Calibration of bi-planar radiography with minimal phantoms

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Introduction

- Computer Tomography (CT) is the gold standard for 3D reconstructions and accurate measurements of bone structures.
- However, not adequate for large bone structures (high radiation)
- Alternative: using Plain Radiography (2D)
  - Requires 2+ radiographs from different views.
  - Accomplished for: spine, pelvis, distal and proximal femur.
Introduction: Calibration

- For mapping $2D \rightarrow 3D$ coordinates the imaging system must be **calibrated** on every examination.
- Calibration is usually performed using large calibration apparatus.
- Attempts have been made for using smaller calibration objects, but:
  - reconstruction errors are higher
  - a considerable number of undesirable objects is still visible in radiographs
Goal

Minimising the impact of calibration objects
Introduction: Goal

Goal
Minimising the impact of calibration objects

How?
Using a distance measuring device to estimate some of the calibration parameters
Radiography calibration

Projection of a 3D point into a radiograph:

\[
\begin{bmatrix}
  w \cdot u \\
  w \cdot v \\
  w
\end{bmatrix}
= M \cdot
\begin{bmatrix}
  X \\
  Y \\
  Z \\
  1
\end{bmatrix}
\]
Radiography calibration

Projection of a 3D point: bi-planar radiography

\[
\begin{bmatrix}
w_i \cdot u_i \\
w_i \cdot v_i \\
w_i
\end{bmatrix} = M_i \cdot \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \text{for } i = 1, 2
\]
Calibration Matrix (for flat detectors)

\[ M_i = \begin{bmatrix} f_i/s & 0 & u_{p_i} & 0 \\ 0 & f_i/s & v_{p_i} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} R_i \\ 0^T \\ 1 \end{bmatrix} \]

- \( f_i \) - focal length
- \( (u_{p_i}, v_{p_i}) \) - principal point
- \( R_i \) - geometrical transformation
Radiography calibration

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  0 & 0 & 1 & 0 \\
\end{bmatrix} \cdot \begin{bmatrix}
  \mathcal{R}_i \\
  t_i \\
  0^T \\
  1
\end{bmatrix} \]

- \( f \) - focal length
- \( (u_p, v_p) \) - principal point
- \( \mathcal{R}, t \) - geometrical transformation
Radiography calibration

Calibration Matrix (for flat detectors)

\[ M_i = \begin{bmatrix} f_i/s & 0 & u_{p_i} & 0 \\ 0 & f_i/s & v_{p_i} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} R_i & t_i \\ 0^T & 1 \end{bmatrix} \]

- \( f \) - focal length
- \((u_p, v_p)\) - principal point
- \( R, t \) - geometrical transformation

Calibration Goal
Finding the values for these parameters for both views.
Determining the parameters values

Inputs:
- Initial solution for the parameters
- A set of point matches
Determining the parameters values

1 Inputs:
   - Initial solution for the parameters
   - A set of point matches

2 Optimisation process (NLSQ):
   1. Triangulate point matches $\rightarrow$ 3D points
   2. Project 3D points $\rightarrow$ projected 2D points
   3. Minimise residuals between the original and the projected points

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Determining the parameters values

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```
minimise difference
```

```
2D point on image 1
2D point on image 2
3D point
Projected 2D point
Projected 2D point
```
Determining the parameters values

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Problem

Optimisation gets easily trapped in local minima.
For narrowing the search space of solutions we propose using a **distance measuring device**.

**X-Ray imaging system**
Narrowing the search space of solutions

- For narrowing the search space of solutions we propose using a distance measuring device.

X-Ray imaging system representation

X-ray source

X-ray device

Table

Detector
For narrowing the search space of solutions we propose using a distance measuring device.

- **f** – Focal distance
  - Distance between the x-ray source and the detector
  - Can’t be measured directly
Narrowing the search space of solutions

- For narrowing the search space of solutions we propose using a **distance measuring device**.

![Diagram of X-ray device setup]

- $t_z$ – Z translation
  - Distance between the x-ray source and the object
  - Can’t be measured directly

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Narrowing the search space of solutions

For narrowing the search space of solutions we propose using a distance measuring device.

A distance measurer may only read $d_m$.
Narrowing the search space of solutions

For narrowing the search space of solutions we propose using a distance measuring device.

We proposed a procedure elsewhere to determine $d_s$ and $d_d$, which are constant for a given system.
Narrowing the search space of solutions

For narrowing the search space of solutions we propose using a **distance measuring device**.

Knowing $d_s$ and $d_d$ enables to determine $f$ accurately with a distance measurer...
Narrowing the search space of solutions

For narrowing the search space of solutions we propose using a distance measuring device.

... and to have an initial guess of $t_z$. 
Correcting scale

- Even with this extension, this method is only able to calculate up to scale solutions.
- A reference distance (visible in both x-rays) is needed to determine the scaling factor.
- We propose using a small calibration object composed by 2 radiopaque parts.

\[
\text{scaling factor} = \frac{\text{known reference distance}}{\text{reconstructed distance}}
\]
Experiments

- The method was tested with a phantom object
  - Stainless steel grid (380x380x1mm)
  - Laser cut squares of 20.0 ± 0.1mm

- 8 radiographs at the same \( f \)
  - \( d_m = 909 \text{mm} \) measured with a laser distance measuring device (typical error ±1.5mm)

- Combined in a total of 17 pairs (out of 28 possible combinations)
Experiments: Inputs

Example points on image 1  Example points on image 2
Experiments: Inputs

(0, 0, 1183)  
(0°, 0°, 0°)

(0, 0, 993)  
(60°, 0°, 0°)
Experiments: Inputs

Example of a reference distance on image 1

Example of a reference distance on image 2
First Experiment

- **Conditions:**
  - Initial parameters: roughly estimated
  - Point matches: no noise
  - Reference distances: 1 (tested 50 different distances of 40mm)

- **Evaluation:**
  1. Phantom grid reconstructed with the optimised parameters;
  2. Scaled with the reference distance
  3. Automatically aligned with ground truth model
  4. Error (per point): euclidian distance between the reconstructed 3D coordinates and the known coordinates of the phantom.
First Experiment: Results

3D absolute error (mm)

- Error count
- Mean = 0.31mm
- RMS = 0.36mm
- Mean + SD = 0.49mm
- Mean + 2SD = 0.66mm

Number of observations

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Second Experiment

- Goal: testing the sensibility of the algorithm to noise on point matches
- Uniformly distributed noise, up to ±15 pixels.
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- Goal: testing the sensibility of the algorithm to noise on point matches
- Uniformly distributed noise, up to ±15 pixels.
Conclusions

- The proposed calibration method achieved sub-millimetric accuracy, even when:
  - initial guess is rough
  - noise is added to point matches (up to $\pm 5$ pixels)

- It requires:
  - a distance measurer,
  - a small calibration object for giving only one reference distance (if absolutes measurements are needed).

- These requirements are low when compared with other methods...
  - ... however, the method should be accessed with radiographs of anatomical parts.
Future work

Experiments with spine radiography

- Initial guess is easy to estimate
- Semi-automatic detection of point matches in vertebrae.
Thank You!

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