We view a switched behavior as a set of trajectories of the form $w(t) = \{ w(t) \mid t \in \mathbb{R} \}$. The trajectories of a behavior can be represented by a set of solutions of a finite system of constant-coefficient differential equations, i.e., if there exists a polynomial matrix $A$ such that

$$\dot{w}(t) = A w(t),$$

then we call the behavior autonomous.

The autonomy of a behavior can be characterized by a behavioral Lyapunov function also ensuring the stability of the switched behavioral system. In this contribution, we propose a notion of switched behavioral system essentially characterized by a bank of behaviors described by higher order differential equations and switching signals.

### Definition 2

A switched behavioral system is a tuple $\Sigma = (B, F, S, J)$ such that

- $B = \{ B_1, B_2, \ldots, B_N \}$ is a bank of behaviors, with $B_i = (F_i, S_i)$, $i = 1, 2, \ldots, N$.
- $F_i$ is the set of smooth functions representing the behavior $B_i$.
- $S_i$ is the set of switching signals.
- $J = \{ J_1, J_2, \ldots, J_N \}$ is a sequence of indices.

Where $J_i$ are the switching signals, $F_i$ represents the behavior, and $S_i$ are the switching signals.

We denote with $w_i(t) = \{ w_i(t) \mid t \in \mathbb{R} \}$ the set of solutions of a finite system of constant-coefficient differential equations, i.e., if there exists a polynomial matrix $A_i$ such that

$$\dot{w}_i(t) = A_i w_i(t),$$

then we call the behavior $B_i$ autonomous.

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### Theorem 1

For $i \in [1, N]$ the following statements hold:

1. There exists a behavioral Lyapunov function $V_i$ for $B_i$.
2. $V_i$ is piecewise constant and right-continuous.
3. The trajectories of $B_i$, $w_i(t)$, are autonomous.

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then we call the behavior $B_i$ autonomous.

The autonomy of a behavior can be characterized by a behavioral Lyapunov function also ensuring the stability of the switched behavioral system. In this contribution, we propose a notion of switched behavior essentially characterized by a bank of behaviors described by higher order differential equations and switching signals.
A system of switched behavioral systems is free, otherwise that none of the components of the system 2) and equation and has therefore minimal state dimension equal to that satisfy the following two conditions:

- \[ 0 > (\varepsilon) (n) \cap \mathcal{E} \]
- \[ \varepsilon \not\in \mathcal{E} \] for all \( \varepsilon \) for which

\[ \{ ((1, 1), (1, 1)) \cap \{ 1, 1 \} \cap \mathcal{E} \} = \emptyset \]

Example: For all \( \varepsilon \not\in \mathcal{E} \) there exists \( (1, 1) \) such that

\[ ((1, 1), (1, 1)) \not\in \mathcal{E} \] for all \( \mathcal{E} \) and \( (1, 1) \) for which

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common references. Instead, for the references in \( \varepsilon^{\alpha} = (m)A^{\alpha} \)
which is not positive, since for instance for \( \varepsilon^{m} = (m)A^{m} \).

Then

\[
\begin{bmatrix}
\tilde{\varepsilon} \\
\varepsilon
\end{bmatrix}
= \begin{bmatrix}
1 & \varepsilon \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\eta & 0 \\
\varepsilon & \tilde{\eta}
\end{bmatrix}
= (\varepsilon^{\alpha})A
\]

and

\( \eta > 0 \) and \( \eta \) is positive definite, so that only if \( \varepsilon^{\alpha} \). 

A behavior does not imply the positivity of \( \varepsilon^{\alpha} \) in general, the positivity of \( \varepsilon^{\alpha} \).

Example 6. Let \( \varepsilon = \varepsilon^{0} \). We consider a behavior \( \varepsilon^{0} \) for which the values of \( \varepsilon^{\alpha} \) are nonnegative.

A quadratic differential form is defined as the quadratic form \( \varepsilon^{\alpha} \) and it is positive definite if and only if \( \varepsilon^{\alpha} \) is positive definite, i.e., only if \( \varepsilon^{\alpha} \) is positive definite.

Clearly, \( \varepsilon^{\alpha} \) is positive definite if and only if \( \varepsilon^{\alpha} \) is positive definite.

where \( \varepsilon^{\alpha} \) does not imply the positivity of \( \varepsilon^{\alpha} \). More generally, \( \varepsilon^{\alpha} \) is positive definite if and only if \( \varepsilon^{\alpha} \) is positive definite.

Our aim is to obtain sufficient conditions for the stability of a switched structure in terms of Lyapunov functions. We shall focus on scalar behaviors, i.e., we shall consider that the number of variables is 1.

We define the order of a quadratic differential form \( \varepsilon^{\alpha} \) as

\[
\langle \varepsilon^{\alpha} \rangle = (m) A^{\alpha}
\]

where \( mA^{\alpha} \) is a symmetric real 2-variable (or 2D) polynomial matrix. We say that \( \varepsilon^{\alpha} \) is nonnegative definite if

\[
\langle \varepsilon^{\alpha} \rangle = \begin{bmatrix}
\varepsilon^{(\alpha)} & \varepsilon^{(\alpha)} \\
\varepsilon^{(\alpha)} & \varepsilon^{(\alpha)}
\end{bmatrix}
\]

and therefore \( \varepsilon^{(\alpha)} \) is nonnegative definite.

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and therefore \( \varepsilon^{(\alpha)} \) is nonnegative definite.
where $\Phi$ is the Hurwitz polynomials of $\frac{(w_p')}{p}$ and $\Phi$ is the Hurwitz polynomials of $\frac{(w_p')}{p}$. Thus, we have that $m = \frac{w_p'}{p}$ and hence
Applying Theorem 9 we can conclude that the switching yield the common Lyapunov function where

\[
\mathbf{Q} = \frac{p}{2} \quad \mathbf{P} = \frac{1}{2} \\
\mathbf{F} = \frac{1}{2} \\
\mathbf{G} = \frac{1}{2} \\
\mathbf{S} = \frac{1}{2} \\
\mathbf{Z} = \frac{1}{2} \\
\mathbf{B} = \frac{1}{2} \\
\mathbf{A} = \frac{1}{2} \\
\mathbf{C} = \frac{1}{2} \\
\mathbf{D} = \frac{1}{2} \\
\mathbf{E} = \frac{1}{2} \\
\mathbf{F} = \frac{1}{2} \\
\mathbf{G} = \frac{1}{2} \\
\mathbf{H} = \frac{1}{2} \\
\mathbf{I} = \frac{1}{2} \\
\mathbf{J} = \frac{1}{2}
\]

Remark 11: We illustrate these results in the following example.

Theorem 10 provides a criterion for the existence of a common Lyapunov function for scalar behaviors. Since this Lyapunov function satisfies the conditions of Theorem 9 the following corollary follows readily.