Optical shock waves in nonlinear dispersive amplifying media

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We show that optical shock-wave solutions are possible in nonlinear dispersive amplifying media that exhibit a frequency-dependent gain and background loss. These shock-wave domains exist at lasing threshold and are permitted in both the normal and the anomalous dispersive regions.

Rare-earth-doped fiber amplifiers are expected to play an ever-increasing role in tomorrow's light-wave communication networks.[1] As a result, various aspects associated with their performance are currently under intense investigation. Among them, their nonlinear properties have received considerable attention. Recent experimental studies have successfully demonstrated coherent π-pulse propagation[2] as well as accumulated photon echoes[3] in erbium-doped fibers at liquid-helium temperatures. Moreover, nonlinear dispersive wave propagation in such amplifying media has also attracted a great deal of interest because of its potential implications in long-haul, high-speed soliton-based communication systems.[4] Modeling of nonlinear wave propagation in dispersive amplifying optical fibers typically proceeds by incorporation into the well-known nonlinear Schrödinger equation of two additional terms that describe the parabolic frequency dependence of the gain profile.[5-7] In fact, the resulting evolution equation is a special form of the so-called Ginzburg–Landau equation[5] (GLE), which is known to admit of chirped solitary-wave solutions,[6,8] both bright and dark. It is important to note that the latter class of wave exists only at the wavelength of maximum gain. The stability of these chirped solitons[5] and the modulational instability properties[9] of the GLE have recently been considered in various studies.

In this Letter we show that optical shock-wave solutions are also possible in nonlinear dispersive amplifying media. This family of solutions is permitted, provided that one takes into account the effects arising from the slope and curvature of the frequency dependent gain curve as well as those of background loss, dispersion, and nonlinearity. Our analysis indicates that these shock-wave domains exist at lasing threshold wavelengths and that they can propagate in both the normal and the anomalous dispersive regions. As was recently pointed out by Agrawal,[10] the occurrence of optical shock waves (or kink solitons) in nonlinear optics is rather rare. Thus far, such shock-wave domains have been identified only in a limited number of systems in which there is either a direct Raman power exchange between waves[11] or intrapulse Raman scattering.[12,13] In our case, these kinklike solitons can propagate undistorted by balancing dispersive and self-phase-modulation effects and by counteracting the process of carrier frequency shift arising from the frequency dependence of the gain.

Let us now consider an amplifying nonlinear dispersive optical fiber whose gain varies with frequency in a fashion similar to that shown in Fig. 1. In the vicinity of the wave's carrier angular frequency ω_0, the net gain g_{net} can then be expanded in a Taylor's series in Ω, i.e., $g_{\text{net}}(\omega_0 + \Omega) = g(\omega_0) + g_0'\Omega + (g_0''/2)\Omega^2 - \alpha_B$, where $g_0' = (\partial g/\partial\omega)_{\omega_0}$, $g_0'' = (\partial^2 g/\partial\omega^2)_{\omega_0}$, and α_B represents the background loss of the fiber amplifier. Cubic- and higher-order terms of the gain dispersion as well as gain saturation effects[5] have been neglected in this analysis. If the carrier frequency ω_0 of the optical wave is taken at the wavelength of lasing threshold $[g(\omega_0) = \alpha_B]$, that is, where the gain curve meets the line of the background losses (points 1 and 2 in Fig. 1), then the net gain dispersion is given by $g_{\text{net}}(\omega_0 + \Omega) = g_0'\Omega + (g_0''/2)\Omega^2$. By employing standard techniques,[5] one can then show that the envelope dynamical evolution equation that describes nonlinear dispersive wave propagation at the lasing threshold wavelength of this fiber amplifier is given by

$$iU_x + (\epsilon - id)U_{\tau\tau} + mU_\tau + |U|^2U = 0, \qquad (1)$$

where $U_x = \partial U/\partial x$, etc. In the above dimensionless equation, the coordinate x is related to the ac-

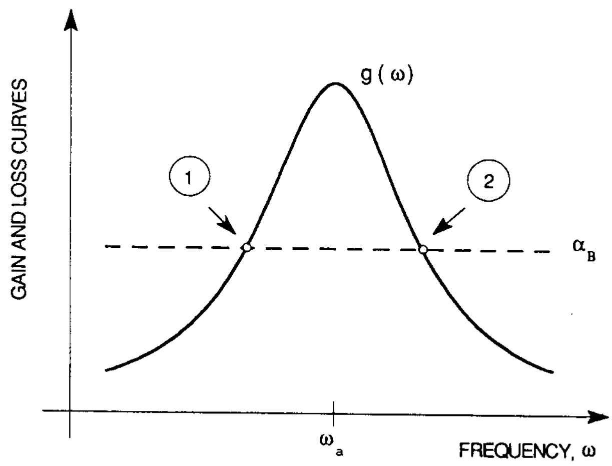

Fig. 1. Gain and loss curves as functions of angular frequency ω. Lasing threshold conditions occur at points 1 and 2.

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tual propagation distance z through $x = (|k_0''|z/2\tau_0^2)$, where τ_0 is an arbitrary time scale that is associated with the wave's temporal pulse width and k_0'' is the fiber dispersive coefficient evaluated at ω_0. The normalized time coordinate frame τ is given by $\tau = [t - (z/v_g)]/\tau_0$, where v_g is the group velocity. The dimensionless envelope U is related to the actual electric-field envelope E through the expression $E = (\lambda_0|k_0''|/2\pi n_2\tau_0^2)^{1/2}U$, where n_2 is the nonlinear Kerr coefficient and λ_0 is the vacuum wavelength corresponding to ω_0. The parameter ϵ in Eq. (1) is defined as $\epsilon = -k_0''/|k_0''|$, and thus $\epsilon = +1$ in the region of anomalous dispersion and $\epsilon = -1$ in that of the normal. The quantity $m = (2g_0'\tau_0/|k_0''|)$ is related to the slope (at the lasing threshold wavelength) of the frequency-dependent gain curve, whereas $d = -(g_0''/|k_0''|)$ is related to its curvature. Even though this is not immediately obvious, Eq. (1) is in fact equivalent to the special form of the GLE previously encountered in other studies.[5,6] As we show below, this equivalence can be established by use of a frequency-shifting transformation.

A straightforward calculation shows that Eq. (1) admits of the following shock-wave solution:

$$U = A\{1 + \tanh[p(\tau + \nu x)]\}\exp(i\theta), \tag{2}$$

where the chirped phase θ is given by

$$\theta = \lambda x + \mu\tau + \alpha \ln\{1 + \tanh[p(\tau + \nu x)]\} \tag{3}$$

and the real constants involved in Eqs. (2) and (3) can be obtained from

$$A^2 = -(3p^2\alpha/d)(d^2 + 1), \tag{4}$$

$$\alpha = -(3\epsilon/2d) \pm [(3/2d)^2 + 2]^{1/2}, \tag{5}$$

$$p = \frac{\left(\mu d + \dfrac{m}{2}\right)(1 + \alpha^2)}{\epsilon(1 + \alpha^2) - \dfrac{6\alpha}{d}(d^2 + 1)}, \tag{6}$$

$$\nu = 2p(\alpha\epsilon - d) - 2\mu(\epsilon + \alpha d) - m\alpha, \tag{7}$$

$$\mu = 0 \quad \text{or} \quad \mu = -(m/d), \tag{8}$$

$$\lambda = 4A^2 - \mu^2\epsilon. \tag{9}$$

Equation (8) shows that Eq. (1) exhibits in reality two distinct shock-wave solutions. The first one, $\mu = 0$, represents a local solution with a carrier angular frequency of ω_0, whereas the second solution has its carrier wavelength translated to a different operating point. To understand this better, let us assume that ω_0 is taken initially at the first lasing threshold frequency, say, point 1 in Fig. 1. It is then easy to show that for $d > 0$ the second solution branch ($\mu = -m/d$) will shift the carrier frequency to point 2, where $g(\omega_0) = \alpha_B$, i.e., at the other lasing threshold wavelength. Without any loss of generality, let us now restrict our attention to local solutions only, in which case $\mu = 0$. Given a physical system for which the quantities ϵ, d, and m are known, α can be obtained from Eq. (5), and subsequently the other parameters A, p, ν, and λ can be determined by setting $\mu = 0$. Evidently, for A^2 to be positive, the ratio (α/d) in Eq. (4) must be a negative quantity. If, for example, $d > 0$ (or $g_0'' < 0$), which is true for frequencies near that of maximum gain, then the minus must be selected in Eq. (5) so that $\alpha/d < 0$. Otherwise, if $d < 0$ or $g_0'' > 0$, then the plus must be used. Note that d can assume negative values whenever the loss line α_B meets the gain below the points of inflection of the gain curve. Moreover, if we assume for a moment that the dispersion of the system is anomalous ($\epsilon = +1$), then it follows from Eq. (6) that the values of p are of opposite sign at points 1 and 2 of Fig. 1. The different character of the solutions that correspond to these two points is depicted in Fig. 2. In this case the solution associated with point 1, where $p > 0$, has the shock front leading the cw tail, whereas the situation is reversed for that of point 2. Unlike the bright/dark solitary waves previously identified in amplifying nonlinear media,[5-8] the shock-wave solutions of Eq. (1) are neither symmetric nor antisymmetric in τ. Clearly, the presence of the term mU_τ, which accounts for the effects arising from the slope of the gain curve, spoils the symmetry of Eq. (1). Moreover the nonlinear shift of the propagation constant as well as that of the frequency could have been obtained by consideration of the dynamics of the wave's cw tails under zero net gain. By assuming an equilibrium pattern of the form $U = 2A\exp[i(\mu\tau + \lambda x)]$ in Eq. (1), one obtains $\lambda = 4|A|^2 - \mu^2\epsilon$ and $\mu(\mu d + m) = 0$, which are precisely Eqs. (8) and (9).

As mentioned above, Eq. (1) is actually equivalent to a GLE. This can be easily shown by the following frequency-shifting transformation:

$$U(\xi, \eta) = \Phi(\xi, \eta)\exp\left(-i\frac{m}{2d}\eta\right)\exp\left(i\frac{m^2\epsilon}{4d^2}\xi\right), \tag{10}$$

where $\xi = x$ and $\eta = \tau + (m\epsilon/d)x$. In the new coordinates ξ and η, the envelope Φ obeys

$$i\Phi_\xi + (\epsilon - id)\Phi_{\eta\eta} + |\Phi|^2\Phi = i(m^2/4d)\Phi, \tag{11}$$

which is the special form of the GLE encountered in earlier studies. It is noteworthy to point out that solutions similar to those of Eqs. (2) and (3) were

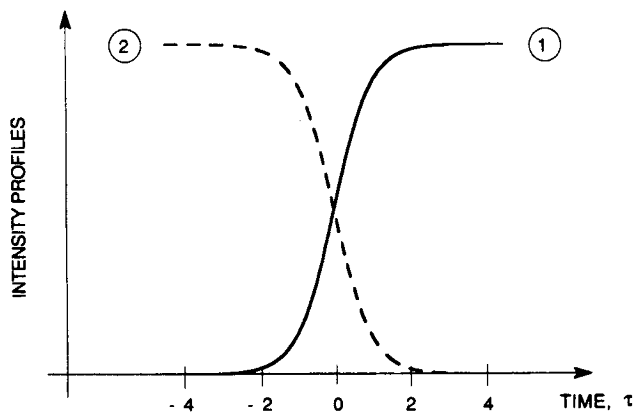

Fig. 2. Intensities of the shock-wave solutions that correspond to points 1 and 2 of Fig. 1, when $\epsilon = +1$.

previously obtained by Nozaki and Bekki[14,15] within the context of the generalized GLE. However, their solutions apply in the case in which the coefficient of nonlinearity is complex with a nonzero imaginary part. In the optical case, this last condition requires the strong presence of a nonlinear absorption process such as that originating from two-photon absorption.[16,17] Our analysis indicates that this sort of solution still persists even in the absence of such nonlinear absorption effects. Whether a spatiotemporal transition to chaos is also possible in connection with Eq. (1) remains an issue that requires further investigation.

As an example let us consider an erbium-doped fiber amplifier whose frequency-dependent gain exhibits a Lorentzian-like profile,[5] i.e., let $g(\omega) = g_m/[1 + (\omega - \omega_a)^2 T_2{}^2]$, where g_m is the gain peak, T_2 is the dipole relaxation time, and $\omega_a = 2\pi c/\lambda_a$, with λ_a the wavelength of maximum gain. For the erbium dopant, $\lambda_a = 1.536$ μm and $T_2 \approx 100$ fs. If we assume that $g_m = 3.45 \times 10^{-2}$ m^{-1}, which corresponds to 30 dB/100 m, and if the background loss α_B is taken to be $\alpha_B = g_m/2$, then the two wavelengths where the net gain of the system is zero are $\lambda_0 = 1.524$ μm and $\lambda_0 = 1.549$ μm. Let us consider solutions at the lower wavelength λ_0 and let us also assume that the fiber dispersion is normal, i.e., $\epsilon = -1$, so as to avoid any modulational instability at the cw tails. This can be accomplished by a dispersion-shifted fiber. If we take the arbitrary time scale $\tau_0 = 1$ ps and if the fiber dispersion at $\lambda_0 = 1.524$ μm is $D(\lambda) = -2$ ps/(km nm), then $d = -7.07 \times 10^{-2}$ and $m = -1.414$. Any small contributions to the fiber group-velocity dispersion arising from the erbium resonance have been neglected in this example.[18] For the values of these parameters, Eqs. (4)–(6) give $\alpha = 0.047$, $p = -0.236$, and $A^2 = 0.11$. The effective temporal pulse width of this wave, $\tau_{0e} = \tau_0/|p|$, can then be obtained, and it is $\tau_{0e} = 4.24$ ps. If we assume that the effective cross-sectional area of this fiber is $S_e = 60$ μm^2 and that $n_2 = 0.6 \times 10^{-22}$ (m/V)^2, then the maximum power of the cw tail of this shock wave is $P_{\max} = 0.52$ W.

In conclusion, we have shown that optical shock waves are possible in nonlinear amplifying dispersive media. These waves exist at lasing threshold wavelengths, and they are possible in the normal as well as in the anomalous dispersive regions.

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