Vector photorefractive spatial solitons

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Received June 12, 1995

We derive vector soliton solutions for photorefractive media in which the refractive-index perturbations must be treated as tensors. The two polarizations of the vector solitons may be coupled through the space-charge field alone and/or through cross terms in the electro-optic tensor, which requires phase matching and specific configurations. We analyze bright planar vector solitons and verify their stability. © 1995 Optical Society of America

Scalar spatial solitons in photorefractive materials have now been predicted and observed by numerous groups. Three generic types are known at present, which we refer to as quasi-steady-state, screening, and photovoltaic solitons. The quasi-steady-state solitons were predicted[1] and observed[2] first and exist in bright and dark realizations in one and two transverse dimensions. They exist during the screening process of an applied field during which the refractive-index perturbation is proportional to the transverse derivatives of the optical intensity, causing compression and collapse of an input Gaussian beam.[3] Screening solitons are found at steady state when an external voltage is applied to a photorefractive material. Iturbe-Castillo *et al.*[4] reported on steady-state self-focusing effects, and Segev *et al.*[5] presented explicit bright and dark solitary wave solutions in one transverse dimension. A recent observation confirmed the existence of screening solitons and demonstrated self-trapping in both transverse dimensions.[6] Photovoltaic solitons in steady state are predicted[7] and observed[8] in photorefractive materials with a strong photovoltaic current, such as $LiNbO_3$. In general, none of these solitons has properties similar to those of Kerr solitons. Perhaps the most important distinction from Kerr solitons is the existence of photorefractive solitons at microwatt and lower power levels and in two transverse dimensions.[6] This implies the practicality of using photorefractive solitons for beam steering, optical interconnects, and nonlinear optical devices.

Vector solitons have been widely discussed for Kerr media.[9,10] In this Letter we find vector solitons in photorefractive materials, which are crystalline media with nonlinear optical properties governed by tensorial relations. The electro-optic tensor effect gives rise to a number of unique cases of vector solitons that have no analogies in other nonlinear media. We find vector solitons whose two polarizations may be coupled through the space-charge field alone and/or through cross terms in the electro-optic tensor. The cross-coupled solitons require phase matching between the polarizations, which can be obtained in specific phase-matched geometries. For these cases we derive bright planar vector solitons with profiles related to those of scalar screening solitons.[5]

We start with the equation for the optical field $\mathbf{E}$:

$$\nabla^2 \mathbf{E} = \mu_0 \tilde{\varepsilon}_{\text{total}} (\partial^2 \mathbf{E}/\partial t^2), \tag{1}$$

where $\mathbf{E}(x, z, t) = A_x(x, z)\exp[i(k_x z - \omega t)]\hat{x} + A_y(x, z)\exp[i(k_y z - \omega t)]\hat{y}$ + c.c. and we assume $\partial/\partial y = 0$ and steady state. The permeability $\tilde{\varepsilon}_{\text{total}}$ may be expressed in a tensorial form as $\tilde{\varepsilon}_{\text{total}} = \tilde{\varepsilon} + \Delta\tilde{\varepsilon}$. The equations for the individual components of $\mathbf{E}$ depend on the displacement vectors of the normal modes of propagation.[11] Here we discuss the special cases that occur when the (x, y, z) axes coincide with the (1, 2, 3) principal axes of the crystalline media. A more general analysis of solitons propagating in an arbitrary direction is beyond the scope of this Letter. Here the only nonzero components in the matrix $\tilde{\varepsilon}$ are $\varepsilon_{xx} = \varepsilon_0 n_x^2$, $\varepsilon_{yy} = \varepsilon_0 n_y^2$, and $\varepsilon_{zz} = \varepsilon_0 n_z^2$, where n_x, n_y, and n_z are the refractive indices for light polarized along x, y, and z, respectively.

The nonlinear changes in the permeability are[11]

$$\Delta\tilde{\varepsilon} = -\{\tilde{\varepsilon} \cdot [(\tilde{r} \cdot \mathbf{E}_{\text{sc}}) \cdot \tilde{\varepsilon}]\}/\varepsilon_0, \tag{2}$$

where $\mathbf{E}_{\text{sc}}(x, z)$ is the photorefractive space-charge field and $\tilde{r}$ is the Pockels tensor. For a beam propagating primarily along the z axis and for solitons in one transverse dimension (x), we write $\mathbf{E}_{\text{sc}}(x) = E_{\text{sc}}(x)\hat{x}$ and the relevant components of $\Delta\tilde{\varepsilon}$ are $\Delta\varepsilon_{xx} = -\varepsilon_0 n_x^4 r_{xxx} E_{\text{sc}}$, $\Delta\varepsilon_{yy} = -\varepsilon_0 n_y^4 r_{yyx} E_{\text{sc}}$, and $\Delta\varepsilon_{xy} = \Delta\varepsilon_{yx} = -\varepsilon_0 n_x^2 n_y^2 r_{xyx} E_{\text{sc}}$. Defining $k_x = kn_x$, $k_y = kn_y$, and $k = 2\pi/\lambda$ (λ is the vacuum wavelength) and using the paraxial approximation about the z axis

0146-9592/95/171764-03$6.00/0

leads to coupled equations for $A_x(x, z)$ and $A_y(x, z)$:

$$2ik_x \frac{\partial A_x}{\partial z} + \frac{\partial^2 A_x}{\partial x^2} = -k^2\{\Delta\varepsilon_{xx}A_x + \Delta\varepsilon_{xy}A_y \times \exp[i(k_y - k_x)z]\}/\varepsilon_0 ,$$

$$2ik_y \frac{\partial A_y}{\partial z} + \frac{\partial^2 A_y}{\partial x^2} = -k^2\{\Delta\varepsilon_{xy}A_x \exp[-i(k_y - k_x)z] + \Delta\varepsilon_{yy}A_y\}/\varepsilon_0 . \quad (3)$$

Here we investigate a *single* nondiffracting beam with $A_x(x, z) = u(x)\exp(i\gamma_x z)$ and $A_y(x, z) = v(x)\exp(i\gamma_y z)$, where $v(x) = \alpha u(x)$ and α is a constant. Substitution into Eqs. (3) yields

$$-2k_x\gamma_x u + u'' = -k^2[\Delta\varepsilon_{xx}u + \Delta\varepsilon_{xy}\alpha u \times \exp(i\delta z)]/\varepsilon_0 , \quad (4a)$$

$$-2k_y\gamma_y u + u'' = -k^2[(\Delta\varepsilon_{xy}/\alpha)u \times \exp(-i\delta z) + \Delta\varepsilon_{yy}u]/\varepsilon_0 , \quad (4b)$$

where $\delta = \gamma_x - \gamma_y + k_y - k_x$ is the phase mismatch that combines the material birefringence and the difference between the propagation constants. Note that solitons in which the two polarizations have different transverse profiles may also be possible. Next we consider self-coupled vector solitons and cross-coupled vector solitons separately with the note that combinations of them may exist.

Class I (self-coupled vector solitons) includes vector solitons coupled only through the dependence of the space-charge field on the intensity. It occurs when $\Delta\varepsilon_{xy} = 0$, or for a finite propagation distance $L \gg \pi/\delta$ (where the contribution of the cross terms averages out). This case is somewhat similar to that of the Kerr-type vector soliton.[9,10] Equations (4) lead to the relation $2(k_x\gamma_x - k_y\gamma_y) = k^2(\Delta\varepsilon_{xx} - \Delta\varepsilon_{yy})$, which can be satisfied only if $k_x\gamma_x = k_y\gamma_y$ and $\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy}$. If these are satisfied, one equation is obtained for $u(x)$:

$$\gamma_x u - u''/2k_x = -k^2 n_x^4 r_{xxx} E_{sc} u/2k_x . \quad (5)$$

It is now convenient to transform the equation to dimensionless variables.[5] We write u in units of the equivalent dark irradiance (I_{dark}) and define $\xi = x/L_s$ and $E = E_{sc}(qL_D/k_BT)$, where $L_s = (\pm 2kb)^{-1/2}$, $b = (k/2)n_x^3 r_{xxx}(k_BT/qL_D)$, $L_D = (k_BT\varepsilon_s/q^2N_A)^{1/2}$ is the Debye length, q is the charge on the electron, k_B is Boltzmann's constant, T is the absolute temperature, N_A the number density of acceptors, and ε_s is the low-frequency dielectric constant (note that only the relative sign between E_{sc} and the crystalline axes is meaningful). The resulting equation for $u(\xi)$ is

$$\ddot{u} = \pm(\gamma_x/b + E)u , \quad (6)$$

where the upper (lower) sign indicates that $b > 0$ ($b < 0$). Following the main result of Ref. 5, we recall $E[I(\xi)]$ ($I = u^2 + v^2$ is the intensity of the beam):

$$E(\xi) = \frac{-\eta}{u^2(|\alpha|^2 + 1) + 1} \frac{qVL_D}{k_BTL_s} , \quad (7)$$

where V is the voltage across electrodes separated by a distance l and, assuming that $l \gg L_s$, we approximate[5] $\eta \approx L_s/l$. We integrate Eq. (6) and obtain

$$p^2 - p_0^2 = \pm\left\{(\gamma_x/b)(u^2 - u_0^2) - \frac{C\eta}{|\alpha|^2 + 1} \times \ln\left[\frac{u^2(|\alpha|^2 + 1) + 1}{u_0^2(|\alpha|^2 + 1) + 1}\right]\right\} , \quad (8)$$

where $p = du/d\xi$, $p_0 = p(\xi = 0)$, and $u_0 = u(\xi = 0)$. Fundamental bright solitons satisfy $p_0 = 0$, $u(\infty) = p(\infty) = 0$, exist for the lower sign only, and are subject to the boundary condition [obtained by substituting $\xi \rightarrow \infty$ into Eq. (8)]

$$(\gamma_x/b)u_0^2 = \frac{C\eta}{|\alpha|^2 + 1} \ln[u_0^2(|\alpha|^2 + 1) + 1] , \quad (9)$$

where $bC\eta \approx kn_x^3 r_{xxx}V/l$. Solving for v rather than for u gives a condition identical to Eq. (9), since $\gamma_x/(n_x^3 r_{xxx}) = \gamma_y/(n_y^3 r_{yyx})$ from the requirement outlined before Eq. (5). Dark solitons may be calculated similarly. The propagation constants γ_x and γ_y are not uniquely defined, and their values depend on the boundary condition [Eq. (9)]. Using that, one can integrate Eq. (6) numerically and obtain the waveform $u(\xi)$.[5] An important property of the self-coupled vector soliton is its arbitrary polarization (no restriction on α); it may be linear, circular, or elliptical.

Realization of a self-coupled photorefractive vector soliton can be made in various schemes. First, the electro-optic tensor must contain two nonzero components of the form r_{iii} and r_{jji} ($i \neq j$), which permits a polarization component parallel to E_{sc}. This excludes all cubic crystals and crystals of the $\bar{4}2m$, 422, 222, and 622 classes. For all other noncentrosymmetric crystals, one still needs to satisfy $\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy}$. This can be achieved by external means (temperature tuning) or by off-axis propagation. KTN in its ferroelectric phase ($4mm$ class) is one option, in which $x \parallel 3$, $y \parallel 1$, and $z \parallel 2$ and consequently $n_x = n_e$, $n_y = n_o$, $r_{xxx} = r_{33}$, and $r_{yyx} = r_{13}$, and we require that $r_{33}/r_{13} = (n_o/n_e)$ be satisfied by use of temperature tuning above the (room-temperature) phase transition. Another option is $LiNbO_3$ ($3m$ class; $m \perp x_2$) and propagation at an angle θ with respect to the 3 (c) axis in the 1–3 plane. In this configuration $\hat{x} = (\cos\theta, 0, -\sin\theta)$, $\hat{y} = (0, 1, 0)$, and $\hat{z} = (\sin\theta, 0, \cos\theta)$. For typical parameters[11] we find that $\theta \approx 11.92°$.

Class II (cross-coupled vector solitons) includes vector solitons coupled through the electro-optic tensor and the space-charge field. The simplest case occurs when $\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy} = 0$, i.e., no self-coupling. This case does not have a Kerr-type equivalent. In Eqs. (4) the coupling term contains the phase-mismatch factor $\exp(i\delta z)$. Therefore, stationary solutions require that $\delta = 0$ and that Eqs. (4) reduce to one equation. This implies that $\alpha^2 = 1$ and $k_x\gamma_x = k_y\gamma_y$. Requiring $\alpha^2 = 1$ means that cross-coupled vector solitons must be linearly polarized at a 45° angle to the x and y axes and that this polarization be maintained during

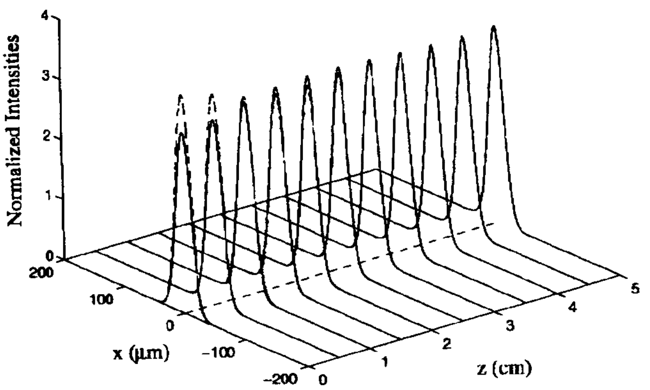

Fig. 1. Evolution of a cross-coupled vector soliton when its x-polarized input component (dashed curve) is perturbed by 10% in its amplitude from the soliton solution (identical to the unperturbed input y component; solid curve).

propagation. The second condition can be satisfied if $k_x = k_y$ and $\gamma_x = \gamma_y$, which implies that $n_x = n_y$ (there is another alternative: $k_x = \gamma_y$ and $k_y = \gamma_x$, which is not physical since γ_x and γ_y must be small compared with k_x and k_y; otherwise the paraxial approximation is violated). Then Eqs. (4) reduce to

$$\gamma_x u - u''/2k_x = -(\pm k/2) n_x {n_y}^2 r_{xyx} E_{sc} u\,, \qquad (10)$$

where the upper (lower) sign represents $\alpha = 1$ ($\alpha = -1$). Transforming Eq. (10) to dimensionless variables, with a different parameter $b = (k/2) n_x {n_y}^2 r_{xyx} (k_B T/qL_D)$, leads to an equation identical to Eq. (6), where $E(\xi)$ has the same form as Eq. (7) with $\alpha^2 = 1$.

The requirements for the existence of cross-coupled vector solitons are as follows. First, the electro-optic tensor must have a nonzero component of the form $r_{iji} = r_{jii}$ ($i \neq j$), which enables a polarization component (i) parallel to E_{sc} to be coupled directly to the perpendicular polarization component (j). Consequently the crystal must possess r_{42}, r_{43}, r_{51}, r_{53}, r_{61}, or r_{62} nonzero components. This excludes all cubic crystals and crystals of the $\bar{4}2m$, 422, 222, and 622 classes. For all other noncentrosymmetric crystals one still needs to satisfy the phase-matching condition. This can be done by choice of a crystal in which $n_x = n_y$, which implies that $\gamma_x = \gamma_y$. This choice can be realized, for example, in uniaxial crystals, where x and y are parallel to the 1 and 2 axes, which results in $n_x = n_y = n_o$, and r_{xyx} is either r_{61} or r_{62}. This occurs for the 3, 32, $3m$, $\bar{6}$, and $\bar{6}m2$ classes. For example, in $LiNbO_3$ ($3m$ class; $m \perp x_2$) the beam would propagate along the c axis, form a soliton in the x_2 direction, and be polarized at 45° between x_1 and x_2. Propagation directions that go out of the crystalline principal planes may permit other options.

Stability of photorefractive vector solitons is an issue of interest. The basic form of the screening nonlinearity is $\Delta n \sim (1 + I)^{-1} = 1 - I(1 + I)$. The first term modifies the propagation constant. The second term is identical to that considered in Refs. 12 and 13 and shows that for this nonlinearity scalar solitons are stable. For vector solitons specifically, we analyze numerically both classes and find them both stable against perturbations as high as 20% in the amplitude or the width of either one of the polarizations in the vector pair. If the input polarization components are identical to the soliton solution of Eq. (5) or (10), the vector pair remains unchanged and stable for very large distances (meters). For example, a 28.5-μm-wide (FWHM) cross-coupled vector soliton in $LiNbO_3$ in the configuration described above, at $\lambda = 0.633$ μm and a total soliton intensity at $x = 0$ equal to $6I_{dark}$, requires $V/l \approx 10$ kV/cm. Figure 1 shows the evolution of this vector soliton when its x-polarized input component (dashed curve) is perturbed by 10% in its amplitude from the soliton solution (identical to the unperturbed input y component; solid curve). The vector soliton does not break up, and both polarizations coincide after a propagation distance of 2 cm. Similar plots are obtained also for class I vector solitons for practically the whole range of physical parameters, very large propagation distances (meters), and perturbations as high as 20%. We conclude that photorefractive vector solitons are stable.

In conclusion, we have found spatial photorefractive vector solitons of two classes: self-coupled and cross-coupled vector solitons.

M. Segev gratefully acknowledges the generous support of a Sloan Fellowship and of Hughes Research Laboratories.

References

1. M. Segev, B. Crosignani, A. Yariv, and B. Fischer, Phys. Rev. Lett. **68,** 923 (1992); B. Crosignani, M. Segev, D. Engin, P. DiPorto, A. Yariv, and G. Salamo, J. Opt. Soc. Am. B **10,** 449 (1993).
2. G. Duree, J. L. Shultz, G. Salamo, M. Segev, A. Yariv, B. Crosignani, P. DiPorto, E. Sharp, and R. R. Neurgaonkar, Phys. Rev. Lett. **71,** 533 (1993); **74,** 1978 (1995); Opt. Photon. News **4**(12), 8 (1993); **5**(12), 9 (1994); Opt. Lett. **19,** 1195, 1296 (1994).
3. D. N. Christodoulides and M. I. Carvalho, Opt. Lett. **19,** 1714 (1994).
4. M. D. Iturbe-Castillo, P. A. Marquez-Aguilar, J. J. Sanchez-Mondragon, S. Stepanov, and V. Vysloukh, Appl. Phys. Lett. **64,** 408 (1994).
5. M. Segev, G. C. Valley, B. Crosignani, P. DiPorto, and A. Yariv, Phys. Rev. Lett. **73,** 3211 (1994).
6. M. Shih, M. Segev, G. C. Valley, G. Salamo, B. Crosignani, and P. DiPorto, Electron. Lett. **31,** 826 (1995).
7. G. C. Valley, M. Segev, B. Crosignani, A. Yariv, M. M. Fejer, and M. Bashaw, Phys. Rev. A **50,** R4457 (1994).
8. M. Taya, M. Bashaw, M. M. Fejer, M. Segev, and G. C. Valley, "Observation of dark photovoltaic spatial solitons," Phys. Rev. A (to be published).
9. D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13,** 53 (1988); N. Akhmediev and A. Ankiewicz, Phys. Rev. Lett. **70,** 2395 (1993).
10. A. W. Snyder, S. J. Hewlett, and D. J. Mitchell, Phys. Rev. Lett. **72,** 1012 (1994).
11. P. Yeh, *Introduction to Photorefractive Nonlinear Optics* (Wiley, New York, 1993).
12. S. Gatz and J. Herrmann, J. Opt. Soc. Am. B **8,** 2296 (1991).
13. S. R. Singh and D. N. Christodoulides, Opt. Commun. **118,** 569 (1995).