



# Seeking proof

Anastássios Perdicoulis

Assistant Professor, ECT, UTAD (<http://www.tasso.utad.pt>)  
Senior Researcher, CITTA, FEUP (<http://www.fe.up.pt/~tasso>)

## Abstract

Proof helps create certainty, and this carries great responsibility. Of the many methods of proof, the challenge is to use the appropriate one in each case.

## 1 Introduction

Proof<sup>1</sup> often requires evidence<sup>2</sup>, demonstration<sup>3</sup>, or an argument<sup>4</sup> to establish a fact or the truth of a statement. Proof may be sought *ex-ante*, or prior to the substantiation of that statement, as in planning, or *ex-post* as in the courts of law. The value of proof is not knowledge itself, but *certainty* — a property of knowledge that makes it trustworthy. In other words, proof helps resolve doubt, uncertainty, or ambiguity, so that one may proceed — e.g. begin a course of action.

Across the various contexts of academic, business, or social life, proof comes in many ways (methods) and degrees of certainty on the issuing side, and another so many on the receiving side to accept that proof. The failure of proof could be as benign as letting doubts persist, or as severe as creating erroneous statements, with potentially grave consequences — e.g. financial, physical, or ethical. Hence, proof holds grand responsibility.

## 2 Hypothesis

The statement to be proven is generally known as a *hypothesis*,<sup>5</sup> which is practically a tentative truth. In *ex-ante* situations, hypothesis often takes the form of a proposition such as a suggested scheme or plan of action in a business context. Propositions in mathematics or physics usually carry an explicit request for proof. And other times the hypothesis is really an assumption, such as when two parties make an agreement: honouring the agreement has a very subtle and delicate ‘live’ proof.

<sup>1</sup>From *probare* [L], to test, as in the theatrical or musical final rehearsals: ‘prova generale’.

<sup>2</sup>From *evident-* [L], obvious to the eye or mind; ‘obvious’ derives from *ob viam* [L], in the way.

<sup>3</sup>From *demonstrare* [L], to point out, explain, describe, or show.

<sup>4</sup>From *arguere* [L], to claim, assert, and also accuse, blame — set of reasons given in support of an idea.

<sup>5</sup>Ἵ�πόθεσις [Gk], sub-position or supposition; plural: hypotheses, or ὑποθέσεις [Gk].



absurd. *Reductio ad absurdum* is still considered as a valid form of argumentation in various contexts such as mathematics, politics, and law.

## 4 Mathematical proof

While the scientific method is paramount for the subject matter of the physical sciences, the abstract nature of mathematics often requires methods more suitable to their ‘insubstantial’ subject matter — from algebra and calculus, to geometry and logic — such as the ‘mathematical induction’ (Example 4.1).

### EXAMPLE 4.1 MATHEMATICAL INDUCTION

*Let:*  $\mathbb{N} = 1, 2, 3, 4, \dots$  and  $n \in \mathbb{N}$

*Statement (S):* ‘All integers of the form  $P_{(n)} = 2n + 1$  are odd’

*Base case ( $n = 1$ ):*  $P_{(1)} = 2(1) + 1 = 3$  (odd)  $\therefore S = \text{true}$

*Any integer ( $n = k$ ):*  $P_{(k)} = 2(k) + 1$  (odd)  $\therefore S = \text{true}$

*Next integer ( $n = k + 1$ ):*  $P_{(k+1)} = 2(k + 1) + 1$  (odd)  $\therefore S = \text{true}$

*Conclusion:*  $S = \text{true}$  for all  $n \in \mathbb{N}$

A proof such as that of Example 4.1 may be credible in the uniform and controlled conditions of the abstract reality of mathematics, but it would be quite amusing to try to replicate it with kiwis: let  $\mathbb{N} = \text{kiwis}$  (from Australia or Tasmania), and prove that ‘All kiwis [from this place] are big’ (e.g.  $W_k > 50g$ ). This, of course, could be made feasible with sampling, but even identified and individually numbered kiwis have not the regularity of natural numbers — e.g. ‘any kiwi’, ‘next kiwi’ — so the generalisation of Example 4.1 would not apply convincingly.

## 5 Official statements

A certain kind of formal but ‘non-scientific’ argumentation is commonly used in contexts of public affairs (e.g. politics) and business (e.g. commerce), with official statements issued by people of authority — Example 5.1. The responsibility for the truth lies with the issuer, who had better researched well before making statements. In the same example, the ‘proof’ — or support to the statement — comes from (a) ‘statistics’, and (b) the plausible ‘explanation’ about the favourable culture conditions.

### EXAMPLE 5.1 OFFICIAL STATEMENT

‘Kiwis in Australia grow bigger than in Tasmania, as statistics report (and I have verified myself in many occasions)’ — stated Mr. Wallaby, the Chairman of the Chamber of Commerce of Australia. ‘Our soil and climate are more favourable for their culture’, he argued.

Official statements may convince by the credibility or authority of the issuer, but the actual statement is usually never examined for its veracity. A common response to the arguments of official statements is a counter-argument, issued by another authority with competing interests, as it often happens in parliamentary debates.

Curiously, academia lends itself as an interesting example in which the truth of a key statement — e.g. ‘this is a good article’ — is rarely judged for its veracity. The peculiarity of the case arises from the fact that academics are not meant to be convinced easily, and that they have the function to be scientifically examining any hypothesis before agreeing or not. It is odd, therefore, that scientific publications are largely judged for their value *indirectly*, often by the credibility or ‘reputation’ of the publishing houses, or — even more accentuated — by ‘official reports’ (or statistics) regarding their popularity (e.g. number of citations or ‘impact factor’), and rarely through direct appreciation — i.e. through reading (Perdicoulis, 2013a).

## 6 Looser alternatives

In certain environments such as the arts, business, medicine, and the military, where people often must venture into unknown circumstances, act quickly, or deal with the qualitative nature of their subject matter, proof faces special limitations — e.g. there is not enough time for full testing, the required *ex-ante* proof is not as good as the *ex-post*, or proof cannot be sharply defined. In such circumstances, some looser alternatives to proof are being used individually or to form arguments (Example 6.1), similar to those also used in the official statements — except they appear to lose their gravitas when issued by common mortals.

### EXAMPLE 6.1 LOOSER ALTERNATIVES

*Evidence:* Customers say that Australian kiwis are bigger than the ones from Tasmania

*Principle/ Axiom/ Assumption/ Belief/ Tenet:* Customers know best

Evidence is generally acceptable in the courts of law, in the arts, as well as in marketing, and is powerful enough to convince people through demonstration to their senses — mostly seeing and hearing. Of course, this works for the truth as much as for folly, and there is no definite way to tell.

The ‘softer’ alternatives such as convictions, beliefs, assumptions (whether stated or hidden) provide conditioning that leads to accepting or rejecting a proposition, so there is not much work there to be done by ‘proof’. Nonetheless, certainty can indeed be obtained through any of the softer alternatives, and can power either commendable or reprehensible acts such as heroic actions or bigotry, respectively.

## 7 Discussion

Making the ‘wrong’ or inappropriate choice for the method of proof in any given context can be detrimental. For instance, opting for an informal way of ‘proof’ such as evidence in an environment with established formal protocols of proof (e.g. a science PhD thesis) could lead to a weak defence — in fact, the candidate would be practically defenceless.

On the other hand, applying the scientific method to choose ice cream — which would actually elevate the choice to a proof which ice cream is the best among a given batch, according to commonly agreed criteria, or which ice cream meets the ISO 22000 food safety standards — would look plainly ridiculous.

Looser alternatives of proof are not as thorough as their formal counterparts, so they are prone to errors, but they do have their use in looser contexts — e.g. social. In any case, experience (Figure 1) helps to formulate good hypotheses, and also to test them fast and reliably.

## 8 Challenges

Methodologically, proof requires an appropriate method in each situation or context. But the challenge is unique, full of responsibility, and quite difficult: just prove it!

## References

- Chalmers, A.F. (1999) *What is This Thing Called Science?* (3rd ed). Buckingham: Open University Press.
- Fowler, J., L. Cohen, and P. Jarvis (1998) *Practical Statistics for Field Biology*. Chichester: John Wiley & Sons.
- Hammond, R., and P.S. McCullagh (1978) *Quantitative Techniques in Geography*. Oxford: Clarendon Press.
- Kuhn, T.S. (1996) *The Structure of Scientific Revolutions* (3rd ed). Chicago: The University of Chicago.
- Perdicoulis, A. (2013c) Research questions. *oestros*, **13**.
- Perdicoulis, A. (2013b) The scientific qualifier. *oestros*, **11**.
- Perdicoulis, A. (2013a) Educated readership. *oestros*, **7**.
- Popper, K.R. (2002) *The Logic of Scientific Discovery* (2nd ed.). London: Routledge. (English version originally published in 1959 by Hutchinson, New York.)
- Watts, S., and L. Halliwell [Eds.] (1996) *Essential Environmental Science*. London: Routledge.

