\xi)\exp\left[-\frac{\eta^2}{2a^2(\xi)}\right] \times \exp\Big\{i[\theta(\xi) + \eta\mu(\xi) + \eta^2F(\xi)]\Big\}, \quad (6)$$

where

$$\eta = s + \nu(\xi) \quad (7)$$

and A, a, θ, μ, F, and ν are all real functions of the normalized distance ξ and are given by

$$a^2(\xi) = 1 + (1 + 8\beta)\xi^2, \quad (8)$$

$$A^2(\xi) = u_0^2/a(\xi), \quad (9)$$

$$F(\xi) = (1 + 8\beta)(\xi/2)\big[a(\xi)\big]^{-2}, \quad (10)$$

$$\mu(\xi) = -2\gamma(1 + 8\beta)^{-1/2}\tan^{-1}\Big[(1 + 8\beta)^{1/2}\xi\Big], \quad (11a)$$

$$\mu(\xi) = -2\gamma(|1 + 8\beta|)^{-1/2}\tanh^{-1}\Big[(|1 + 8\beta|)^{1/2}\xi\Big], \quad (11b)$$

$$\nu(\xi) = -\Big\{\mu(\xi)\xi + 2\gamma(1 + 8\beta)^{-1}\ln\big[a(\xi)\big]\Big\}, \quad (12)$$

$$\theta(\xi) = \theta_0 + \frac{1}{2}\int_0^\xi d\xi'\,\mu^2(\xi') + \frac{(1 + 4\beta)}{4\gamma}\mu(\xi). \quad (13)$$

In the above expressions u_0 and θ_0 are, respectively, the initial (at $\xi = 0$) amplitude and phase constants of this Gaussian beam. Furthermore, Eq. (11a) is to be used whenever $1 + 8\beta > 0$, whereas for $1 + 8\beta < 0$ Eq. (11b) has to be employed. The $\mu(\xi)$ function that appears in Eqs. (12) and (13) has to be appropriately selected from Eq. (11a) or (11b), depending on whether $(1 + 8\beta)$ is positive or negative. In obtaining the above solution we assumed (without any loss of generality) that the Gaussian beam enters the PR crystal at its minimum waist point.

Let us now physically interpret these results. Equation (8) clearly shows that the normalized spot size $a(\xi)$ of this Gaussian beam tends to decrease for $\beta < -1/8$, whereas it increases when $\beta > -1/8$. Thus, if the externally applied electric field is above the threshold value necessary for observation of spatial solitons ($\beta = -1/8$), then the Gaussian beam tends to experience spatial compression. This result is in agreement with previously reported experimental observations.[3] On the other hand, if $\beta = -1/8$, then the Gaussian beam behaves as a solitary wave, because its spot size and amplitude remain unchanged. In fact, by setting γ to zero, one obtains the Gaussian PR spatial soliton of Ref. 2. In the range $-1/8 < \beta < 0$ the beam starts to diffract, but now at a somewhat slower pace than what would have been anticipated if the external bias field were switched off ($\beta = 0$). Reversing the polarity of the externally applied field (i.e., $\beta > 0$) produces a more pronounced diffraction spreading. In all cases the amplitude $A(\xi)$ increases or decreases depending on whether the beam undergoes contraction or expansion, and always in such a way so that the integral $\int |u|^2 ds$ remains a constant of the motion. Another interesting consequence of our results is that the model equation (5) permits a self-focusing collapse when $\beta < -1/8$. In this case, a collapse is expected to occur at a critical distance $\xi_c = (|1 + 8\beta|)^{-1/2}$, or at $z_c = kx_0^2(|1 + 8\beta|)^{-1/2}$, where $A \rightarrow \infty$ and $a \rightarrow 0$. Keeping in mind that our nonlinear diffraction theory is one dimensional, this sort of behavior is relatively rare[9] within the context of nonlinear optics. Self-focusing collapse in $\chi^{(3)}$ media is possible only when the dimensionality of the nonlinear diffraction problem is 2 or higher.[10,11] Clearly this predicted collapse will never happen in reality, because it is an artifact of our paraxial assumptions employed in developing Eq. (5). Moreover, Eq. (5) was derived under the assumption that the PR space-charge field varies only along the x coordinate. In reality, however, the evolution of the beam induces a z change as well, which in turn perturbs the validity of Poisson's equation in Kukhtarev's model.[8] By checking the self-consistency of our solution we have found that this error is very small, provided that $|1 + 8\beta|^{1/2}/[kx_0 a(\xi)] << 1$. In a typical case, where $kx_0 >> 1$, this error becomes important only near $\xi = \xi_c$. These effects provide an additional argument as to why the self-focusing collapse is not indeed expected to occur. Nevertheless, spatial compression is still anticipated when $\beta < -1/8$. The role of the power-exchange process that takes place among the spectral components of this Gaussian beam is also self-evident. Equations (7) and (12) show that, for finite γ's, the center of the beam leaves the origin (at $x = 0$) and shifts to a new position, $x_d = -x_0\nu(\xi)$. Furthermore, the central $\mathbf{k}_c$ vector of the Gaussian beam is no longer parallel to the z axis. The angle θ_d between $\mathbf{k}_c$ and the z axis can be obtained from Eqs. (6) and (11) and is given by $\theta_d = (kx_0)^{-1}\mu(\xi)$. It is important to note that our solution is valid as long as θ_d lies within the paraxial cone.

Let us now illustrate our results with an example. We consider a PR crystal of the SBN type, which is oriented as previously suggested.[2] For this particular orientation $r_{\text{eff}} \approx r_{33} = 224 \times 10^{-12}$ m/V. This crystal's refractive index is $n_1 = 2.35$, and its static relative permittivity is $\varepsilon_r \approx 1100$. Moreover, the trap density is taken here to be $P_d = 1 \times 10^{15}$ cm^{-3}. The optical wave is polarized along the x axis, and its vaccum wavelength is $\lambda_0 \approx 0.5$ μm. The initial spot size of this Gaussian beam is assumed to be $x_0 = 50$ μm. With these parameters, the critical electric field intensity necessary for observation of PR solitons is $E_c \approx 330$ V/cm. If a bias field $E_0 = 800$ V/cm of proper polarity is applied to this crystal (in the x direction), then $B = -1.16 \times 10^{-4}$, $d = 5.18$ μm, and $\Gamma = 6 \times 10^{-4}$ μm. The normalized quantities β and γ can then be evaluated and for these values are $\beta \approx -1.16$ and $\gamma \approx 11.13$. At a distance $\hat{z} = 2.22$ cm from the origin, the spot size of this Gaussian beam is reduced to half of its original value, i.e., $x_0(\hat{z}) = 25$ μm. At this point, the transverse deflection of this beam is found to be $x_d = 58$ μm, and the angular deviation from the z axis is $\theta_d = 6.8$ mrad.

It is also interesting to compare the theoretically predicted self-bending angle θ_d of a Gaussian beam with experimental observations. An experimental study of this effect was reported in Ref. 6. In that work, an unbiased ($\beta = 0$) $BaTiO_3$ crystal of length 5 mm was used, and the Gaussian beam was launched in the sample at an angle of 12.5° with respect to the z or c axis. From the data of Fig. 5 of Ref. 6, one finds that $\Gamma \approx 1.39 \times 10^{-4}$ μm. In this case, our model predicts a self-deflection angle $\theta_d = -2k_0\Gamma \tan^{-1}(\xi)$ of approximately 3.18 mrad, or 0.18°, which is in good agreement with that of 0.2° experimentally observed.

In conclusion, we have shown that a Gaussian beam can undergo spatial compression in PR materials when the externally applied bias field exceeds the critical value necessary to establish PR solitons. Beam self-bending effects that arise from the $\pi/2$-phase-shifted component of the PR grating have also been taken into account in our analysis. We have also found that in the paraxial limit the PR nonlinear evolution equation can also permit a collapsing solution. It is worth pointing out that, at this point, is not quite clear whether this compression–collapse behavior is generic to all beam profiles or whether it is specific only to Gaussian beams. To formally prove or disprove this statement may require the development of a virial-like theorem[11] for Eq. (5).

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