Vector interactions of steady-state planar solitons in biased photorefractive media

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A theory describing the steady-state propagation of orthogonally polarized planar bright beams in biased photorefractive media is developed. Interactions between soliton states of each polarization in a strontium barium niobate photorefractive crystal are then investigated numerically. Our results indicate that such vector interactions can lead to a number of interesting effects such as beam compression and beam steering.

Recently it was demonstrated that spatial optical solitons can be supported in photorefractive (PR) media.[1-6] Because such self-trapped beams are possible at microwatt power levels,[7-9] they are currently a topic of considerable interest. These solitons occur when the process of diffraction is exactly balanced by the confining effect that arises from the light-induced index change in the photorefractive material.[1-6] Recent theoretical studies have shown that a particular kind of soliton, the so-called screening soliton, can exist under steady-state conditions, provided that the PR material is externally biased.[5,6] Thus far, bright, dark, and gray steady-state domains have been predicted.[5,6] Note that in all previous investigations it was implicitly assumed that the optical wave front of these self-trapped beams is singly polarized, i.e., parallel to any one of the crystal's principal axes.

In this Letter we study, for the first time to our knowledge, the polarization dynamics of bright planar beams in a biased PR material under steady-state conditions. The evolution equations obeyed by the vector components of such a beam in a PR medium are first derived. Assuming bright soliton solutions as the input intensity distribution in each polarization, the interaction between the orthogonal components is then investigated numerically. Pertinent examples are provided in which the PR crystal is taken to be of the strontium barium niobate (SBN) type.[10] We show that the interplay between the cross-polarized components can give rise to a number of interesting effects such as beam compression, beam confinement, and beam steering.

To study the interactions between orthogonally polarized steady-state planar solitons in PR media, let us first consider an optical beam that propagates along the z coordinate and is allowed to diffract only along the x direction. Thus, in essence, our diffraction theory is one dimensional. For demonstration purposes the PR crystal is assumed to be SBN with its optical c axis oriented in the x direction. The electric field polarization of the optical wave front ($\mathcal{E}$) is then expressed in terms of its transverse vector components, i.e., $\mathcal{E} = \mathcal{E}_x\hat{x} + \mathcal{E}_y\hat{y}$. The external bias electric field is also assumed to be parallel to the c axis. Under these conditions the perturbed refractive index along the x direction is given by $n_x^2 = n_e^2 - n_e^4 r_{33} E_x$ and in the y direction by $n_y^2 = n_o^2 - n_o^4 r_{13} E_x$, where n_e and n_o are the extraordinary and ordinary unperturbed indices of refraction, respectively, r_{33} and r_{13} are the electro-optic coefficients involved, and $\mathbf{E}_x = E_x\hat{x}$ is the total static field, a sum of the applied dc electric field and that induced from the space charge in this material.[11] For simplicity, the PR material is also assumed to be lossless. The optical field can then be expressed as usual in terms of slowly varying envelopes (U, V) such that $\mathcal{E} = (U\hat{x} + V\hat{y})\exp(ikz)$. k is an average propagation constant given by $k = k_0 n_{\text{av}} = (2\pi/\lambda_0)n_{\text{av}}$, where $n_{\text{av}} = (n_e + n_o)/2$ and λ_0 is the free-space wavelength of the incident beam. In that case, it can be readily shown that the (U, V) envelopes obey the following evolution equations:

$$iU_z + \frac{1}{2k}U_{xx} + \gamma_1 U - \frac{n_e^4 k_0^2 r_{33} E_x}{2k} U = 0, \tag{1}$$

$$iV_z + \frac{1}{2k}V_{xx} + \gamma_2 V - \frac{n_o^4 k_0^2 r_{13} E_x}{2k} V = 0, \tag{2}$$

where $U_z = \partial U/\partial z$, etc. and the constants $\gamma_{1,2} = k_0^2(3n_{e,o}^2 - n_{o,e}^2 - 2n_o n_e)/8k$. The polarization intensities $|U|^2$ and $|V|^2$ of the optical beam are assumed to be well confined within the x width W of the PR crystal. Moreover, these intensities are also assumed to be of the bright type, in which case they are expected to vanish at infinity, i.e., $|U|^2, |V|^2 \to 0$ as $x \to \pm\infty$. If the applied bias voltage is appreciable enough that one can ignore diffusion effects,[5,6,12] it is then straightforward to show that the space-charge field E_x in this crystal is given by

$$E_x(x, z) = \frac{E_0}{\dfrac{R_e}{R_d} + \dfrac{1}{W}\displaystyle\int_{-W/2}^{W/2} dx \frac{I_d}{I(x, z) + I_d}} \times \frac{I_d}{I(x, z) + I_d}, \tag{3}$$

where $I(x, z)$ is the total optical power density and I_d is the so-called dark irradiance.[12] In Eq. (3), $E_0 = \mathcal{V}_{\text{ext}}/W$, where $\mathcal{V}_{\text{ext}}$ is the potential of the constant voltage source, R_e is the external circuit resistance, and R_d is the dark resistance of the PR sample. Note that for typical photorefractive materials[11] $R_d \gg R_e$, and thus the (R_e/R_d) factor in Eq. (3) can be neglected. Moreover, for relatively well-confined optical beams the integral[5,6] $W^{-1} \int dx I_d (I + I_d)^{-1} \approx 1$. As a result, the space-charge field E_x is given approximately by[5,6]

$$E_x(x, z) = E_0 \frac{I_d}{I(x, z) + I_d} \cdot \quad (4)$$

Equation (4) shows that the static electric field E_x attains a constant value in the dark regions away from the optical beam, i.e., $E_x(x \to \pm\infty) = E_0$. It is also interesting to note that I_d can be artificially elevated, as demonstrated in Refs. 8 and 9. The optical power density $I(x, z)$ is related to the polarization components (U, V) through the expression $I(x, z) = (n_e/2\eta_0)|U|^2 + (n_o/2\eta_0)|V|^2$, where $\eta_0 = (\mu_0/\epsilon_0)^{1/2}$ is the vacuum intrinsic impedance. Furthermore, for convenience, let us also adopt the following transformations: $\xi = z/kx_0^2$, $s = x/x_0$, $U = (2\eta_0 I_d/n_e)^{1/2} \exp(i\gamma_1 z)X$, and $V = (2\eta_0 I_d/n_o)^{1/2} \exp(i\gamma_2 z)Y$, where x_0 is an arbitrary spatial width and the beam power density in each polarization has been scaled with respect to the dark irradiance I_d. From this, the normalized envelopes X and Y are found to satisfy

$$iX_\xi + \frac{X_{ss}}{2} - \frac{\beta_1 X}{1 + |X|^2 + |Y|^2} = 0, \quad (5)$$

$$iY_\xi + \frac{Y_{ss}}{2} - \frac{\beta_2 Y}{1 + |X|^2 + |Y|^2} = 0, \quad (6)$$

where the dimensionless quantities $\beta_1 = (k_0 x_0)^2 n_e^4 r_{33} E_0/2$ and $\beta_2 = (k_0 x_0)^2 n_o^4 r_{13} E_0/2$. Clearly, Eqs. (5) and (6) show that the vector nonlinearity is of the saturable type. Hence this system of equations may also be applicable to electrostrictive saturable Kerr media, especially in the case when $\beta_1 = \beta_2$. The above set of two coupled equations reduces to a single equation if the incident wave front exhibits only one polarization, which was previously found to permit soliton solutions.[5,6] To study vector interactions of these steady-state planar solitons in biased photorefractive media, one solves Eqs. (5) and (6) numerically, using a standard split-step Fourier method.[13] By use of the lowest-order conservation laws it can be directly shown that the two components do not in fact exchange any power, i.e., $\int_{-\infty}^{\infty} ds|X|^2$ and $\int_{-\infty}^{\infty} ds|Y|^2$ are constants of the motion. Nevertheless, they can greatly influence the evolution of each other through cross-phase-modulation effects.

As an example the PR material is taken to be SBN:60 with the following parameters[10] at a wavelength $\lambda_0 = 0.5\ \mu m$: $n_e = 2.33$, $n_o = 2.36$, $r_{33} = 237$ pm/V, and $r_{13} = 37$ pm/V. If we let the arbitrary spatial scale be $x_0 = 40\ \mu m$ and the external applied electric field be $E_0 = 40 \times 10^3$ V/m, we find that $\beta_1 = 35.3$ and $\beta_2 = 5.8$. The intensity distribution at $\xi = 0$ (at the input) in each polarization is assumed to be its bright-soliton solution. In a given physical system the shape and the width of these steady-state solitons are uniquely determined[5,6] by the magnitude of the externally applied electric field and the ratio of their peak intensity to that of the dark irradiance, i.e., the parameters r_1 and r_2, where $r_1 = |X(\xi = 0)|^2_{max}$ and $r_2 = |Y(\xi = 0)|^2_{max}$. Let us first consider a pump–probe configuration with the x-polarized envelope X much more intense than Y. The two beams are taken to be completely overlapping at the input, i.e., the separation between them is $\Delta s = 0$. Numerical solutions of Eqs. (5) and (6) show that the pump propagates without experiencing appreciable change in its form. On the other hand, the probe beam exhibits a complex evolution pattern. Figure 1 shows the evolution of the probe beam with $r_2 = 0.1$ when it copropagates with an $r_1 = 40$ pump beam at $E_0 = 40 \times 10^3$ V/m. As a result of pump-induced effects, the probe initially undergoes compression and subsequently expands on an average. The compression at $\xi = 0.6$ ($z \approx 3$ cm) is found to be 70%, with its intensity FWHM decreasing from 98 to 30 μm. The amount of compression is found to depend largely on the ratio of the probe and the pump beam widths. Initially the probe tends to concentrate its energy within the pump profile width. If the input width of the probe increases compared with that of the pump, a higher compression factor is achieved. If we let the polarization intensities be spatially separated rather than overlapping at the input, a similar amount of compression is obtained. However, the focusing in this case is asymmetric, and the contracted probe develops a side wing. Apart from compression, we find that the cross-polarized beams are initially attracted to each other.

To observe the attraction force between the X and Y beams in more detail we consider two equal-amplitude $r_1 = r_2 = 1$ solitons in each polarization component of the incident beam. The evolution of these two waveforms is investigated when they are initially separated by $\Delta s = 0.5$, which corresponds to 20 μm. Figure 2 depicts the interaction between the two polarizations. The y-polarized beam, which involves a smaller β ($\beta_2 = 5.8$), does not experience any appreciable change in its form. However, this is not the case for the x-polarized beam where $\beta_1 = 35.3$, which undergoes significant evolution in its width and peak intensity. Moreover, the center of the x-polarized beam initially gets steered toward the center

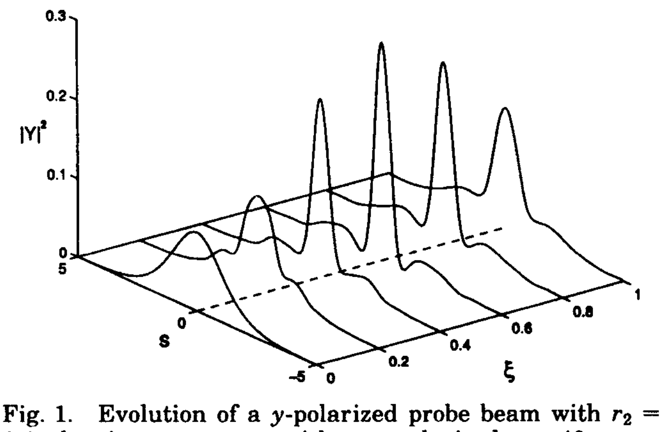

Fig. 1. Evolution of a y-polarized probe beam with $r_2 = 0.1$ when it copropagates with an x-polarized $r_1 = 40$ pump.

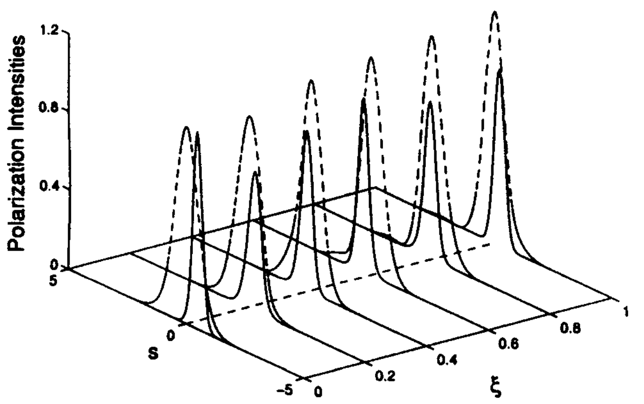

Fig. 2. Interaction between $r_1 = r_2 = 1$ x–y-polarized beams when their initial separation is $\Delta s = 0.5$. The normalized intensities $|X|^2$ and $|Y|^2$ are represented by solid and dashed curves, respectively.

of the other beam and thereafter oscillates around that point; i.e., beam trapping takes place. The X beam, which has a FWHM of 18 μm at the input, is found to shift by 7 μm at $\xi = 0.2$, which corresponds to an actual distance of $z = 1$ cm. Such interactions may be promising in all-optical switching applications.

In conclusion, we have described the evolution of an arbitrary polarized steady-state bright beam in a biased photorefractive medium. The interactions between its vector components were also considered in detail when the input intensity in each polarization was taken to be its bright-soliton solution. We show that such interactions can give rise to interesting effects such as beam compression, beam trapping, and beam steering. In closing, we note that the effect of other processes on vector dynamics may also merit further investigation. These may include loss as well as self-deflection effects.[3] Moreover, it remains to be seen whether the system of Eqs. (5) and (6) will permit vector soliton solutions similar to those previously encountered in Kerr media.[14]

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