Neural Encoding Models in Natural Vision

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Abstract

From the existing medical imagiology tools, it has been progressively more possible to correctly show the association between cognitive sensorial processes and zones in the brain. Recently, the acquisition of Functional Magnetic Resonance Imaging (fMRI) has enabled the partial reconstruction of the images that were being exhibited to the subject in experiments where visual stimuli are presented while the neural activity is recorded - which is to say, technologies are emerging that will allow to extract the visual signals from the brain.

The effort to transform fMRI signals into the original stimulus relies heavily on the construction of decoding models of neural and neuronal brain activity. This work presents a study on the functional characteristics of neurons, in particular from the primary visual motor cortex, and the subsequent implementation of models which attempt to predict the response of neurons of the area to natural visual stimuli. The data used comes from the Neural Prediction Challenge and is presented in a 'contest' environment where users are encouraged to test their results in a blind response test. This data was provided by Gallant Labs from University of California Berkley.

The implementation of the state of the art algorithms and models was performed, and some innovations, notably the use of KNN Classifiers for "Fire or not" classification, and NARX neural networks to model non-ideal characteristics of neurons were attempted. A massive neural network based approach was also attempted but failed to reach fruition because of technical considerations.

The selected improvements can marginally increase performance from the current state of the art by about 2%, which is a relative success considering that in the past 8 years the field only saw a 10% improvement in prediction accuracy. A speedup of $5\times$ can also be reported by implementing some of the critical computations on the GPU as well as reduction of $10\times$ in a critical part of the state of the art algorithm - considering the overhead and other functions this translates into a practical up to 1200% speed improvement, with the approximate same results (equal when converted to single precising floating point).
Acknowledgements

Firstly a word of praise to the researchers at Gallant Labs which so kindly provide access to the public of rather rare and so high quality material. I would like to thank specially my Supervisor, Professor João Tavares, for the disponibility and guidance, as well as his high quality scientific library.

A word of aknowledgment to the great work of Professor Gallant, and Dr. Neselaris and all those which make science fiction possible through their innovative resarch into the vision pro- cesses into the brain.

To Professor Aurélio Campilho, for teaching me the arts of classifiers and machine learning, on which this piece of work mostly settles.

A special thanks to my family for their support during the late hours of work and the early frustrations in particular to my mother, and to my friends, which were always there to help me take my mind off work whenever it became too much. A special thanks to Michael Oliver for providing his code and methods for the improvement of my own work.

Nuno Pedro Pinto Albuquerque Sousa
“One man’s ‘magic’ is another man’s engineering.  
’Supernatural’ is a null word.”

Robert A. Heinlein
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1.2 Reconstruction of the clip from the brain of the subject through the utilization of fMRI. This experiment gained notoriety because through fMRI data acquisition, centered in the occipital lobe (where the visual cortex is located) with high temporal resolution, was possible to observe what the human mind sees. The method for reconstruction lies in an extensive sample database for the subject, with hundreds of hours of film and response. The neural responses were processed into responses to 3D filtering (2D images + time), and then the observed response’s channels (filter responses) were correlated to each of the databank’s responses, the top 100 candidates were averaged to show the predicted image [NVN*11].

2.1 There is a relative myriad of types of neuron, the shown shapes are but a subset of the variety found in a complex being such an highly evolved mammal, which possesses very specific neurons for different functions, as well as neuron like cells in the heart, in the sensory systems, which together make for a wealth of types. [Stu08]

2.2 As shown, neurons can communicate and propagate their signals with one another both through synaptic clefts using chemical neurotransmitters, and the direct connection of dendritic trees. In the above image when the signals reaches the cluster of the four neurons they all fire. Given that each neuron has one and only one axon this allows for the dendritic trees to be the effectors of the one-to-many communication in neural networks. [Stu08]

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### Abbreviations

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<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>fMRI</td>
<td>Functional Magnetic Resonance Imaging</td>
</tr>
<tr>
<td>RF</td>
<td>Receptive Field</td>
</tr>
<tr>
<td>BOLD</td>
<td>Blood Oxigenation Level Detection</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial Neural Networks</td>
</tr>
<tr>
<td>rANN</td>
<td>Recursive Artificial Neural Networks</td>
</tr>
<tr>
<td>FFT</td>
<td>Fourier Fast Transform; Fourier Transform</td>
</tr>
<tr>
<td>SPRF</td>
<td>Spatio Temporal Receptive Field</td>
</tr>
<tr>
<td>rMLP</td>
<td>Recursive Multi Layer Perceptron</td>
</tr>
<tr>
<td>NARX</td>
<td>Nonlinear Autoregressive Network with eXogenous inputs</td>
</tr>
<tr>
<td>NARE</td>
<td>Nonlinear Autoregressive Network with Endogenous inputs</td>
</tr>
<tr>
<td>GD</td>
<td>Gradient Descent</td>
</tr>
<tr>
<td>SD</td>
<td>Steepest Descent, Standard Deviation (depending on context)</td>
</tr>
<tr>
<td>LARS</td>
<td>Least Angle Regression</td>
</tr>
<tr>
<td>PSTH</td>
<td>Peristimulus Time Histogram</td>
</tr>
<tr>
<td>STA</td>
<td>Spike Triggered Average</td>
</tr>
<tr>
<td>STC</td>
<td>Spike Triggered Covariance</td>
</tr>
<tr>
<td>GPU</td>
<td>Graphics Processing Unit</td>
</tr>
<tr>
<td>IPC</td>
<td>Instructions per Clockcycle</td>
</tr>
<tr>
<td>TGD</td>
<td>Thresholded Gradient Descent</td>
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Chapter 1

Introduction

This dissertation work was elaborated in partial fulfillment of the requirements for the award of the Master in Biomedical Engineering at the Faculty of Engineering of the University of Porto (FEUP).

The courses of Biomedical Image Analysis and Computer Aided Diagnosis provided the framework, and are the technical background on which the following work is nested. This work focus on the construction and improvement of neural prediction methods. It’s core component is informatic by nature (in the sense that no practical experimentation was performed), and is the application of insights into computational neurology, computerized classification, machine learning to the particulars of the prediction of the behavior of neurons from the primary visual motor cortex.

The current state of computational neurology has enabled, with recent advancements in the precision and quality of measuring instrumentation, the creation of interesting applications in the image extraction domain, directly from the human brain.

Initially this dissertation started with the broad theme of ‘fMRI Image Analysis’. On that topic the most exciting new developments were being made in the field of computational neuroscience. Being that it’s a new area both because of recent hardware developments in the fMRI machinery and the expansion of techniques and some maturation of algorithms to analyse the data, the topic of neural encoding became a focus. To further enhance the knowledge of the behavior of the neuron, there is very little that fundamental image analysis techniques can do. Neuron behavior analysis doesn’t appear to require edge detection, feature identification, mesh and blob analysis, or others of the sort. It’s challenges rely more on the field of classifiers, machine learning, time series prediction and general statistics. The choice for visual sensor neurons is a natural one, by converging with the initial boundaries of the proposed work, by being tied to some exciting recent fringe experiments, and because within computational neuroscience visual neurons are the ones for which there is more ample data, and for which their behavior closely relates to their inputs, which we can define with precision.

In this field there is one particular academic challenge, the Neural Prediction Challenge, which is closely related to this work, as it provides data, and a competitive environment on which to test
1.1 Motivation

The experiments that are perhaps most interesting to the layman would be the reconstruction of the word ‘neuron’ from the human brain as performed by researchers at ATR Computational Neurostudies Institute in Tokyo, Japan, which in 2008 published [MUY+08] the success of extracting the word ‘neuron’-in a series of letter sized frames- from the human brain whilst it was being displayed to the subject via fMRI (see figure 1.1). Then David Gallant and the Gallant Labs, University of California Berkley, managed to do something of similar impact by reconstructing video as was viewed by individuals while their brain activity was being monitored by fMRI. Though remarkable, the reconstruction is very blunt, both because of the current limitations on the quality of acquisition but also by the simplicity of the decoding models used. The experiment can be seen in figure 1.2.

The quality of these works depends on the quality of the neural models which are used. The process of mapping the stimulus to the response (as observed by fMRI, MEG, direct electrode placement etc.) is called the encoding. The opposite of this, which is to from the observed responses reconstruct the stimuli which provoked them is called decoding. As from an encoding model the corresponding decoding model can be algebraically constructed, to improve the encoding models is too directly contribute to the improvement of the decoding model.

Figure 1.1: These tests were performed on two subjects. In the upper line the exhibited stimuli are shown, and through a decoding model and fMRI mapping, the reserchers from ATR Computational Neurostudies Institute were able to reconstruct the image as shown in the following lines. In the last line the average of the various samples is shown, and can be confirmed to contain the characteristics (letter discerning and aspect) of the original. [MUY+08].
Introduction

Figure 1.2: Reconstruction of the clip from the brain of the subject through the utilization of fMRI. This experiment gained notoriety because through fMRI data acquisition, centered in the occipital lobe (where the visual cortex is located) with high temporal resolution, was possible to observe what the human mind sees. The method for reconstruction lies in an extensive sample database for the subject, with hundreds of hours of film and response. The neural responses were processed into responses to 3D filtering (2D images + time), and then the observed response’s channels (filter responses) were correlated to each of the databank’s responses, the top 100 candidates were averaged to show the predicted image [NVN+ 11].

1.2 Applications

The study of the brain and a better understanding of neuro pathways is in itself a worthy objective in the pursuit of better algorithms for encoding, yet the practical implications of this field include:

1. The improvement of Vision substitute implants to help the blind and the visually impaired - Indeed the challenge of observing the world and converting that into usable information is one long achieved and overcome. The problem still resides in how to effectively transfer that information to the nerve centers of the brain, either in the optic nerve, or by direct implant in the visual cortex areas. Better algorithms imply better image quality for those treatments.

2. The Brain-Computer interface in general, not only for the blind is currently a field of immense research. There is a possibility in the future of using small foci of microwaves to through the incresce of temperature trigger the firing of small neuron clusters, and there are already some successes with Trans-cranial Magnetic Simulation in triggering the brain from
the outside. The idea of having the computer project images onto one’s mind is closely re-
related to neural encoding [WVCPL07].

3. Greater quality video compression techniques, as they are often based on discriminating the
features humans are most sensitive against.

4. Better assessment of neural damage in trauma cases where the vision area was involved.

5. High Dimensional Statistics - Though it’s mostly the other way around (advancements in
High Dimensional Statistics contribute greatly to this field rather than the opposite), but
practical applications of this nature are excellent grounds for the advanced meta-heuristics
of higher tier statistics scientist’s toolbox, and are excellent practical cases of application.

1.3 Objectives

This work has as objectives the study (by review) of the neural systems of vision, as well as the
algorithms and techniques used to analyze their behaviors. The most recent (and best performing
incidentally) are to be implemented, and studied, and should there be margin for so, improved. By
intuition and by biomimetics (in the sense that neural networks approach the behavior of biological
neural networks) a solution using neural networks is going to be attempted again - there were
already several trials which the author found somehow lacking. Time permitting the usage of both
the state of the art solution, and a neural network classifier could be attempted, creating a hybrid
solution, (with hopefully the well established bases of the current state of the art which is based
in gabor filter response wavelet transform in 3D, could meet the good non linear properties of the
neural network.

Initially there was a plan for both Recursive Neural Networks and Massive Neural networks,
meanwhile some thought and critical considerations over the ineffective learning algorithms for
recursive neural networks made this objective discarded. Though it still holds a high potential the
lack of effective tools for training them made them unsuitable for this work.

1.4 Organization

In this first chapter, the introductory preamble attempts to elucidate as to the context and purpose
of the chosen topic, as well as showing it’s applications and explaining the organization of the
structure itself.

Chapter 2 is a short review on neurophysiology, which aims to remind the reader of some basic
principles of neural function, such as how and when a neuron fires, and how it’s all modelled. It’s
a memory freshener which is somewhat useful for the purpose of this work, for the insights into
working properties of neurons and is required to better understand the efforts to encode it.

Chapter 3 is an introduction to the mechanisms of vision, neural pathways to the cortex, from
the retina to the V1 cortex. It contains some insights onto particularities of the vision system that
may be useful for neural prediction.
Chapter 4 is dedicated to explaining how the various algorithms tested work in the various phases of the processing. It starts off by trying to elaborate on the LNP model and what algorithms constitute it’s parts, and then goes into some detail on the various workings of said algorithms: from the preprocessing and STA/STC, Gabor Filters and the Wavelet Transform, the middle layer of machine learning and optimization using linear classifiers, validation, oversampling, jackknifing, bootstrapping, and a brief mention into the later performance enhancers, such as the linear distribution modifiers, and the Hybridization with NARX Neural Networks.

Chapter 5 shows some typical examples of behavior of the methods, as well as the parameter sweep constraints, best values, and the performance in terms of correlation of the methods, as well as the best results achieved.

Chapter 6 is a short critique and assessment of the work done, the challenges, and what is there to improve.

In the Annex there are is the code listing for the main script of the work as well as the profiler screens relating to the improvement of performance.

1.5 Integration Into Scientific Endeavor

This work attempts to improve on the already existing solutions of the state of the art regarding neural coding of the vision stimuli. As it replicates and improves the current state of the art it’s integration into the field falls under the category of theoretical research on the field of computational neuroscience. The contribution of this work to the field is small but solid, and should further validation warrant, suitable for adaptation to scientific publishing. The proposed technical innovations have already been proposed to the maintainer of the leading matlab toolbox in computational neuroscience - strflab, to be delivered to the public in the next release.
Introduction
Chapter 2

Neuron

Neurons are a very particular type of cell present in multicellular organisms, their main particularity being that they can transmit electrical signals along distances - either great as in the nerve neurons or short as in intra-brain processing networks. They do this by generating electrical impulses, known as action potentials, or more simplistically spikes that travel in the nervous system, from neuron to neuron. The very nature of the ‘fire’ or ‘non-firing’ states may elude us to consider these systems as simple binary state machines, however in the sequence of firings neurons convey information, which is to say, neurons encode information in spike trains, in certain ways analogue to a digital communication line (eg: Morse code or Serial Protocol Interface - SPI ). The representation of analogue quantity in neurons comes in the form of the frequency of spiking. To understand and explain how a particular piece of information is represented by a particular neuron one must devise an encoding model, which is thus defined as an approximation of the behavior a neuron is expected to exhibit to a certain stimulus.

Understanding and devising the way neurons communicate with each other requires first some understanding of what are neurons, how they work physiologically, how they fire and are inhibited, and in this chapter a brief review of some neuron related topics is shown, in two distinct parts, the neuron as a biological entity, and the neuron firing as a mathematical construct for better abstraction.

2.1 The Neuron

The standard neuron can be divided in four regions:

- Soma: The cell body, it’s the control center of the neuron, as well as the stage for most cell life supporting functions, including the manufacturing and recycling of the neurotransmitters.
Neuron

- **Dendrites**: Are processes which extend outside the cell and present the input ports of the neuron.
- **Axon**: A Neuron may have many thousands of dendrites, but it will only have one axon. Like the dendrites it’s a process. Can be considerably long, and may or may not be covered by Schwann cells and mielin.
- **Terminal Tree**: The axon terminals, it can present spreading, like a tree. It contains and releases the neurotransmitter vesicles, which fuse with the cell membrane, releasing them into the synaptic gap.

Usually neurons are supported by Glia or glial cells. They can be up to 50 times more numerous than actual neurons in the central nervous system. They provide support and foundation, being responsible for the more mundane tasks of the cerebral and nervous tissue preservation, such as assuring oxygen and glucose supplies, and cleaning up after neuronal death.

There are many types of neurons, varying in the shape of their terminal trees, presence of myelinized axons, number of connections among others. A variety of types can be seen in figure 2.1

![Figure 2.1: There is a relative myriad of types of neuron, the shown shapes are but a subset of the variety found in a complex being such an highly evolved mammal, which possesses very specific neurons for different functions, as well as neuron like cells in the heart, in the sensory systems, which together make for a wealth of types.[Stu08](Image.png)

### 2.1.1 Synapse

There are two types of synapses: electrical and chemical ones. The typical synapse is a junction of a terminal branch of the axon with the dendrite of another neuron, on which there is a gap, the synaptic gap. The transmission of the signal is performed by the release of neurotransmitters from the axon terminal branch into the synaptic gap. Being that this is a neurotransmitter mediated connection, the term "chemical synapse" applies.

Electrical synapses on the other hand are junctions on the dendrites of different neurons which share ion gates allowing for the action potential to spread to each other. Which is to say, the
output of a neuron though unique, can affect several neurons. A scheme of both a chemical and an electrical synapse can be seen in figure 2.2.

![Neuron diagram](image)

**Figure 2.2:** As shown, neurons can communicate and propagate their signals with one another both through synaptic clefts using chemical neurotransmitters, and the direct connection of dendritic trees. In the above image when the signals reaches the cluster of the four neurons they all fire. Given that each neuron has one and only one axon this allows for the dendritic trees to be the effectors of the one-to-many communication in neural networks.[Stu08]

## 2.2 Neuron Firing

Most neurons fire to propagate the electrical impulse, which is to say, they fire because neurons connected to them are providing them with a signal they deem worthy to fire themselves. There are exceptions to that, most notably:

- Sensory Neurons: Certain types of neurons in the human’s sensory organs are original impulse makers, generating polarization on their membranes when they heat up, subject to mechanical efforts, sense a particular flavor or smell, get irradiated by appropriate light.
Neuron

- Pulse Neurons: Some circuits of the human brain are regulated by clocks, and there are neurons which fire at a regular rate which are thought to provide a clock to some systems of our processing, they self excite themselves and fire at regular intervals of time [LH39].

The process of firing takes recourse to the presence of ion gated channels across the membranes of the neuron. The involved ions are predominantly: Sodium ($Na^+$), Potassium ($K^+$), Calcium ($Ca^{2+}$), and Chlorine ($Cl^-$).

2.2.1 Action Potential

Action Potential refers to the electrical dipole established between the outer side of the membrane (extracellular medium) and the inside of the cell. This potential corresponds to the different concentrations of ions inside and outside the cell. At resting conditions the average neuron exhibits a potential of -70$mV$ relative to it’s surroundings, which are set to be the ground or 0$mV$. This polarization of the cell is usually resultant of a higher concentration of $Na^+$ outside of the cell, and a greater intern concentration of $K^+$.

The currents across the membrane which depolarize the cell are the result of Ions flowing in and out through the membrane. These flows are both caused by electrical gradient and concentration gradient, which means that when the gates are open the flow will also generate electrical potential by adjustment of the concentration of ions. If the electrical effect alone were present, the voltage across the membrane would simply drop to zero over time upon discharge and rest at 0$mV$. That is not what happens: indeed the action potential rises to 20$mV$ upon which the gates are again closed, and sodium and potassium pumps exchange the two ions at a three per two ratio in favor of the exterior, restoring the electrical balance to the cell, but not before exhibiting some hiperpolarization from the excess chlorine which is now electrically motivated to leave the cell. The resulting curve of potential can be seen in figure 2.3.

The propagation of the potential across the axon results from the electrically sensitive Sodium/Potassium pumps, which upon exposition to a higher membrane potential(as the electrical potential propagates within the cell) open up, letting sodium rush out. The process of depolarization thus travels across the axon, as seen in fig 2.4.

The refractory period is described as the period of time after a neuron fires in which it is incapable of firing again. There are two distinct periods, the absolute refractory time, and the partial refractory time. The absolute refers to the time while the action potential is still above resting potential, in which the gates are open and thus the neuron is firing. The relative refractory time is the period when the sodium/potassium pumps are still working and the cell enters it’s hyperpolarization fase. During this time whilst the potential doesn’t level with the resting potential the neuron can fire, but it’s very difficult for it to do so, yet if it is very stimulated, (if most of it’s affluent neurons fire) it can still fire.
Figure 2.3: The action potential (A - right) and the synaptic junction or cleft (B - right). A - This is a typical curve of membrane potential across time upon the firing of a neuron. The rest potential is broken and the membrane polarizes (often unto positive values) by the entry of Sodium through $Na^+$ Channels, followed by the action of $Na^+/K^+$ pumps which restore the balance, and with the reentry of chlorine a state of hyper polarization is achieved, which marks the end of the refractory period. The potential stabilizes again into the resting potential and the neuron can fire again.

B - The synaptic cleft is a privileged medium for the transaction of neurotransmitters between the incoming neuron and the outgoing one. In this cleft the emitting neuron releases vesicles filled with neurotransmitters such as: adrenalin, seratonin, Acetylcholin, dopamine among many

2.2.1.1 Measuring the Action Potential

For the purpose of measuring the firing of neurons, the electrical activity caused by the action potential firing is the most accurate way. There are several ways of doing this with electrodes:

- By placing the electrode directly on the soma of the neuron, piercing the membrane with a small needle and measuring with reference to the extracellular medium, capturing the full extent of the potentials developed

- Measuring with a specific tip directly touching the axon without piercing the membrane, allows for the action potential propagation to be felt neatly and with clear distinction

- Placing the electrode near the cell, in the immediate extracellular medium. The non ideal spread of charge across the medium suffers a fluctuation with the flows of ions which is measurable, yet requires much greater instrumentation quality and complete rest from muscular activity and other biosignals and RF noise which may affect the quality of the measurement.
Figure 2.4: The mechanism of propagation relies on the reinforcement effect of the various sodium channels. Being voltage gated, when they sense the change in potential inside the cell they contribute to the effect by opening up themselves thus creating a new focus of entry. Thus they progressively activate each other until the potential reaches the synaptic cleft.

The diagram of acquisition can be seen in figure 2.5. In the interest of studying neurons at a biological and electro-functional level the measurement on the soma is perhaps the best. To accurately measure the firing of the neuron without fear of influence of the environment, nearby neurons, and with great reliability the axon contact method is best. Yet, the measurement most used for in vivo studies is the extracellular recording, for it's ease of usage (piercing a neuron requires observation under a microscope for most neurons and is thus tricky to implement with live animals -including humans)[DA01].

2.2.2 Synaptic Transmission

The synaptic process encompasses two cells, the presynaptic or outgoing cell, which is the one who is already excited and transmits the impulse, and the postsynaptic or incoming cell, which may or may not be triggered by the neurotransmitters received on it’s dendrite.

At the synaptic cleft as shown in figure 2.3-B the membrane wave of action potentials triggers the opening of Calcium $Ca^{2+}$ voltage gated channels, from the incoming neuron. The calcium rapidly enters the cell, and the higher $Ca^{2+}$ concentration activates the calcium responsive proteins connected to the neurotransmitter vesicles.

The neurotransmitter proteins are chemicals (proteins) which have specialized antigens or receptors in the outgoing cell of the synaptic cleft. The vesicles fuse with the membrane of the
Figure 2.5: Types of measurement of action potential. Top - measurement on the soma, with greater detail near the baseline which conveys some info. The dashed line stands for clipping, to better show the lower part of the curves, for the potential is similar to the bottom measurement. Middle - Extracellular recording on the medium. The potential is about $1000 \times$ smaller in amplitude yet can be detected. Bottom - Measurement by contact with axon, full scale potential, and greater ease of usage than measuring in the soma, as some axons are very long and easy to connect with in nerve tissue. [DA01]

synaptic cell, releasing their content into the synaptic cleft, because the calcium sensitive proteins change shape allowing the undocking of said vesicles from the cytoskeleton of the presynaptic neuron.

In the cleft, the relatively shielded environment does permit some of the contents to escape and go into the free extracellular medium, but most stays in the groove, and fuses with the receptor molecules. There are different ways through which different neurotransmitters activate the receptors, both through opening ion channels, protein conformation, indirect protein signaling, and some whose mechanism isn’t entirely understood. The receptor molecule being activated in
some way may allow ion flux which can become critical and trigger the complete fire of the post synaptic neuron.

Eventually the neurotransmitters are either bounced back through thermal shaking and reabsorbed at the presynaptic neuron, or are metabolized. Both cells may use metabolites and neurotransmitters to repackage vesicles with neurotransmitters for future release.

### 2.2.2.1 Implications

Given that the synaptic junction is a chemical and highly chaotic event, some conclusions regarding the nature of the passing of signals can be drawn: The effect of the firing of a neuron is not instantaneous passing to other neurons, thus, there is a time lag, and a period of time in which neurotransmitters are active and abundant in the synaptic cleft in which the fire of the pre synaptic neuron is being felt. This allows for a lesser need for synchronism in natural neural networks. Given the relative imprecision of the method of firing, the speed of propagation and the lengths of the circuits which culminate in a neuron being stimulated by several other neurons, there is allowance for the affluent (the ones which are presynaptic in the connection) neurons to fire at different but similar times, and still trigger the fire of the neuron.

Other phenomena is the loss of neurotransmitters. Cells need to be constantly replenishing their vesicles with neurotransmitters, and given the losses to the exterior, the breakdown of some, and other interaction phenomena, neurons get tired. This is important in the sense that theoretical neuroscience always considers for practical purposes (and because not doing so would greatly increase the complexity of the problem) that neurons don’t have memory - which is to say : the firing of a neuron in previous moments does not impact on the firing in future moments. This is convenient because it leaves only the stimulus as an input variable, and allows some principles of LTI (linear time invariant) systems to be applied.

The tiring of the neuron is a phenomenon which can be felt in short time frames due to inability of the neuron to reposition vesicles and operate at the same speed ( it needs energy and oxygen to support these operations) and at a larger time frame as the progressive loss of neurotransmitters begins to take it’s toll and synthesis begins to occur at full force. Humans feel this in the loss of concentration after periods of time of mental activity, and is corroborated in the sense that depressed individuals (with lower levels of neurotransmitters) loose focus and experience inability to concentrate at a much faster rate than healthy ones.

On other effect contrary to the "no-memory" paradigm is the fact that successive fires are more likely given the presence of neurotransmitters in the synaptic fence. While it’s more difficult for very fast fire rates to occur for the reasons explained in 2.2.1, there is an opposite mechanism in the synaptic cleft which makes the occurrence of repeated firings more effective at triggering the effluent neuron. Which is to say, if the stimulus is very strong, there is a ‘valley’ of fire probability between very high rates and average rates of fire[13].
2.3 Firing rates and Impulse Trains

Although some information can be extracted from the amplitudes and duration of action potentials traveling through neurons (particularly about how many neurons stimulated them and with what synchronism) for the purpose of the information that is conveyed by the neuron to its output, all neuron firings can be treated the same [DA01]. This identical stereotype fire of a neuron is for all intents and purposes just a coordinate in time, and thus the information conveyed by the neuron is expressed as a series of spikes, more precisely as a list of times at which spikes occurred. The ensemble is referred to as a spike train.

Given $N$ spikes, if we denote the times $t_i, i = 1, 2, ..., N$ of their occurrence, we can mathematically express these events on the form of a sum of idealized spikes by means of the Dirac $\delta$ function.

$$\rho(t) = \sum_{i=1}^{N} \delta(t - t_i)$$ (2.1)

As such the function $\rho(t)$, called the neural response, is everywhere well behaved and can be integrated on its entire domain.

For any function $h(t)$, which is well behaved (continuous, everywhere integrable and differentiable, and bound) we can assert that the sum of a list of occurrences of that function at the time of the spikes ($t_i$) can be expressed both in the form of sum, or integral:

$$\sum_{i=1}^{N} h(t - t_i) = \int_{0}^{T} h(\tau) \rho(t - \tau) d\tau$$ (2.2)

Combining equation 2.1 and 2.2, we can express $h(t)$ in the following form, provided the integral limits encompass the point $t$:

$$\int \delta(t - \tau) h(\tau) d\tau = h(t)$$ (2.3)

Knowing that the sequence of action potentials corresponding to a stimulus varies from trial to trial, neuronal responses are handled in a probabilistically manner. As such the variable of interest is the probability of a spike occurring at a given time. Knowing the Dirac $\delta$ function, the probability of a spike occurring on a precisely specified time is zero (it approximates asymptotically) in any distribution [GK98]. In order to get a workable value one must consider the probability of a spike occurring within a time frame $T < t < T + \Delta t$

For clarity of notation, probabilities are indicated with a capital P: $P(0 < t < 100ms)$ - Probability of a spike occurring between 0 and 100ms into the trial. The probability density function is represented by $p(T)$ indicates the probability density at time $T$, which would be equivalent to $P(T - 0.5 < t < T + 0.5)$, if $p(t)$ is constant across $T - 0.5 \rightarrow T + 0.5$.

The probability of a spike occurrence at a given infinitesimal time point is thus described by the probability density function, and is by definition the firing rate of the cell, and so we can define:

$$p(t) = r(t)$$ (2.4)
Neuron

From here, and the relationship between the fire rate and time intervals we can write:

\[ r(t)\Delta t = \int_{t}^{t+\Delta t} \rho(\tau) d\tau \]  

(2.5)

This line of thought is to arrive at the conclusion that both the rate of fire and the spike train are equal when used inside of integrals, based on the above equations:

\[ \int h(\tau) \rho(t-\tau) d\tau = \int h(\tau) r(t-\tau) d\tau \]  

(2.6)

2.3.1 Fire rate nomenclature

In the literature is important to distinguish several quantities which are commonly referred to firing rate. For the purpose of this work the quantity called single-spike probability density, \( r(t) \), is to be the fire rate.

The quantity corresponding to the fire count is also typically represented with an \( r \).

\[ r = \frac{n}{T} = \frac{1}{T} \int_{0}^{T} \rho(\tau) d\tau \]  

(2.7)

And for several trials the notation is as follows:

\[ \langle r \rangle = \frac{\langle n \rangle}{T} \]  

(2.8)

The distinction being that the fire count is usually presented without the time signature: \( r \) vs \( r(t) \). In the literature the term "fire rate" is used in all contexts: \( r, r(t) \) and \( \langle r \rangle \). Both fire rate and firing rate mean the same thing in this work.

2.3.2 From Spike Trains to Firing Rates

There is no unique way of estimating the fire rate \( r(t) \) from a finite set of data [DA01]. Considering that \( r(t) \) is a probability density function, there is no exact result. The spike count \( r \) over a set of time periods can describe the measured spikes.

By sampling the time in discrete intervals \( \{T_1, T_2, T_3, ..., T_N\} \) and creating the time periods between those \( B_1 = t : T_1 \leq t < T_2, B_2 = t : T_2 \leq t < T_3 ... T_{N-1} = t : T_{N-1} \leq t \leq T_N \) we can apply the definition of spike count \( r \) to these time periods, or time bins, and create a count of the number of spikes occurring at the given timer period. Usually the time periods are such that \( T_{k+1} = T_k + \Delta t \), meaning all bins are equally long, except for the last that usually by convention is infinitesimally greater by including the last sample of the measurement.

The bin is thus the sum of the spikes that occur on the corresponding period:

\[ B_i = \int_{T_i}^{T_{i+1}} \rho(t) dt \]  

(2.9)

One of the problems of this time binning is that it’s sensitive to the bin separation times \( T_i \). Other is that with the increase of \( \Delta t \) it progressively loses more and more information. Given
the counts at each bin, the fire rate for each time period can be simply
\[ r_i = \frac{B_i}{\Delta t}, \]
which is a valid approximation of the fire rate for the given times. Usually the neural response is provided in this form, rather than the spike train, a signal where each sample represents the amount of firings that occurred in that time frame.

A more detailed solution (in the sense that there is much more information in the final result) is, for all moments of the considered time domain count how many spikes occurred in a given time frame. This is for all intents and purpose the same thing as the binning but applied to all points instead of the center of the bins, and is equivalent to the operation of convoluting with a box kernel. This usage of a sliding window negates the arbitrariness of bin position. One should note that despite appearing to have better temporal resolution, times separated by less than half the window are correlated as they refer to the same spikes. The equation 2.11 indicates how the fire rate at a given point can be calculated and equation 2.11 is the definition of the whole function. Note that the formula is mathematically equivalent to a convolution, though it originates from the 1st equation.

\[ r(t) = \int_{t-\Delta t}^{t+\Delta t} \rho(t) \, dt \]  
\[ r(t) = \int \rho(\tau)w(t-\tau) \, d\tau \]  
(2.11)

The window used can be a moving average, a box window function, expressed as follows:

\[ w(t) = \begin{cases} 1/\Delta t & \text{when } -\Delta t/2 \leq t \leq \Delta t/2 \\ 0 & \text{otherwise} \end{cases} \]  
(2.12)

One problem with the usage of the moving window is that if too big, the approximation of the true fire rate fails from excess of propagation: e.g. consider a zone of heavy firing followed by a zone of no firing, the no fire values are going to be much affected by the dense firing region, and even though a separation is apparent to the human mind the process results in a slope of decaying probability as the window moves away from the dense zone. If the window is too small it may show abrupt changes due to the stochastic entry and leaving of spikes from the zone of action. To mitigate these problems a solution is to not use a moving average, but a weighted average which decays with distance, and knowing that the distribution of spikes follows a Poisson distribution, the usage of a Gaussian Window is the perfect candidate. The gaussian window allows for the focus on the zone itself, and yet doesn’t neglect the more distant times.

A gaussian window kernel is defined as follows:

\[ w(t) = \frac{1}{\sqrt{2\pi\sigma_w}} e^{-\frac{t^2}{2\sigma_w^2}} \]  
(2.13)

The results of the action of the conversion from spike trains, already on the form of a PSTH (the neural response in the form of a binned function is called a Peristimulus Time Histogram, however the author finds the name misleading, for though it looks like an histogram it’s by no means one - it’s widely used in the literature however) to different estimations of the fire rate are
Table 2.1: Loss of performance by conversion from PSTH to Fire Rate $r(t)$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Correlation to Original Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Sample</td>
<td>1.0000</td>
</tr>
<tr>
<td>Box Window 80ms</td>
<td>0.7611</td>
</tr>
<tr>
<td>Box Window 160ms</td>
<td>0.6456</td>
</tr>
<tr>
<td>Box Window 480ms</td>
<td>0.4381</td>
</tr>
<tr>
<td>Gaussian Kernel 480ms</td>
<td>0.5932</td>
</tr>
</tbody>
</table>

shown in figure 2.6 and the corresponding quality and lack of thereof (by loss of information) are shown in table 2.1

Figure 2.6: Original stimulus and processing to fire rate. From top to bottom: 1 - Original signal for the neuron 206B from the Neural Prediction Challenge, not a number (NaN) values removed. 2,3,4 - Conversion to fire rate through convolution with a box window of specified size. Note the smoothness created, which obstructs the original details, but at the same time is easier to approximate by Optimization algorithms. 5 - Convolution with a Gaussian Kernel, results in more fidelity than the same size box window and is arguably of the same smoothness (Notice the peaks at t=12). Original Image
2.4 Simple Neuron Models

The firing of a neuron is a complex biochemical event, which was first explained by Alan Hodgkin and Andrew Huxley for which they received the Nobel Prize in Physiology or Medicine in 1963. They studied and devised a mathematical model on the initiation and subsequent propagation of ionic action potentials across the membranes of the squid giant axon.

The Hodgkin-Huxley model defines a set nonlinear ordinary differential equations that explains the electrical characteristics of the electrically capable cells, e.g., neurons or cardiac myocytes. Since then better and more accurate models have emerged which detail intricately the process of neuron fire. [VGDSA08]

Sadly those are too complicated for our purposes, the high quality single neuron models establish very well the foundations to the understanding and predicting the fire of a neuron at a sub fire lag time resolution, but we have neither the data of spike voltages with time resolution or the computational resources to take advantage of the more biologically perfect neuron models. The simplest model possible is that a neuron fires based whether the sum of its inputs reaches a given threshold. There is usually a weight associated with each incoming neuron’s response. The probability of fire is thus:

$$P_{\text{NeuronFire}} = \sum_{i=1}^{N_{\text{AfferehtNeurons}}} S_i \dot{w}_i$$  \hspace{1cm} (2.14)

Where $P_{\text{NeuronFire}}$ is the probability of the neuron firing or not (usually converted to 1 or 0 by a threshold), $N_{\text{AfferehtNeurons}}$ is the total number of pre-synaptic neurons, $S_i$ is the state of a given afferent neuron - whether it’s currently firing or not, and $\dot{w}_i$ is an associated weight to that neuron. This is the model which is used for artificial neural networks, for it’s simplicity and low computational requirements. Note that in this model, training the weights becomes a simple task, as they linearly combine to create the result, thus enabling for an easy discrimination of the gradient of the error.
Chapter 3

The Visual Cortex and Vision

The capacity for vision is widespread among most of animalia. However, large animals, in particular mammals have perfected it to great extent.

The organs responsible for vision are the eye, and the brain. The knowledge of how vision works is of importance for the neural code analysis. In this chapter, a brief memory freshner on the process of vision is presented. It’s only suitable that vision is explained in the course of this work, as it is the main motivation, and gives some background into exactly what is being observed, and is the system that we’re trying to model - in neural coding we attempt to get the neural response in function of the stimulus, we are thus creating a system which converges to the concepts shown in this chapter.

3.1 The eye

Eyes are complex sensory organs which evolved from light sensitive points or areas on the surface of primitive invertebrates. Developed complex vision capable eyes usually contain the following aparata: A protective casing, a lens system, a photosensitive layer, and mechanisms for orientation of the organ covering a field of vision.

3.1.1 Global Anatomy

In figure 3.1, the principal structures of the human eye - this structure is similar in all primates, and alike and equivalent in most mammals, which the exception only humans and primates have three different wavelength sensitive pigments (as opposed to two by most vertebrate species). The outer layer of the eye, the protective casing is known as sclera, which is modified on it’s anterior portion (the outer part of eye) to for a transparent process called the cornea. Through the cornea radiation enters the eye.

In the interior of the sclera lays the choroid, a layer of irrigation and vascular tissue that supports most structures. Sheathing the posterior two thirds of the choroid is the retina, the neural
The Visual Cortex and Vision

tissue which contains the photo receptors. The zone in the retina where the highest concentration of photo-receptors and nerve density is called the fovea, and is responsible for the very high detail zone in our field of vision for fine tasks such as reading or precision hand coordination.

In the cornea opening there is a lens, fixated by a series of ligaments called ciliary zonule. The ciliary zonule is fixated to the thick part of the choroid by the ciliary body. In this ciliary body there are radial ciliary fibers, which are muscle fibers which contract and expand the pupil. The changes in the opening of the pupil can decrease the opening area 16 fold, increasing the quantity of light that enters the eye allowing it to adapt to the luminosity field of vision.

The space between the lens and the retina is called the vitreous humor, which is a jelly like substance that in volume constitutes the bulk of the eye. It is transparent and provides mostly structural stability. The liquid humor is the fluid that occupies the spaces around the cornea, bathing the lens, the pupil, and nourishing it. It’s produced in the ciliary body and exits through Schlemm’s duct out of the cavity for re-absorption.

Figure 3.1: Anatomy of the eye, seen from a sagital cut.[WR77]

3.1.2 Retina

The retina is perhaps the most delicate and most complex sensory tissue in all of the realm of life. It’s the functional part of the eye, and in certain aspects it’s not entirely dissimilar from a
The Visual Cortex and Vision

CCD or CMOS sensor of a digital camera. It’s a tissue that spans most of the inner surface of the ocular globe, and some of its particularities are of interest when considering the challenge of neural prediction. The light capturing cells are the rod cells and the cone cells.

3.1.2.1 Physiology

There are several types of cells of interest in the retina. The most important of which are certainly the photo-sensitive cells: the rods, and the cones. These cells contain photo-receptors: protein pigments which change their shape or conformation when stimulated by appropriated wavelength light, triggering sodium channels which initiate the neural firing.

Rod cells are much more abundant - about 120 Million in the retina - and they capture light mono chromatically (they are insensitive to color). They have very fast reaction times, and constitute most of the peripheral and parafoveal vision, identifying contours, providing vision in the dark (very low light) and are very responsive to fast moving stimuli. Cones on the other hand can have one of three types of pigments, which respond to colors red, green and blue (it’s no coincidence we use those colors as the basis for our color systems) and are much more abundant in the fovea centralis. There are about 6 Million cones in the human retina, and sometimes they are coupled to ganglion cells on a 1 to 1 basis, effectively becoming a pixel-like structure in our vision, and are instrumental to fine vision (ganglion cells take many receptors as there are only about 1.2 Million nerve fibers leaving the retina, for about 120 Million photo receptor cells).

The rods and cones transmit the signal to bipolar neuron cells. These cells are so caller for exhibiting two processes, and they are an effective buffer for the low level processing occurring in the retina. Each bipolar cell is connected to several rods or cones (exclusively to the same type: there are rod bipolar cells and cone bipolar cells, they don’t mix). They interface with horizontal cells, amacrine cells and ganglion cells. They carry the signal to ganglion cells. Amacrine and horizontal cells are lateral processing neurons, and together with bipolar cells they make the earlies pre-processing in the vision pathway. The main difference between horizontal cells and amacrine cells is that the first operate on the rod-cone / bipolar cell synapses, and the latter in the bipolar /ganglion interfaces.

Ganglion cells collect the information from bipolar cells, and amacrine cells, or a combination thereof, and their axon goes into the optic nerve carrying the signal to the brain. Ganglion cells may also have photo-receptors (melanopsin instead of rodopsin or cone pigments) to identify simply the amount of light present, both for the control of the pupil and the circadian rhythm. It is thought that they form an independent system from the cones and rods, and explains why people with severe damage to the occipital lobe which has provoked complete blindness can still perceive light and regulate their daily rhythms by daylight.

One of the known effects of the lateral processing is the receptive field typically being having two zones, one of inhibition and another of excitement. Usually on the form of disks. There are two different behaviors to the dispersion of light depending whether the bipolar cell is a naturally "On" or "Off" cell. The behavior of the entire ensamble to a stimulus of the receptive field (the
cones or rods which are connected to the tree which culminates in the observed neuron) can be seen in figure 3.2.

3.1.2.2 Organization

In figure 3.3 the neural composition of the tissue is shown. It’s made out of ten layers, and contains the photo-receptors. The layers are as follows, by order of illumination (first layers are illuminated first by incoming radiation, in other words are more anterior) [Gan05]:

1. Inner membrane - Structural support.
2. Nerve fiber layer - Nerves that exit the retina into the optic nerve.
3. Ganglion Cells - This layer is where the nuclei of the ganglion cells are present. The axons of which become the nerve fiber layer.
4. Inner Plexiform - Is where the synaptic connection between the dendrites of the ganglion and amacrine cells meet the axons of bipolar cells.
5. Inner nuclei - Contains nuclei bipolar cells.
6. Outer Plexiform - Synapses between rods and cones and bipolar cells.
7. Outer nuclei - Nuclei of the rod and cone cells.
8. External membrane - layer that separates the functional part of the rod and cone cells from their nucleus and body.
9. Photoreceptors - where the actual rod and cone like parts of the photo-sensitive cells are. Here the photoreceptors are contained in cellular structures resembling rods and cones.
10. Retinal pigment epithelium - Creates a barrier to light, ending the retina.
Figure 3.2: Typical behaviour of ganglion cells to light on their receptive fields. This sort of processing is a little bit like edge detection, and is very attuned to motion. Note that the light is pulsed in order to constantly provoke response (a steady light even in the right pattern fails to be effective at provoking high fire rates).
Figure 3.3: Organization of the retina. Light propagates from bottom to top. Neural spikes travel top to bottom. The various layers are represented. Simplified mechanism of action: the spike is generated in the cones or rods, is grouped and transmitted to the bipolar cells, suffering lateral processing from horizontal cells. In then travels to ganglions, either directly or through amacrine cells (which also introduce lateral processing), and from the ganglion goes into the optic nerve.
3.1.3 Implications on Neural coding

One other aspect of the biochemical process in the actual cones and rods of the cones and rods’ cells is that as soon as light activates the photo-receptors, they stay activated, and for the cell to fire again they must first return to their dim-lit state. This makes, that for a high firing rate to occur in the photo-receptor cell the light must go off at some times. One continuous strong light will produce one fast firing followed by absence of firing. A continuous low intensity light provokes a mild response, as some photo-receptors manage to return to their unstimulated state and get hit again (statistically a low light will not hit all the pigment, and some of it will manage to unexcite) and every once in a while a firing is triggered. For a maximum fire rate to occur, there needs be a very strong pulsed light. The cone cells have also another mechanism similar: they are sensitive to variations in light/color mor than to static light/color. It is thought (yet mainly unresolved) that microsaccades - very small rapid twitches by the eye, usually 40-120 arc-minute for few microseconds - exist to avoid the image to fade, from lack of activity from photo-receptors.

This combined with the effects of lateral processing as seen referent to figure 3.2 make the retina much more effective at spotting things in movement. Indeed the entire pre-processing occurred in the retina suggests a focus on the derivatives of the image over time rather than it’s continuous zero frequency components - which is to say, the retina works very well at tracking a moving object against a fixed background, which is only natural, considering the evolutionary pressures exerted on predators.

3.2 From the Optic Nerve to the Visual Cortex

The optic nerve leaves (in the case of most vertebrates) the eyes, and divides itself in four fascia - groups or bundles of nerve fibers corresponding to different areas of vision experienced by each eye. These bundles then proceed to cross the brain to reach the posterior part of it, in the occipital cortex, reaching the Primary Visual Cortex, which is located in Brodman’s Area 17 of the cortex, more simply the V1 cortex.

In fig. 3.4 the division between the areas of the retina and the subsequent nerve path are shown. For each eye there is a division into the part of the retina which observes the perifery of the field of vision, and the center or nasal portion of the field of view. The information of the right eye’s periferity is complemented by the left eye’s nasal field, and together they detail the left half of the field of vision, and are processed on the right hemisphere. The same holds true for the right half of the field of vision being processed in the left hemisphere.

Implications of this are that if the image is presented to one eye, and a given V1 neuron is being analyzed, only half of said image is present in the respective hemisphere, and so there is a separation line in the image domain. Thus the receptive field of that neuron can only reach so far into the image.

However, the optic nerve doesn’t connect directly with the V1 area of the brain, before that the signal is processed by the the Lateral Geniculate Nucleus, which mixes the components from
The Visual Cortex and Vision

Figure 3.4: In this figure it’s possible to see what the various nerve bundles carry in terms of information from the visual field. Notice the nerve crossover at the optic chiasm, where the nasal looking parts of the retina are sent to complement the vision from the periphery of the other side.[Fix94]

the right and left eye, of their nasal and peripheral component, aggregating them. And the neurons involved in the NPC are from the fovea cortex.

3.2.1 The Visual Cortex

The Visual Cortex contains different areas which are responsible for different functions in vision. The zone of interest for this work is only the V1 cortex which has a direct equivalency to the ganglion cells, and which maintains relatively rough information on what is being observed by the retina.

3.3 The V1 Neuron

Neurons of the V1 cortex are of two main types: simple and complex. Simple neurons appear to exhibit STRF responses to well defined stimuli in terms of orientation, spatial frequency and phase. Complex neurons on the other hand are much more varied, they appear to be less selective to those parameters and can be thought as an accumulation of the behaviour of simple V1 neurons. They can have a well defined preferred orientation but be lax in terms of slight spatial offset of the
The Visual Cortex and Vision

Figure 3.5: Various areas responsible for the processing of vision. Not labeled: above V3A in purple - V7 zone[Log99]

stimulus in relation to the receptive field, or they may respond to any direction of a given stimulus, and they combine excitatory behaviour with inhibitory to different simple V1-like responses.

Experiments probing the primary visual cortex of macaques found that there is a pattern in terms of the column and the depth of the neuron in the cortex, as can be seen in figure 3.6.

(a) Orientation Angles of V1 Neurons in the visual field

(b) Preferential Orientations of V1 Neurons row-wise, as a function of the displacement of the probing needle

Figure 3.6: Study on the preferred orientations of the response of V1 Neurons performed in macaques. It was determined that orientations varied 3° to 10°, and that there was an organization in the cortex, with similar orientations close together. The probing needle was displaced along an axial cut of the cortex and the preferential orientation was determined by showing various orientations of flickering bars and finding the one with maximum response. It was also found that a similar pattern emerges with depth into the cortex, but to a lighter extent. Modified from [HW74]
### Table 3.1: Function of the visual projection areas of the human brain [Log99]

<table>
<thead>
<tr>
<th>Area</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>Primary Visual Cortex : Is the one that receives the stimuli from the lateral geniculate nucleus. Contains both the mapping and earlier processing part in terms of STRF orientation, edge detection, blob detection and earlier texture analyzes.</td>
</tr>
<tr>
<td>V2, V3, VP</td>
<td>Extended processing, Larger Visual Fields</td>
</tr>
<tr>
<td>V3A</td>
<td>Dedicated to motion</td>
</tr>
<tr>
<td>V4v</td>
<td>Function Unknown - suspected to handle color flickering</td>
</tr>
<tr>
<td>MT/V5</td>
<td>Motor coordination, movement processing, hand to eye coordination</td>
</tr>
<tr>
<td>LO</td>
<td>Large object recognition</td>
</tr>
<tr>
<td>V7</td>
<td>Function Unknown - suspected to handle hierarchical processing and interfacing with memory for object identification</td>
</tr>
<tr>
<td>V8</td>
<td>Color Processing</td>
</tr>
</tbody>
</table>

As often complex cells receive the outputs from simple cells, it’s expected to be more likely that the combinations of preferred orientations for complex cells share common directions than not. It’s not written in stone however, because the axons that connect them can be quite long, and complete different directions can be mixed in a complex cell.
Chapter 4

STRF Methods

4.1 STRF

The Spatio Temporal Receptive Field is a function which maps the visual stimuli to neural responses. The Receptive Field is the subset of the stimulus which provokes a response in the neuron. The Spatio Temporal part adds the information of how the neuron responds to changes in its receptive field over time, and to particular combination of subsets of its receptive fields.

The stimulus and the response can be represented as two time-variant signals. Let $s(X_i, t)$ be the spatio-temporal field and $r(t)$ the instantaneous fire rate (whose approximations are defined in chapter 2 at time $t$). The discreet space points are $X_i \in X_1, X_2, X_3, \ldots, X_N$ and time instants are $0 \leq t = 0 \ldots T$

Sensory neurons develop a fire rate usually explained by the linear rectified model [TDS+01]:

$$r(t) = |\sum_{i=1}^{N} \sum_{u=0}^{U} h(X_i, u) s(X_i, t-u) - \theta + \epsilon|^+$$  \hspace{1cm} (4.1)

Where $h(X, u)$ is a linear filter applied to the various points in space $X_i$, and the time delays $0 \ldots U$ influence how a stimulus present in $t - u$ influence the firing rate at time $t$. The notation $|x|^+$ describes half wave rectification:

$$|x|^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$  \hspace{1cm} (4.2)

The term $-\theta$ in 4.1 is a Thresholding agent (forcing the response to be 0 if the other components fail to meet the threshold) and the $\epsilon$ is an unknown parameter representing the stochastic nature of neuron fire - this makes it the non linear part of the response not modeled by the linear model, and can also incorporate the measurement noise. That makes $\epsilon(t)$ the residual or error signal.

The delays are always bound to $U$, and we assume for the purpose of computing that $s(X, t), \quad t < 0 = 0$ which means the system is causal. The neuron in this model has memory of its inputs up to $U$ but has no memory of it’s outputs, though given that they are derived from the same expression, it
STRF Methods

has a partial memory of its outputs being that its past outputs are progressively more correlated
with the current one as they approach the current time (as more and more terms of the sum are
equal between the two).

This model is then of the form:

\[ r(t) = f(s(X, (t - U) \rightarrow t)) + \varepsilon \quad \text{Where:} (t - u) \rightarrow t = (t - u) \leq x \leq t \quad (4.3) \]

On this work one innovation proposed is the explicit assumption of the memory of the neuron
of its past outputs. This allows for it to model phenomena as tiring - after a very heavy fire rate
the subsequent fire rates are temporarily diminished. And we'll aim to create a model of the form:

\[ r(t) = f(s(X, (t - U) \rightarrow t))g[r((t - V) \rightarrow (t - 1))] + \varepsilon \quad (4.4) \]

By adding a function taking into account previous responses we're greatly increasing the
amount of information present. We're considering the U time lags of the stimulus plus V time
lags of the response to the stimulus. One problem with this approach is that it is chaotic in the ac-
cumulation of error: if a previous segment was poorly predicted, the poor predictions will empact
the current prediction. Luckily this effect stops accumulating error if a low fire rate region occurs.
Yet, there is biological rational in the workings of both the retina as well as neurons in general to
justify this aproach, and in that sense it's closer to the biological reality of the system.

Finding adequate ways to express the function g is a problem addressed later in section 4.5.4.

4.1.1 LNP Model

To predict spike trains, there is a well established framework on which most neural prediction
algorithms fall, which is the Linear, Non Linear, Poisson Cascade model. The STRF model is an
example of a LNP model. There is firstly the response to a filter (or several) which models the
spatio temporal selectivity of the neuron to the image stimulus, followed by a nonlinearity, which
can be any aggregation function, such as sum and \(1/x\) curve - in the case of the STRF it's just
a threshold curve. The resulting fire-rate then goes through an inhomogeneous poisson process
(seen in image 4.1 as a dice) which stochastically determines where the spikes occur based on the
probability / firing rate of the occurring.

This model has strong biological foundations, given the neuron model seen in 2.4. If there is
a tree that culminates in the observed neuron (making it the trunk of tree), and whose leaf nodes
are photo-receptors, it makes sense that the various simple responses end summing up to a linear
combination, because at each intermediate node the same combination rationale can be applied.

It is conceivable however that this model isn't sufficient to describe the whole process. It's
plausible that there are more complicated relationships that linear combination between V1 neu-
rons, such as basic logic operations, multiplication, reversal, which can be partially approximated
but not fully described.
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Figure 4.1: The LNP Model - The linear combination of responses of the stimulus to a filter are then passed through a non-linearity function and then the spikes occur randomly following a Poisson distribution whose parameters are constrained by the instantaneous firing rate. The diagram shows for simple neurons (A) of the V1 cortex there is only one preferred direction and spatial frequency. For complex neurons often the combination of several filters is needed to approximate their response (B). In the bottom the STA and STC are shown to a stimulus of black and white bars presented to the subject. The idea is to create moving windows. By averaging the resulting windows STA is created, by estimating the covariance STC is calculated [RSMS05].

4.2 Image Domain

The stimulus $s(X_i, t)$ is for all practical reasons in this work, a film: a sequence of image frames shown at periodic periods of time. Another more intuitive representation could be $s(i, j, t)$, where $i$ and $j$ are the pixel coordinates of the image, and $t$ the frame number. The abstraction $X_i$ is just for generalization, and because the stimulus may not necessarily be on the form of an image and the equations of the previous section hold true if $s$ is something else such as sound, or the channel values of a wavelet transform.

This stimulus provided in the Neural Prediction Challenge can be seen in figure 4.2.
STRF Methods

Figure 4.2: Typical frames from the stimulus. Top: This is the "high quality version" at $136 \times 136$ resolution. These are the images shown to the subject primates in their native resolution. Clip size varies from 8000 to 20000 frames - up to 320s. Bottom: Low resolution down-sampled segment of the region of interest - determined by STA

### 4.2.1 STA and STC

In the image domain there are two quantities that can be constructed which give information about the response. If we take the average of all exhibited frames which trigger a neuron spike we have gathered some information about what makes the neuron fire... the zones whose activity provokes a greater response will be more expressed and have greater average values. Its useful to take into consideration the STA of current spike in function of the previous stimuli. Thus usually a STA is defined over a window of $T_{STA}$ samples of length. The definition of STA is expressed in equation 4.5.

$$STA = \frac{\sum_{i=1}^{T} y_i s(t \rightarrow t - T_{STA})}{\sum y_i}$$  \hspace{1cm} (4.5)

Where $y_i$ is the spike count for that time frame (if the data comes in a PSTH form). If we do it with spikes instead of PSTH, $y_i$ simply becomes 1. $s(t \rightarrow t - T_{STA})$ denotes the stimuli in the given time frames. (Can concatenated and extracted at the end, or the operation can be performed to each value of t individually).

The process can be seen in figure 4.1 or in more detail in figure 4.3.

A problem with simple STA is that if there are constant characteristics in the stimuli (such as
a white or corner) it is going to get expressed in all the frames and thus be in the final STA despite having contributed nothing (or at least the contribution cannot be verified) to the response. If the stimuli is white noise, or at least well distributed in terms of variance and average everywhere, then the average over time is 0.5 (from 0 to 1 in luminosity of the pixel) and that yields great information in the STA, because inhibition zones are also shown by being lower than average in the STA. To try and counter this, one can "whiten" the STA, trying to make each channel average and variance similar to white noise. To that purpose we can divide by the covariance matrix of the stimulus:

$$STA = \frac{1}{T} \left( \sum_{i} s(t \rightarrow t - T_{STA}) s(t \rightarrow t - T_{STA})^T \right)^{-1} \sum_{i=1}^{T} \frac{y_{i}(s(t \rightarrow t - T_{STA})}{\sum y_i}$$

Where $A^T$ denotes the transposed matrix - the first term of equation 4.6 is the Covariance Matrix.

This makes STA applicable in natural vision movies such as our data.

Spike Triggered Covariance is a related analysis tool. Having the advantage of finding a multi-dimensional feature space for the STRF (as opposed to STA which only provides the tri-dimensional space of the movie), the eigenvectors of $STC - C$ (where $C$ is the Covariance matrix such as the one defined in equation 4.6 - not the inverse though) inform over the excitatory or inhibitory nature by being positive or negative respectively.

The definition of STC is as follows:

$$STC = \frac{1}{n_X - 1} \sum_{i=1}^{T} y_{i}(s(t \rightarrow t - T_{STA}) - STA)(s(t \rightarrow T_{STA})^T$$

In figure 4.4 sample results of the application of these methods to our data yield the following.
Figure 4.4: Top Row: STA results for $T_{STA} = 4$ for neuron 206B. RTA and RTC mean the same thing as STA and STC but are used when instead of a spike train we use a PSTH to calculate - Response Triggered Average instead of Spike Triggered Average. For the various delays it’s possible to see a gabor like structure forming, with low spatial frequency - This STA can be used to generate predictions by convolving it with the stimuli (though with very low quality). Bottom two rows: STC Projections : STC is a matrix whose orthogonal projection STC1 and STC2 are used as values for a latter stage of optimization and can be used for creating predictions by linear combination. The visualization of the projections elucidates as to the direction vectors which most affect the response.

STA and RTC as neural response predictors have long been surpassed and provide poor results in the later stages of combination and thresholding, however they are important tools in the determination of the parameters for other algorithms, most notably they indicate well the areas of excitatory and inhibitory behavior as well as preferred directions.

4.3 Gabor Wavelet Transform

Representing visual information of a film on the form of images across time is very intuitive and practical, yet information in the frequency domain is more linearly related to the observed response [DG05]. With that in mind some algorithms emerged which first transform the stimulus to it’s Fourier power components and only then try to apply linear combination, optimization and further stages of processing. The current state of the art approach consists of a discrete wavelet transform with Gabor Wavelets - by other words - the response to a filter bank. A Gabor wavelet is a sine plane wave multiplied by a gaussian. It is well known that Gabor Wavelets model very well the responses of simple V1 neurons [Dau80]. Another proof of the adequation of this type of filter to video description is the fact that it’s a video compression technology used with effect in some of today’s codecs [ERK90][LK92] for it’s capacity to describe visual structures in the way in which we perceive them.
A gabor wavelet or filter is of the form:

\[
g(x, y, \lambda, \theta, \phi, \sigma, \gamma) = e^{-\frac{(xcos\theta + ysin\theta)^2 + (ysin\theta - xcos\theta)^2}{2\sigma^2}} \times e^{i(2\pi \frac{xcos\theta + ysin\theta}{\lambda} + \phi)}
\]

Where \(\lambda\) is the spatial wavelength, \(\theta\) is the orientation, \(\phi\) is the offset, \(\sigma\) is the standard deviation of the gaussian, and \(\gamma\) is the spatial aspect ratio. An example gabor filter is shown in figure 4.5.

![3D visualization of a Gabor Filter](image)

Figure 4.5: 3D visualization of a Gabor Filter - This is a single specimen, a filter bank is made out of thousands like these, varying in orientation, placement, frequency amongst other parameters. This filter is multiplied together with a motion function by the video to generate a single channel, a signal varying over time.

The responses of the neurons are to ‘activity’ in a given preferred orientation/zone, so we create 3D filters consisting of a 2D Gabor Wavelet appearing and disappearing across time at a given velocity.

For the Gabor transform of the video we use the following parameters for the wavelets (the Gabor filters):

- Orientation
- Spatial Frequency
- Position
- Velocity
- Time Span
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- Size
- Imaginary representation

These amounts of parameters make the filter bank very large in terms of channels. It’s easy enough to create huge banks even with small subsets of these parameters. In section 3.3 we’ve established that orientation selectivity is around 3° to 10°, and only 180° should be covered (the response is simply negative if the direction is opposite), therefore we can estimate the filter bank to require between 18 and 60 divisions.

One good solution to obtain large representation of frequencies, is to use logarithmically spaced frequencies from covering the entire image to a sharp line.

We establish a grid of position of application of the filter, rendering the offset obsolete (it’s essentially the same thing but with more freedom, as we can also translate transverse, instead of just in the direction of the orientation). The size of the filter bank reached 40000 in some tests (which is rather large comparing with the 16 × 16 image at hand), but good values were found at size 6000. Too large filter banks increase the dimensionality of the optimization and have downsides, though some algorithms handle it better than others. In figure 4.6 an example of a used gabor filter is shown.

![Gabor Filters Example](image)

Figure 4.6: A single gabor 3D filter, with a temporal size of 9. From left to right temporal indexes: -4 to 4. The Gabor filters have a sensitivity to changing features. This particular one has a short temporal velocity and a low spatial frequency. This is a channel representing the half wave rectified real part $|\text{Re}(g)|^+$

4.4 Optimization Algorithms

The middle part of the LNP model requires iterative processes of optimization - these vary in terms of speed, final result, resistance to noise, resistance to undersampled values, handling of high-dimensionality, and can make all the difference in terms of the final quality of the achieved solution. sub

4.4.1 Gradient Descent

Finding the best way to combine linearly all the stimulus channels (in whatever form they may be, such as image pixels, responses to STA and STC or wavelet channels) is a problem of optimization. The Gradient Descent concept simply describes a method of finding local minima from a starting point by going down the gradient. Very much alike being at the top of a mountain blinded - if
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one wishes to get to the lower ground, or more precisely reach the bottom of the valley below, a simple method is at each step to determine the orientation of the slope of the ground, and give the next step in that direction - it’s so intuitive most of us would proceed that way without thinking about it. It may not guarantee the shortest path (the shortest path may be over a lower mountain in the way, and we would go around by using this method) but it’s the simple base to the problem of regression at hand.

The minima we’re trying to find is the error of the solution, and the slope can be estimated by the current position.

Let $W_i$ be the array of weights for all $N$ channels of the feature space $S_i \ i = 1 \rightarrow N$, such that:

$$r_p(t) = \sum_{i=1}^{N} W_i S_i \tag{4.9}$$

Where $r_p(t)$ is the predicted response for time $t$.

Then we can define the error of that prediction as the difference between the predicted value and the real observed response:

$$\varepsilon(t) = r_p(t) - r(t) \tag{4.10}$$

Let’s start by initializing the Weight Vector to 0. For each sample in the response we can add to the Weight vector the features which triggered that response:

$$W_i^0 = 0, 0 < i \leq N \tag{4.11}$$

For the general case Gradient Descent, the approximation is as follows:

$$W^{k+1} = W^k + \gamma \nabla \varepsilon(W^k) \tag{4.12}$$

Where $\gamma$ is the step size, $\nabla$ the gradient operator and $\varepsilon(X^k)$ denotes the error function in respect to the weight vector $W$ (as opposed to $\varepsilon(t)$ which is the error of a given time frame).

Applying equation 4.12 to a single channel / weight, and using the estimation of gradient $\nabla \varepsilon(X_i) = \varepsilon . S$ we arrive at the following expression for tor an individual feature:

$$W_i^{k+1} = \sum_{t=0}^{N_{\text{samples}}} W_i^k + S_i(t) \cdot \varepsilon(t), 0 < k \leq N \tag{4.13}$$

$W_i^k$ denotes the Weight of the $i$'th channel after iteration $k$. Equation 4.13 can be read as: in each iteration add to the weight the average of the error of the current solution multiplied by the stimulus which produced that error. That means that after each iteration each weight will get adjusted by the average of how that weight affected the prediction.

In our particular case we have a stimulus and several time delays which may affect the response, mathematically it’s the exact same thing, it can be worked by extending the feature domain over the chose time frames of delay, but for clarity consider the following method of computation as implemented in this work.

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Figure 4.7: Gradient Descent - here is shown a simple gradient descent on two dimensional feature space. Our actual case is highly dimensional but it shows the principle of progressing in favour of the gradient (or against it if we consider the gradient to be positive when rising) [Wik13a]

Let $S$ be a 2 dimensional ($N$ Features and $N_{\text{samples}}$ Time) stimulus matrix.

$$S = \begin{pmatrix}
S_{1,1} & S_{1,2} & \cdots & S_{1,N_{\text{samples}}} \\
S_{2,1} & S_{2,2} & \cdots & S_{2,N_{\text{samples}}} \\
\vdots & \vdots & \ddots & \vdots \\
S_{N,1} & S_{N,2} & \cdots & S_{N,N_{\text{samples}}}
\end{pmatrix} \quad (4.14)$$

Where $S_{i,j}$ refers to the $i$'th channel at time frame $j$. Given a delay window from $t = 1 \rightarrow U$, we construct the error and delay matrix as follows:

$$D = \begin{pmatrix}
\varepsilon(1) & \varepsilon(2) & \cdots & \varepsilon(U) & \cdots & \varepsilon(N_{\text{samples}}) \\
0 & \varepsilon(2) & \cdots & \varepsilon(U) & \cdots & \varepsilon(N_{\text{samples}}) \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \varepsilon(U) & \cdots & \varepsilon(N_{\text{samples}})
\end{pmatrix} \quad (4.15)$$
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By multiplying \( D \times S^T \) we have for every sample in row \( j \) the proposed gradient estimation based on that sample:

\[
\nabla \varepsilon = D \times S^T = \begin{pmatrix}
\sum_{c=1}^{N_{\text{sample}}} S_{1,c} \varepsilon(1) & \sum_{c=1}^{N_{\text{sample}}} S_{2,c} \varepsilon(2) & \cdots & \sum_{c=1}^{N_{\text{sample}}} S_{N,c} \varepsilon(N) \\
\sum_{c=2}^{N_{\text{sample}}} S_{1,c} \varepsilon(1) & \sum_{c=2}^{N_{\text{sample}}} S_{2,c} \varepsilon(2) & \cdots & \sum_{c=2}^{N_{\text{sample}}} S_{N,c} \varepsilon(N) \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{c=U}^{N_{\text{sample}}} S_{1,c} \varepsilon(1) & \sum_{c=U}^{N_{\text{sample}}} S_{2,c} \varepsilon(2) & \cdots & \sum_{c=U}^{N_{\text{sample}}} S_{N,c} \varepsilon(N)
\end{pmatrix}
\]

(4.16)

In matrix \( D \times S^T \) we have the estimation of the gradient by multiplication of every sample’s error with the features of that sample at a given time delay. Alternatively we could have condensed the time delays in a single line and handled the entire thing as a greater extent feature space, but this description is faithful to the MATLAB implemented version - which takes greater computational advantage of this form as it allows MATLAB’s highly optimized matrix computation to be used and is significantly faster than iteratively calculating all values involved with for loops.

4.4.1.1 Momentum

Momentum in the gradient descent is the concept of using the past gradient combined with the current one.

\[
\nabla_m \varepsilon^{k+1} = \nabla \varepsilon^{k+1} + \nu \nabla_m \varepsilon^k
\]

(4.17)

The inclusion of momentum is convenient cases where the hyper-surface of the error function contains some noise. By including moment the algorithm passes over small pieces of noise that would otherwise cause a slowdown. On the approach with the minima momentum also helps correcting the zig zag of going back and forth around the depression (as they average a little). A typical value for the momentum would be \( \nu = 0.5 \). By comparison with simple feedback LTI closed ring systems we can clearly see that this is a simple low pass filter upon the gradient.

4.4.1.2 Norm of the Gradient

The step size used for each new iteration is an important parameters of the optimization algorithm. A small step size assures precision in getting near the true minimum, but is very slow to arrive there. A large step size will quickly arrive at a solution approximate from the minimum but will not approach it completely going back and forth over the minimum in a zig zag pattern. One possible solution to this problem is to divide by the error, this ensures that the step size is governed by the defined parameter and will minimize overshoo. One problem with this approach is that the minimum error is not zero, only if everything were ideal would this mechanism be feasible. In fact at the start of the algorithm we know that the minimum is less than or equal to the initial estimation but it can be any value between 0 and that value. Another candidate for an estimation of "how close" we are to the minimum is the magnitude or Norm of the gradient. If we divide the
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step by the gradient’s norm we’ll get step sizes which are large enough when the gradient is very low, and small enough when the gradient is very high, rendering the algorithm resilient to high spatial frequencies.

The expression for the update of the coefficients becomes:

$$W^{k+1} = W^k + \frac{\gamma \nabla \varepsilon(W^k)}{||\nabla \varepsilon(W^k)||}$$

(4.18)

Where ||X|| denotes the euclidean norm of vector X.

4.4.2 Coordinated Gradient Descent

In very high dimensional cases of "exploratory" linear combination (exploratory in the sense we only use a high number of features because we don’t know which might actually be used, rather than we expect them all to contribute somehow) it may be useful only to add the greatest component of the gradient. This is called Coordinated Gradient Descent because effectively we’re only going along a coordinate at a time. The gradient is thus reduced to it’s maximum component as seen in equation 4.19.

$$\nabla \varepsilon_{\text{Coordinated}} = \max(\nabla \varepsilon)$$

(4.19)

One limitation of CGD in high dimensional data is that it has some trouble getting the important coefficients adjusted right. It will detect them and get them near their perfect value early on, but then because they are close it is not likely to touch them again, favoring noisy alternatives best, and in high dimensional data is easy to find uncorrelated features louder than the required adjust at the important ones.

4.4.3 LARS

The Least Angle Regression method is a good tool from high-dimensional statistics. It’ has "feature selection" like operation, and is computationally as efficient as the forward selection algorithm. It is stable and well correlated features with the response will get their coefficients updated at roughly the same rate.

The algorithm relies on the correlation between each feature vector and the output, adding the best. And as soon as the information achieved by others makes them redundant, removing them.[EHJT04]

The algorithm is as follows:

1. Start with all coefficients $W_i$ equal to zero.

2. Find the predictor $S_a$ most correlated with $r(t)$ by taking the correlation of all features vs response

3. Increase the coefficient $W_a$ in the direction of the sign of its correlation with $r(t)$. Take residuals $\varepsilon = r(t) - r_p(t)$;
STRF Methods

4. Stop when some other predictor $S_b$ has as much correlation as $W_a$.

5. Increase $(W_a, W_b)$ in their joint least squares direction, until some other predictor $S_c$ has as much correlation with the residual $\varepsilon$.

6. If any $S_iW_i$ is now less predictive than a member outside the ensemble, drop it.

7. Continue until: all predictors are in the model, the stopping set has started diverging, or maximum iterations reached.

4.4.4 Thresholded Gradient Descent

This is the state of the art method in terms of results [NPC]. It’s a simple gradient descent (implementing momentum, adaptative steps) where we define two thresholds : $\theta$ and $\theta_G$, respectively the channel threshold and the group threshold.

We start with an empty set of active channels. In the gradient estimation matrix as obtained by equation 4.16 we add values above $\theta \times \max(\nabla)$ to the active set. Furthermore for each time frame (each line in the matrix) we remove those that are below $\theta_G$ of the maximum of the group. This allows for the selection of the most relevant features, and to prune those in each time frame which have sub-average performance (they can be above the global threshold because the frame which generated them was very active and so most components got dilated - it’s a common phenomena at image change moments with high motion, most components trigger highly above average for those frames, and less important ones are "dragged" up).

This allows for the formation of a good basis of features early on with high correlation with the response and makes the early high values of the error function not propagate so much to the noise / poorly correlated features.[GL05]

In our case’s notation this simply translates into:

$$\nabla \varepsilon_{i,j} = 0 \text{ if } \{i, j\} \notin A$$

and:

$$\{i, j\} \in A \text{ if } \nabla \varepsilon_{i,j} \geq \theta \max(\nabla \varepsilon) \land \nabla \varepsilon_{i,j} \geq \theta_G \max(\nabla \varepsilon_i)$$

4.5 Other Methods

This section details non-optimization related techniques used.

4.5.1 KNNC

An different aproach to the steepest gradient was also used: classifiers. If we consider the various time points as samples with an associated feature array, we can train a classifier with it. One natural choice given the high number of samples is the KNN Classifier. KNN’s principle of operation is tremendously simple : a new sample is classified by the majority of the $k$ closest samples.
STRF Methods

more visual explanation can be seen in figure 4.8. The advantages of this classifier over linear combiners is that it can incorporate complete non-linear effects, being dependent only of the quality of the data and the distribution of samples to fall on the zone of interest.

The pixels themselves of the untreated stimulus can be used as features for this classifier, and it still yields interesting enough results. However our problem requires not a single 2 class classification, but a multi class one. Fire counts at each bin of the PSTH can reach 11, so we have a 12 class classification problem. This doesn’t suit KNNC very well, because classes need to be well represented, and equally represented in interclass boundaries for the method to be reliable. That is not the case because high spike counts are much rarer than single spikes or no spikes at all. However, if we consider only whether the neuron fires anything or not at all, then knnc is a formidable classifier, and can yield important information in particular about the moments when the neuron is not expected to fire, and be combined with other methods.

![Figure 4.8: KNN classification - For a new sample the class of the $k$ closest neighbours of that sample determines the classification, majority rules. From http://www.statistics4u.com/fundstat_eng/cc_classif_knn.html](http://www.statistics4u.com/fundstat_eng/cc_classif_knn.html)

4.5.2 Adaptative Threshold Fitting

This very simple technique is a way of estimating the appropriate threshold at which to clip the response to zero. The threshold is simply varied from 0 to the max of the predicted response, and we assess the correlation for each clipping, and chose the threshold which fits best. Often best correlations are obtained without threshold (0) but it contributes to improving the response sometimes.
4.5.3 Resampling

It’s important when performing the classification / optimization to use both a early stopping set: this is a subset of the sample whose data is not used in the gradient, but whose error is measured and determines when we’re over-fitting. Two methods of re-sampling were used: jackknifing and bootstrapping. They are very similar: in jackknifing we remove a segment from the image and optimize without it (using it as early stopping is fine, and actually convenient). In bootstrapping we do the same thing but randomly, by creating a substitution segment and randomly substituting parts of the training signal with subsets of the substitution segment. In both cases we used 5 iterations - 5 fold jacknifing and 5 fold bootstrapping.

The resulting filters can be averaged to yield a strf with very little biasing and over-fitting indeed, and generally improves results upon a completely independent testing segment left out and not used in anything from the beginning of the processing.

Two methods of re-sampling were used: jackknifing and bootstrapping. They are very similar: in jacknifing we remove a segment from the image and optimize without it (using it as early stopping is fine, and actually convenient). In bootstrapping we do the same thing but randomly, by creating a substitution segment and randomly substituting parts of the training signal with subsets of the substitution segment. In both cases we used 5 iterations - 5 fold jacknifing and 5 fold bootstrapping.

4.5.4 NARX Networks

Nonlinear Auto Regressive Exogenous networks are a type of model, which aims to predict a time series, based on itself, and a different time series [Dia08]. The mathematical model is of the following form:

\[ y(t) = F(y(t - i), y(t - 2), y(t - 3), \ldots, y(t - T_1), x(t), x(t - 1), x(t - 2), \ldots, x(t - T_2)) + \varepsilon \] (4.22)

If we look closely this is an analogous form of the model proposed in equation 4.4 (by different notation - this one is the canonical when describing NARX and is only applied to 1D), which means this method fits perfectly into the idea of assigning output memory to the prediction function, and thus modeling the response’s auto correlation. The applications of this model excel in time-series prediction (the response is a time series) from another time series (the predicted response). Thus we can have in our notation:

\[ r_p(t) = F(r_p(t - i), r_p(t - 2), r_p(t - 3), \ldots, r_p(t - U), r(t), r(t - 1), r(t - 2), \ldots, r(t - V)) + \varepsilon \] (4.23)

Where U and V are the maximum delays to be considered in the feedback and exogenous signal respectively. This equation can be used for training the function F, which is a Neural Network with the layout shown in figure 4.9. The topology in question is called open loop, because we allow
for the feedback and exogenous signal to be fed directly to the classifier, because we possess both. This is necessary to simplify training, because as we insert a true feedback circuit the complexity is going to increase exponentially in the backpropagation learning algorithm [mata].

![Diagram of Neural Network Training](image)

Figure 4.9: Matlab’s ntrain tool, showing the network model begin trained - open loop, in the closed loop the output \( y(t) \) is connected to the incoming \( y(t) \)

One thing of mention regarding the use of NARX, is that it may be somewhat stochastic. The values used for the initialization of the neuron weights and bias are randomly generated. We’ve used a constant seed to maintain results consistent, but when using the tools, the small changes of performance from trial to trial are to be expected, if the reader attempts a parallel implementation.

### 4.6 Computational Implementation

The chosen language and environment for this work was Matlab \( @/\) v2013a. For the tests with neural networks and classifiers of which knn emerged as the best, prtools - Pattern Recognition Tools library v 4.2.4 28 was used. It consists of a good abstraction layer for handling machine learning, through a powerful "dataset" general class to describe data, and appropriate tools for training, visualization, validation and deploying created solutions.

The strflab library v1.45 was used, it implements most of the concepts, it borrows heavily from netc statistical models library, and implements most of the functions described in this work. Certain enhancements were made. strflab also contains the 2nd best method-state of the art, which is from a colleague of the library’s developer. The current state of the art code was supplied on request by the involved researcher which is also the creator and maintainer of strflab.

In previous works by the author, Matlab Distributed Computation Server was used to take full advantage of the author’s computers computational resources (see table 4.1). Sadly the evaluation license expired mid-way so there was need to devise a "poor-man’s" work distribution center. For
### Table 4.1: Computational Resources Available for Processing

<table>
<thead>
<tr>
<th>Machine 1 - Panther</th>
<th>Server</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>Intel Core 3570k @ 4.2Ghz</td>
</tr>
<tr>
<td>Memory</td>
<td>32 GB DDR3 1600MHz</td>
</tr>
<tr>
<td>GPGPU</td>
<td>Nvidia GTX 660 Ti @ 1100 MHz 2.6 TFLOPS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Machine 2 - Königstiger</th>
<th>Client</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>Intel Core 3930k @ 4.5GHz</td>
</tr>
<tr>
<td>Memory</td>
<td>32 GB DDR3 1600MHz</td>
</tr>
<tr>
<td>GPGPU</td>
<td>ATI 6970 @ 950 MHz 2.7 TFLOPS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Machine 3 - PzKw III</th>
<th>Client</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>AMD Phenom II 955 @ 4GHz</td>
</tr>
<tr>
<td>Memory</td>
<td>4 GB DDR2 1333MHz</td>
</tr>
<tr>
<td>GPGPU</td>
<td>None - Graphics card doesn’t support</td>
</tr>
</tbody>
</table>

In this case the full data files, scripts, functions and libraries were placed on each of the computers and a small client-server architecture was devised.

#### 4.6.1 Distributed Computing

The main focus of the distributed computing was to increase the speed of parameter sweeping, by far the most time consuming activity of computation in this work. In this light the clients simply connect to the server and listen for coordinates - a given trial’s parameters. The architecture is first come first served, with a client connecting to the server, getting the next coordinates (a set of parameters to test) and then returns the results. It does so endlessly until the server tells it to end the program. The server stores the results relayed by the computing nodes. This approach is certainly not as noble and perfect as MATLAB’s implementation of simulated annealing on a distributed computing platform, but it’s good enough to speed up somewhat the parameter sweeps. The server itself is running two instances of MATLAB, allowing it to compute as well, at the same time as it gathers data - however there seems to be some bugs associated with this approach as there were some random crashes which caused some annoyance.

#### 4.6.2 GPU Optimization

The strflab library is very well done, with immense flexibility and excellent coding practices, and the author is to be commended on its quality, it does not however implement any type of computational acceleration by GPGPU - General Purpose Graphics Processing Unit. Modern graphics cards (and some dedicated computing cards) aside from rendering graphics, have established themselves as titans of computation - a high-level graphics card has nowadays about $400 \times$ the computation resources of a top class CPU. Graphics cards achieve this high performance in terms of FLOPS (floating point operations per second) by using programmable SIMD (single instruction multiple data) arrays which use low intelligence processing units, not unlike CPU cores, but much
STRF Methods

more limited in terms of instruction set and advanced conditionals like branch prediction. A diagram of the two can be seen in figure 4.10. For simple operations the sheer number of cores at a much slower speed and much lower IPC (instructions per clock cycle) can make them incredibly more potent than a typical 4GHz Quad Core CPU.

To leverage this computing power the designed applications must be highly parallel, such is our case. Although the algorithms used for optimization are iterative, the great computational effort at each iteration is the convolution and multiplication of matrices, which are extremely parallel (one can divide the work load of matrix multiplication or convolution by different workers). Matrix multiplication can be parallelized into single multiply and add (MAD) operations over two perpendicular vectors for each element of the resulting matrix, and that task can be done by a different processing core of the graphics card. Likewise convolution can be divided in time frames with each core handling the convolution of a smaller segment of time.

In terms of convolution a speedup ceiling of $205 \times$ is possible in our system, and in terms of matrix multiplication up to $110 \times$ of improvement was achieved in ideal settings. The real speedups of the process are not that great however, still significant but within order of magnitude of the original performance. One particular problem is the movement of data in and out of the GPU (graphics processing unit) to the RAM of the computer, which despite being very fast in terms of bitrate and transmission speeds, has a slow initiation and synchronization delay. The implementation of the heavy workload functions of matricial computation was done trying to minimize as much as possible these fluxes of data, but this, in conjunction with the fact that most of the algorithms still rely on CPU and in particular matlab’s JIT based aproach instead of native code, turn the improvements of said algorithms into $5 \times$ the original version. In very long computations with a small step, long signals, a high number of channels and the use of resampling, the overall efficiency speedup is near the $5 \times$ achieved in the algorithm, yet for lesser intensive trials the other stages of the process dominate the spent time.

The two most critical functions of computation were optimized in this work, being the functions that calculate the gradient and which evaluate the STRF with data error towards the response - respectively linGrad and strErr - this was done by modifying these functions to use GPU optimized subroutines also adapted in this work which perform the calculations -linGradTimeDomain and linFwd.

To assess and improve the performance of the various stages of the algorithm Matlab’s profiler tool was used. This tool discriminates exactly how much time was spent in each line of code, and at which functions and child functions all the way down to Matlab’s core essentials.

Seen in annexes, are the differences between the same exact test run on GPU optimized calculation of the gradient and standard CPU one.

Another, and perhaps more fortuitous performance improvement was discovered in a 2272s trial: it was discovered using the profiler that the Threshold Gradient Descent - not included in strlab, still unpublished work by the library’s author and state of the art current holder, which he made available for the purpose of this work - spent exactly 2000.65s in a poorly placed "find" operation which when substituted by a more convenient direct index mapping reduced to 200s the
Figure 4.10: Illustration of the differences between a CPU and a GPU. The higher core count makes processing on the GPU much faster. The data transmission between both must however be kept to a minimum, as it can slow the process significantly.

Given that the author of the strf lab library has not given express permission to print his work, the code listings of strf lab changed functions are presented in minimal scope. In annex the performance enhancement is shown from a printscreen of profiler.
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Chapter 5

Results

In this chapter the results from the conducted trials are presented. The pValues represent the probability of the given result being a random event between two uncorrelated distributions, and are a measurement of the statistical reliability of the result. The pValue is obtained by the pairwise comparison of the measurements using matlab’s `corr`.

5.1 Measurement of Performance

The chosen criteria by the Neural Prediction Challenge is the correlation between the predicted neural response and the observed one. In particular Pearson’s product-moment correlation coefficient (also known simply as Pearson’s R). It was formulated by Karl Pearson, as the covariance of the two variables over the product of their standard deviations.

$$r = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$  \hspace{1cm} (5.1)

Which on a sample by sample base can be derived by the estimators:

$$r = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}}$$  \hspace{1cm} (5.2)

This measure of performance can be criticized, firstly because it’s insensitive to the mean of the variables, and because it assumes a normal distribution. The first is completely counter purposeful in the given situation, it’s important not only to predict when the spikes occur but as well the magnitude (in terms of number of spikes per time bin). Ignoring the average ranks equally two predictions, with the same shape, but translated. The second, is well known not to be the case, the firing of neurons in a given time frame behaves in a Poisson Process manner [KV01], and thus has a Poisson Distribution of probability density function.
Results

Regardless the authors of the challenge claim, that being that this measure is the most often employed one in the literature and previous tests (and everyone is probably already familiar with it) it is the one to be used.

5.2 Parameter Sweep

The overall free parameters and variations of methods make such a great extent of combinations that parameter sweeps, although extremely thorough in the opinion of the author, and taking advantage of the various speedups and relatively sophisticated domestic computational resources of the author, could not plausibly browse through.

5.2.1 Available Methods

5.2.1.1 Resolution

Starting with methods combinations, one has the opportunity of using the low quality $16 \times 16$ resolution, the full resolution stimuli ($136 \times 136$) or a cropped center of an arbitrary dimension (it was used by other works with agreeable results at dimension $32 \times 32$). The size of the stimuli doesn’t affect the greatest extent of the computation time which is in optimization. Once the Gabor Wavelet is done, the number of channels is the same, yet to obtain the same spatial frequency detail if one used an image which covers a greater part of the image, the density of application of Gabor Filters must be increased, therefore a greater resolution will force the use of more channels for the same detail. With the low performance algorithm STA/STC because the pixels themselves can be used, resolution does affect proportionally computation time, and the high resolution stimuli becomes a heavy computation, yet there appear to be no improvements in results.

Regardless the variations defined for the parameter of resolution are:

1. Downsampled $16 \times 16$ stimulus
2. Cropped $32 \times 32$ center of high resolution stimulus
3. Full $136 \times 136$ stimulus

5.2.1.2 Optimization Method

The Optimization Methods are described in the Methods section, they are by crescent order of performance:

1. Simple Gradient Descent - With Momentum and Norm factoring and adaptive step
2. Coordinate Descent
3. LARS
4. Threshold Gradient Descent
Other methods including direct fit (similar to an averaged KNN) and Neural Networks were attempted but without promising results that warrant being pursued any further. The idea behind SGD and CGD are that if the result could be improved by getting better features they might combine them well enough - if the lacking part of the solution was detail, or amount of channels, they would be able to leverage the information into better results.

### 5.2.1.3 Train Percentage

The amount of data used for validation and training actually seems to influence the results more than anticipated. It was to be expected that the higher the percentage of data used for training the better the result, but with less statistical validity. However, as we increase the percentage of training above 70%, the validation correlation rate starts decreasing. The proposed explanation is that by reducing the validation set, we’re increasing the impact of noise. As we average over more frames the actual qualities of the method statistically compensate.

The sweep constraints are $step = 0.5 \rightarrow 0.9$ at $\Delta step = 0.1$. 

### 5.2.1.4 Delays to use

The information taken from STA/STC shows that most of the response is governed by the first nine frames of the response, thus we propose that the delays screening as follows.

$$U = 7 \rightarrow 15 \text{ with } \Delta U = 2$$

Though several experiments we delays of 20 and 30 were used. 30 being the number of time frames between image change (0.5s) of the stimulus. Nine was indeed found to be the perfect value for all but 3 of the neurons in which it was 7.

### 5.2.2 NARX delays

The delays of the response to consider in the NARX network. Too much and it becomes noisy and hampers learning, too little and there is no sufficient information for a basis of decision. The other delay ($U$ in our nomenclature regarding NARX) is set to 2, the algorithm only considers current prevision and past prevision, that way, information on the derivative is present to the classifier, allowing it to know not only the predicted fire rate, but how it’s varying. This allows to model things like pre-emptive fire. If the response is rising rapidly the network tends to increase the peaks quicker, allowing for a good piece of non-linearity closely related to biological behavior.

### 5.2.3 Thresholds

Both the global and the group threshold used in the Threshold Gradient Descent are very data dependent.

The search window used was: $\theta, \theta_G = 0 \rightarrow 1$ with $\Delta \theta, \Delta \theta_G = 0.1$
5.2.4 Gabor Parameters

The Gabor Wavelets consist of the various parameters seen in Methods, in the Gabor section. The sweeping parameters are defined:

1. Direction Divisions: Given that neurons can vary 10° a good sweep is from 6 to 18 (30° to 10° spacing). These are spread over a 180°.

2. Frequency Divisions: 3-8, as an inverse function of the overall window size, more than 8 would result in 2 pixels.

3. Velocity Divisions: 2:5, given that movement occurs within a five frame window, more than this would be extraordinarily correlated and would be pointless.

4. Temporal Size: Size of wavelet in terms of time. 5 to 11 for reasons given above.

5. Grid Spread: Various gaussians are applied in various point of space in a grid. In relation to the standard deviation of the gaussian of the wavelet. Suggested 2.5, searched until 1.5 with 0.5 intervals.

5.2.5 Overall Sweeping Interpolation

The resulting field of parameters consists of 48384 combinations. This number is completely prohibitive, and the full extent of the parameter sweep was not performed. Even when tried, the simple fact that there are computer crashes, and the long term stability of the program is not perfect, made the complete sweep impossible. What was done were some partial coverages of smaller zones among the parameter field, and by inspection of performance values, progressively constraining until the ideal values were found. The various interrupted trials were manually patched to form a reasonable knowledge of performance, which allowed an heuristic approach unto these final results, yet, if there are very localized foci of parameter combinations with better results, it is possible they have not been found. It should be referred that NARX parameters, resampling and optimization were optimized separately. Given that they are expected to exhibit some independence.

5.3 Results

In the following section, typical results from the action of the proposed methods are shown. And a comparative of the application of the proposed innovations and the current state of the art is shown, as well as a comparative table of the found correlations, for the best parameters for each method (the maximum, not the mean).
5.3.1 Wavelet Transform

The diversity of parameters can make a graph of the evolution of them all difficult to comprehend, essentially in each line is a channel, and these channels are the convolution of a filter of Gabor kernels which describe the image (becoming a form of wavelet transform) - seen in figure 5.1 is a clipped subspace of the stimulus after being decomposed into wavelet channels. It’s easy to see the problem of dimensionality and computation cost when we consider that we obtain that from a $16 \times 16$ stimulus across 15s in time.

![Figure 5.1: 3D graph of 1000 channels over 15s time period (900 samples), notice the periodic spikes across most channels. When there is motion in the image most channels rise, this makes the channels extremely correlated amongst themselves, and causes to consider if some dimensionality reduction by joining similar channels wouldn’t be positive for something else apart from performance.](image)

5.3.2 Adaptive Threshold

The curve of performance for a threshold array is always computed, and the maximum is chosen as the final response. Typical curve shown in fig. 5.2.

5.3.3 Legacy STA / STC

With correlation values in the 0 to 0.25 range, STA STC shows it’s obsolescence. KNN does a better job for a image domain algorithm in terms of results. Yet this algorithm was an absolute must for it’s the historical way (since the 50’s) that the visual cortex has been analyzed. An example of training can be seen in figure 5.3.

Yet there are some merits to it, in terms of allowing the visualization of the field of view responsible for the activation of the neuron STA/STC still gives practical results as seen in figure 5.4.
Results

Figure 5.2

Correlation vs Threshold

(20Gb, 10 time lags, gradient descent, 1396 channels, 70% training, 30% validation)
Figure 5.3: The predicted response from the STA/STC predictor - Notice the little correlation present. However, some spaces of inhibition and some stimuli peaks do align.

Figure 5.4: On the left it’s visible in blue the regions which trigger the V1 neuron response, and in red the regions which inhibit it. It’s related to the graph on the right, showing how the prediction by the magnitude of the STA/STC at given delay.
5.3.4 Gradient Descent

Gradient Descent results were somewhat disappointing. Expectations weren’t high to begin with, but with large numbers of features it was somewhat expected that it’s simplicity and greater speed would actually turn up some decent results.

Figure 5.5: Typical training graph of a Simple Gradient Descent - on the left, the resulting weights - on the center: the training set error, on the right: the early stop set error - As soon as the error starts to increase in the early stop set, it means that the algorithm is overfitting in the training set, and the solution is no longer improving for an unbiased sample.

It does converge pretty fast, and is an excellent pivot point to anchor comparisons between parameters.

5.3.5 Coordinated Gradient Descent

As expected CGD converges slowly but steadily onto solutions more perfect than simple GD. This comes for it’s method of only editing one weight a time. Given the high dimensionality of the problem, with thousands of channels it’s no surprise this algorithm takes thousands of iterations to reach completion. Typical convergence of CGD is shown in figure 5.6.

5.3.6 LARS

LARS path towards the solution looks a little more ragged than most, yet it’s remarkable how it converges in so little iterations. This is the result of using the best correlation rather than the gradient descent estimation as source of information on which parameters to update. This comes at the cost that each LARS iteration is much more heavy than the corresponding gradient descent approaches, and takes much more time. The process is seen in figure 5.7.
Results

Figure 5.6: It can be seen the difference of emphasis given in the Coordinate Descent, by the relative scarcity of the Weights, and it’s also visible that this optimization takes a lot of iterations to reach it’s minimum.

Figure 5.7: The correlation approach makes the LARS algorithm show a weight array not unlike the Coordinate Descent, with few weights but meaningful. It converges rapidly.
Results

5.3.7 Thresholded Gradient Descent

The training with the Thresholded Gradient Descent method is quick (iterations per second), fast (reaches minima in little iterations), and effective (minima is lower than other methods). As seen in figure 5.8.

Figure 5.8: Optimization using Thresholded Gradient Descent. Notice the quick progression and little drop of performance on the early stopping set once the minima has been reached. The algorithm quickly activates most of it’s weights, here in the end result all are activated, but the noisy components being activated late manage to now be overexpressed, and the weights are less noisy than an equivalent Simple Gradient Descent.

A comparison of other methods to this can be seen on table 5.2. And the various prediction responses can be seen in figure 5.9.

Figure 5.9: Shown in this graph are the ampliations of a given part of the validation response. In green we see the peak enhancing power of the NARX. In purple we can see the KNN profile, predicting only whether or not a zone is a firing zone.
Table 5.1: KNN’s response confusion matrix

<table>
<thead>
<tr>
<th>True Labels</th>
<th>Estimated Labels</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>603</td>
<td>329</td>
</tr>
<tr>
<td>1</td>
<td>344</td>
<td>512</td>
</tr>
<tr>
<td>Totals</td>
<td>947</td>
<td>843</td>
</tr>
</tbody>
</table>

5.3.8 KNN

The performance values of KNN are not great, averaging 0.35 correlation, however the correlation is exceptionally high because for the given testing algorithm it’s as bad to miss from 0 to 1 as from 1 to 7. Yet we know that it carries predicting power distinguishing fire zones with effectiveness as seen in table 5.1. The discriminating power can also be seen in figure 5.10.

Figure 5.10: Shown in this graph are the original response in blue, vs the predicted by the KNN classifier in red. This high quality temporal information can be of great use in future models.
Results

Table 5.2: Comparison of Relative Performance among methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Relative Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>STA/STC</td>
<td>34%</td>
</tr>
<tr>
<td>SGD</td>
<td>74%</td>
</tr>
<tr>
<td>CGD</td>
<td>79%</td>
</tr>
<tr>
<td>LARS</td>
<td>89%</td>
</tr>
<tr>
<td>TGD</td>
<td>100%</td>
</tr>
</tbody>
</table>

5.4 Final Results

The results using the best parameters, and including the contribution, Threshold Fitting, NARX processing, Thresholded Gradient Descent. The best parameters are the best overall parameters, there was no adaptation for each individual neuron - for the idea is to establish a solid functional algorithm, not waste the user’s time performing very large time consuming parameters sweeps.[ZD13]

Parameters for maximization: \( \theta_G = 0.6 \quad \theta = 0.8 \quad Dir_{Div} = 16 \quad V_{Div} = 3 \quad F_{Div} = 4 \quad K = 7 \quad \gamma = 3E^{-4} \quad Res = 16 \times 16 \quad U = 9 \quad V = 9 \)

The results can be seen in table 5.3. The very low p Values should be perceived in the light that there are many samples (15000 mean). The results for neuron “r0220A” were excluded from the computation of the mean improvement result. It’s not only the doubtful p-value, which could have been artificially increased by using cross validation, it’s the fact that by inspection there looks to be only incidental fitting in both algorithms. There was no actual improvement, and the superior values are but random coincidence, as all algorithms ever tried in the neural prediction challenge fail to predict at all (like ours) with any discriminant power for this neuron. Therefor, the author feels it’s more honest to exclude the extra 1% improvement this part of the dataset would provide.
Table 5.3: Final Results from the best solution found

<table>
<thead>
<tr>
<th>Neuron</th>
<th>Correlation value</th>
<th>State of the Art Correlation</th>
<th>Improvement</th>
<th>p Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>r0206B</td>
<td>0.6694</td>
<td>0.6631</td>
<td>1%</td>
<td>0.000</td>
</tr>
<tr>
<td>r0208D</td>
<td>0.4032</td>
<td>0.4062</td>
<td>-1%</td>
<td>0.001</td>
</tr>
<tr>
<td>r0210A</td>
<td>0.5049</td>
<td>0.4902</td>
<td>3%</td>
<td>0.003</td>
</tr>
<tr>
<td>r0211A</td>
<td>0.4728</td>
<td>0.4669</td>
<td>1%</td>
<td>0.001</td>
</tr>
<tr>
<td>r0212B</td>
<td>0.4642</td>
<td>0.4576</td>
<td>1%</td>
<td>0.003</td>
</tr>
<tr>
<td>r0217B</td>
<td>0.4091</td>
<td>0.4102</td>
<td>-3%</td>
<td>0.001</td>
</tr>
<tr>
<td>r0219B</td>
<td>0.5249</td>
<td>0.5050</td>
<td>4%</td>
<td>0.001</td>
</tr>
<tr>
<td>r0220A</td>
<td>0.1654</td>
<td>0.1428</td>
<td>16%</td>
<td>0.1204</td>
</tr>
<tr>
<td>r0221A</td>
<td>0.6164</td>
<td>0.6037</td>
<td>3%</td>
<td>0.000</td>
</tr>
<tr>
<td>r0222A</td>
<td>0.4563</td>
<td>0.4694</td>
<td>0%</td>
<td>0.002</td>
</tr>
<tr>
<td>r0223A</td>
<td>0.4918</td>
<td>0.4411</td>
<td>3%</td>
<td>0.001</td>
</tr>
<tr>
<td>r0225C</td>
<td>0.4156</td>
<td>0.4127</td>
<td>1%</td>
<td>0.001</td>
</tr>
<tr>
<td>Average</td>
<td>0.4662</td>
<td>0.4557</td>
<td>2%*</td>
<td></td>
</tr>
</tbody>
</table>

*-Doesn’t include r0220A as it’s clearly ineffective there
Results
Chapter 6

Conclusions and Future Work

The outcome of this work is definitively a most positive one. The results explicit a small but consistent improvement over previous methods. This improvement is only natural as it relies on what may be derogatorily called "post-processing tricks" and the already state of the art method, yet, given the evolution that the field of neural prediction has taken over the last few years, these tricks seem appropriate. Another thing that makes the novel methods suggested on this work more worthy, is that the field of regression, optimization and statistical analyzes in high dimensionality is very well established with a great many types of algorithms and statistics dominating the field. Innovating on the fundamentals of neural prediction thus becomes hard. The major innovation in the author’s perspective in this is the usage of NARX networks, because they appeal to the idea that the current model which does not include previous response memory is wrong. That is the way to perfect the current model, to find the intuition on the working of the brain, and transpose it to the needs of better models.

Another success was the proposed successful speedup of the methods by acceleration on the GPU. From a proof of concept a sizable improvement in speed was achieved, which including the corrected performance drops in the state of the art code amounts to $12 \times$ the original performance, which is in itself a result to be satisfied about.

It should be noted however, that the performance values described for performance by the author of the current state of the art method couldn’t be achieved using the code provided for the effect. The results of the proposed methods are compared to the state of the art as evaluated by the author of this work.

6.1 Additional Work

Across the time span that this dissertation took, many different technical ideas were thought and tested, like Massive Neural Networks, Variable Power Non Linearity, Support Vector Machine classification, Gabor Channel description through polynomial coefficients, which either far
Conclusions and Future Work

fetched, computationally impossible - more frequent poor performers, didn’t get detailed in this work, or didn’t get brought to fruition because of computational complexity.

One idea which failed, on which there was good basis for believing there would results was the neural network approach.

The idea behind massive neural networks (networks that cover the entire range of the input video over a given time lag, and have the number of layers necessary to support such a large input structure, about the same number of layers as inputs) was that since they are akin to biological networks, there would be somehow the learning algorithm that would make them approach the biological network somehow and give high quality responses. Even if the LNP model was a true perfectly accurate descriptor of neural activity to visual stimuli or the state of the art the best is ever going to be possible, the neural network would converge to it with sufficient training, and model the gabor wavelets from the direct pixel input, combine them, apply thresholding, simulate auto-recursivity through delays on some layers, and eventually come to a solution as good as the state of the art at least.

The truth is that networks of those size crashed the author’s computational resources, demanding far too much memory. Initially the computational price was underestimated in face of the abundance of resources present, as well as the possibility of using a grid computing for the effect of training the network. It was not the complexity of running the network that undermined the effort, the training algorithms did. Either they were unfit for very large networks, or too complex. Even in network with only 6 layers (about 200 times less than the projected) training of such a large input network became prohibitive in time, with trains lasting for hours for results not worthy of mentioning, similar to STA’s.

6.1.1 Further Computational Optimization

The speed gains were highly satisfactory, yet, if dedicated kernels implemented in native language were used for GPGPU acceleration, implementing the entire process on the GPU, the speed gains would be as high as 50 fold the ones obtained. By creating a complete dedicated algorithm implemented with attention to the full use of the graphics hardware, it is possible to contemplate very fast parameter sweeps which may lead to better results, as more space can be sampled. This optimization was not attempted as the "mood" for this work was mostly exploratory in terms of solutions, quickly changing small things in every iteration - and the effort of implementing this algorithms natively on the hardware would have been rather final and were not attempted. Now that we’ve established well what the best algorithms are, a robust, down to earth implementation in the hardware is an absolute must.

6.2 Current State of Neural Coding

As a general observation, neural prediction is extremely unreliable, neurons are far too complex and unpredictable (in the stochastic sense) to accurately predict from the stimulus alone. The existing methods though highly specialized and optimized fail to do more than associate high rate
Conclusions and Future Work

of fire zones with a particular set of visual event families which trigger and inhibit. Indeed if we look at the graphs of predicted responses vs. actual response, we notice that the prediction is only meaningful at certain regular intervals. This is due to the image changes that occur every half second, and during that change, it’s probable and expected for there to be a neural response.

Some neurons were harder than others, from the implausibly silent number eight 220A, which appears to have little to no correlation with the stimulus to trigger happy R206B which kindly fires at nearly everything that moves. This makes the challenge even more complex, because to find a good general method or algorithm it must be universally applicable. One approach which in hindsight was something that ought to have been explored was the how to estimate good parameters quickly and from the data. It’s perfectly adequate to expend half a second computing the best threshold for the adaptive threshold method, but if we tried to vary the number of samples it would involve complete trials, and lots of them, extend that over a couple other parameters and one has got months of computation ahead in most systems. Yet the curves from STA/STC could be used to indicate the best values instead of using a fixed one. That is only an example of parameter estimation without requiring sampling the parameter space with large numbers of trials. There are a large number of early statistics about the stimuli and the response and there is some intuition in the author that it may lead to more "adaptive" things, which might increase the performance.

Neuron 220A was a puzzle indeed, it fired very little, and there was nothing on the STA/STC to indicate what region in particular triggered it. It’s firings were unsynchronized with the motion in the image unlike all other neurons. There is the possibility that this neuron, and others, have external things apart from the visual stimuli driving their inputs. Consider as an example the possibility that this neuron is somehow involved in the mechanism of the cogniscence and feedback of blinking. We have little knowledge of how the three neuron layers of the retina work, how can we establish beyond doubt that the V1 cortex is comprised of the typical complex and simple V1’s and be done with it? Detailed analysis of this one in particular leads the author to conclude that it might be only indirectly related to vision, or be part of a chain of processing yet misunderstood.

6.3 Future Improvement

One method which sparked my hope for actual meaningful predictions everywhere, not just on the high ‘momentum’ spaces of the film, is the KNN classifier. The highly ‘nervous’ behavior” it exhibits indicates that it can do more than just a global prediction of activity, but can go as far as detail the inter "momentum" zones, where the firings are sparser. Sadly it’s correlation is rather disappointing as seen, but the confusion matrices indicate something that feels entirely different. It predicts whether the neuron fires or not at all with statistically relevancy. The statistic simplicity of the KNN make it a well suited competitor for the "black box" approach - if we consider we know nothing about the visual system and only consider the provided data, the only significant thing we have is what happens when a stimulus is given: then neuron fire rates go up, down, become zero, it’s this information by experience that the KNN system so convincently conveys: let’s find what happened in the training set, when a stimulus like this occurred and it will probably behave the
same way. The approach was limited by the abundance of data, some neurons were perfect for KNNc with a good distribution of fire rates and equally abundant classes, whilst others provided too much biasing in the distribution of class. If most samples say the neuron won’t fire, it’s only natural most predictions also say it won’t fire.

Yet another path which may have some fundament for pursuit is the usage of evolutionary algorithms. If changing only the parameters doesn’t seem to be improving performance further, improving the algorithm with automatic adaptation can also be worth exploring, yet the high dimensionality kills it from nearly the very beginning, as evolutionary algorithms excel at extreme non-linearity and semi-supervised learning, yet if there is some trick to adapt this class of solutions to this problem, it will likely bridge the missing gap.

An alternative hypothesis is that the data itself doesn’t contain any more useful information than what is already shown by the current methods, but although the likelihood that some neurons are indeed limited, for the general case the sampling rate and the repeatability before similar stimuli shows that most of the behavior is deterministic and can likely be predicted further.
Appendix A

Matlab Code Listing

A.1 Main algorithm - unicase

This is the main algorithm, expressed with plenty of options that let the user control the usage of the implemented functionalities. This particular version is the standalone, and executes a full processing cycle from data loading do displaying intermediate and final results, with or without figures. Doesn’t include STA/STC functionality - it was never included in this main version, as it was built for testing the more advanced options. The full code directory which includes the routines for visualization, optimized code, the needed libraries, and some other specific code fragments (such as the STA/STC trials) will be made available at http://paginas.fe.up.pt/~bio08062, the inclusion of this particular piece of code is to allow the understanding of exactly what was done, and how was it done.

```matlab
% Wavelet Transform Trial
% ------------------------------------------------------------------------%
% This matlab script loads the data from the neural Prediction Challenge, 
% and applies a gabor wavelet transform - read transforms the pixel inputs 
% into channels corresponding to the responses to a filter databank 
% an actual video compression technique).
% The gabor filters are 3 dimensional (2D image x time frame) and correspond 
% to a series of spatial frequencies and velocities in time. The said filter 
% are then applied in a mesh grid in different points of the original 
% image, and convoluted. 
% Filter options are defined in the parameters section. 
% The algorithm then tries to obtain a linear combination of these filters 
% that best fits the data. 
% A post processing stage involving Threshold adaptation, NARX neural 
% networks and KNN classifers is also present. Parameters for which are 
% provided in the options.
```
Matlab Code Listing

% Options for oversampling, jacknifing and validation are also present.
% Uses functions from the strflab toolbox and prtools toolbox
% Original Work by Nuno Albuquerque Sousa - in the context of Master's
% Thesis in Biomedical Engineering
% August 2013 - Ver 2
%--------------------------------------------------------------------------%
%%
%Loading Data (Except full stims)
clear;
clc;
% Loads all data
global globDat;
data_path = 'v1_nvm_data';
run([data_path '\cellinfo.m']);

strflabDir = get_function_dir('get_function_dir');
if isempty(strflabDir)
    error('Cannot find strflab directory!');
end
addpath(genpath(strflabDir))

% Initializes the global initial data holders
Cstim ={};
Cfstim={};
Cvstim={};
Cresp ={};
CrespRaw={};
% Reads all signals from every file and places them in their respective
% cells
for i=celldata;
    t=load([data_path '\i.datafile','stim', 'resp', 'vstim', 'resp_raw']);
    Cstim{end+1}=t.stim;
    Cresp{end+1}=t.resp;
    Cvstim{end+1}=t.vstim;
    CrespRaw{end+1}=t.resp_raw;
    disp (['Loaded ' i.cellid]);
end

%%
close all;
clear params, globDat;
tic;

%--------------------------------------------------------------------------%
% HDmode=0; % 0- use low res 16x16 image , 1 - use high res 132x132
% image , 2- use cropped section from the center of the
% image

cropx=32; % Cropping parameters for the window to be taken from
% the center of the high res image

cropy=32;

currentNeuron = 2; % The correspomding neuron as in the list celldata

% The ideal delay would be small, but given the fact
delayWindow = 0:9; % The number of frames used to create each sample

pTrain=0.7; % Percentage of the overall samples to use in training
cutnan=0; % Cut Not a Number (NaN) responses (trimmed or invalid
% frams from aquisition from the stimulus as well
% (might provoke incongruences in gabor filtering)
dispFigures=-1; % Wether or not to display figures 1: Display
% figures, 0: Don't display figures only console output
% -1: Don't display anything but fianl result

resamplingMode=0; % Resampling Mode - Option regarding the use of resampling
% 0 - Don’t use resampling
% 1 - Use Bootstraping
% 2 - Use Jackknifing
optMethod=3; %Optimization method
% 0 - trnGrad - Simple
% 1 - trnGrad - Coordinated Descent
% 2 - LARS
% 3 - Thresholded Gradient Descent
classifierMode=0; % What information to use to train the classifier
% 0 - Use the pixels across a timeframe
% 1 - Use the wavelet channels used by the optimization
% step
% 2 - Something Entirelly different (code holder)

% View preprocWavelets 3D for information on the gabor filter wavelet
% decomposition parameters.

params = preprocWavelets3d;
params.dirdivisions=12;
params.fdivisions=4;
params.tsize=9;
params.phasemode=4;
params.veldivisions=3;
Matlab Code Listing

```matlab
params.normalize=1;
params.local_dc=1;
params.std_step=2.5;
params.f_step_log = 0;

% set the options for the Optimization step
switch(optMethod)
    case 0
        options = trnGradDesc;
        options.nDispTruncate = 0;
        options.stepSize = 3e-04;
        options.adaptative=1;
    case 1
        options = trnGradDesc;
        options.coorDesc=1;
        options.nDispTruncate = 0;
        options.stepSize = 3e-02;
        options.adaptative=1;
    case 2
        options = trnLARS;

        %LARS specific Options
        options.maxChan=150; %LARS adds and removes channels as it goes
        % max channels is necessary to keep him from "diverging"
    case 3
        options = trnThreshGroupGradDescStepShrink;

        %Threshold Descent specific Options
        options.threshold=0.8; %Sparsity of the Wavelets used
        options.thresholdGroup = 0.2; % Sparsity of delays used for wavelet
end

options.earlyStop = 1;

switch(dispFigures)
    case 1
        options.display = -5;
    case 0
        options.display = 5;
    case -1
        options.display = 0;
end

switch(resamplingMode)
    case 1
end
```

optionsBs.resampBootstrap;  
optionsBs.optimOpt=options;  
optionsBs.testFrac=0.20;  
optionsBs.nResamp=5;  

  case 2  
    optionsBs = resampJackknife;  
    optionsBs.useHoldOut = 1;  
    optionsBs.nResamp = 5;  
    optionsBs.jackFrac = 0.1;  
    $Use$ threshold gradient descent  
    optionsBs.optimOpt = options;  
end  

%---------------------------End-Options-----------------------------------%  

%Load appropriate data  
switch HDmode  
  case 0  
    rawStim=Cstim{currentNeuron};  
    resx=16; % Used resolutions  
    resy=16;  
  case 1  
    % It’s not feasible to load the entire array to memory as done in  
    % the low resolution case, given the high data volume and RAM usage  
    % considerations. Full resolution is done loading one at a time.  
    rawStim=loadimfile([data_path '\ celdata(currentNeuron).fullstimfile]};  
    resx=132;  
    resy=132;  
  case 2  
    rawStim=loadimfile([data_path '\ celdata(currentNeuron).fullstimfile]};  
    rawStim=rawStim(floor((end-cropx)/2)+1:floor((end+cropx)/2),floor((end-cropy)/2)+1:floor((end+cropy)/2,:)};  
    resx= cropx;  
    resy= cropy;  
  otherwise  
    error('Invalid Resolution Option');  
end  

resp=Cresp{currentNeuron};  
pVal=0.7;  
rawStim = single(rawStim);  
nnanIdx=find(~isnan(resp));  
resp=resp(nnanIdx);  
if(cutnan==-1)
Matlab Code Listing

```
rawStim=rawStim(:,:,nnanIdx);
   
if(cutnan~=1)
    stim=stim(nnanIdx,:);
end

% Creates the gabor filters and convolves them with the stimulus
[stim,params] = preprocWavelets3d(rawStim, params);

if (cutnan~=1)
    stim=stim(nnanIdx,:);
end

% Separate a validation set and a training+early stopping set
sepIdx = floor(length(resp)*pTrain);
stimVal = stim(sepIdx+1:end,:);
respVal = resp(sepIdx+1:end);

% Use a window - has been disabled by poor results
% resp = conv(resp,gausswin(5)/5,'same');

% KNN Classifier Section
% ------------------------------------------------------------------------%
delayWindowC = 0:10;
nSamples=size(rawStim,3);
switch classifierMode
    case 0
        data=zeros([resx*resy*length(delayWindowC) nSamples]); % Allocate memory
        for i=1:nSamples
            data(:,i)=reshape(rawStim(:,,(i-delayWindowC-1).*((i-delayWindowC)>0) +1),[resx*resy*length(delayWindowC) 1]);
        end

        multipleSpikeMode=0;
        switch multipleSpikeMode
            case 0
                labels=double(resp>0);
            case 1
                labels=double(resp>0);
                for k=find(resp>1);
                    for m=1:resp(k)
                        data(:,end+1)=data(:,k);
                        resp(end+1)=resp(k);
                    end
```
end

case 2
    labels=floor(resp);
end

data=data(:,nnanIdx);

% Transposes the data - Prtools expects m rows by k columns, in which m is
% the number of samples and k the number of channels or characteristics per
% sample. The opposite was used in the strflab convention.
data=data';

    case 1
    data=zeros([nSamples size(stim,2)*length(delayWindowC)]);
    for i=1:nSamples
        data(i,:)=reshape(stim((i-delayWindowC-1).*((i-delayWindowC)>0)+1,:),
            size(stim,2)*length(delayWindowC) 1));
    end
    multipleSpikeMode=0;
    switch multipleSpikeMode
        case 0
            labels=double(resp>0);
        case 1
            labels=double(resp>0);
            for k=find(resp>1);
                for m=1:resp(k)
                    data(:,end+1)=data(:,k);
                    resp(end+1)=resp(k);
                end
            end
        case 2
            labels=floor(resp);
        end
    end

sepIdx=floor(pTrain*length(labels));
trainDS = dataset(data(1:sepIdx,:),labels(1:sepIdx));
testDS = dataset(data(sepIdx:end,:),labels(sepIdx:end));

% Train the classifier
W=knnc(trainDS,3);

V=testDS*W;
confmat(V);
tresp=V*labeld ;
if(dispFigures~=0)
    figure;
    plot(resp); hold on; hold on; plot(length(resp)-length(tresp)+1:length(resp),
        tresp,'r');
    corr(resp(end-length(tresp)+1:end),tresp)
% End of KNN classifier
stim=stim(1:sepIdx,:);
resp=resp(1:sepIdx);
% Globalize stimulus and response
strfData(stim,resp)
% Create new linear strf
strf = linInit(size(stim,2), delayWindow);
strf.b1 = mean(resp);
strf.params = params;
if(~resamplingMode)
    % Selects a fraction of the training data to be used for actual training
    trainingIdx = [1:floor(.8*globDat.nSample)];
    % And the remaining 20% for the stopping set
    stoppingIdx = [floor(.8*globDat.nSample)+1:globDat.nSample];
    %Trains the strf
    strfTrained=strfOpt(strf,trainingIdx,options,stoppingIdx);
else
    trainingIdx = [1:globDat.nSample];
    strfTrained_tmp=strfOpt(strf,trainingIdx,optionsBs);
    strfTrained = strfTrained_tmp(1);
    strfTrained.b1 = mean(cat(2, strfTrained_tmp.b1), 2);
    strfTrained.w1 = mean(cat(4, strfTrained_tmp.w1), 4);
end

[strfTrained,predresp]=strfFwd(strfTrained,1:globDat.nSample);
nnanIdx=find(~isnan(predresp));
netInput=predresp(nnanIdx);
netTarget=resp(nnanIdx);

% NARX network section
inputSeries = tonndata(netInput,false,false);
targetSeries = tonndata(netTarget,false,false);
inputDelays = 1:2;
feedbackDelays = 1:12;
hiddenLayerSize = 12;
net = narxnet(inputDelays,feedbackDelays,hiddenLayerSize);
[inputStates,layerStates,targets] = preparets(net,inputSeries,{},targetSeries);
Matlab Code Listing

```matlab
359 net.divideParam.trainRatio = 70/100;
360 net.divideParam.valRatio = 15/100;
361 net.divideParam.testRatio = 15/100;
362
363 % Train the Network
364 [net,tr] = train(net,inputs,targets,inputStates,layerStates);
365
366 % Test the Network
367 outputs = net(inputs,inputStates,layerStates);
368 $errors = gsubtract(targets,outputs);
369 performance = perform(net,targets,outputs)
370
371 % View the Network
372 $view(net)
373 $figure, plotperform(tr)
374
375 netc = closeloop(net);
376 netc.name = [net.name ' - Closed Loop'];
377 $view(netc)
378 [xc,xic,aic,tc] = preparets(netc,inputSeries,{},targetSeries);
379 yc = netc(xc,xic,aic);
380 closedLoopPerformance = perform(netc,tc,yc)
381
382 nets = removedelay(net);
383 nets.name = [net.name ' - Predict One Step Ahead'];
384 $view(nets)
385 [xs,xis,ais,ts] = preparets(nets,inputSeries,{},targetSeries);
386 ys = nets(xs,xis,ais);
387 earlyPredictPerformance = perform(nets,ts,ys)
388
389 % END NARX section --------------------------------------------------------%
390
391 %-----------------------------Testing-------------------------------------%
392 % Now let's globalization our validation data and use the trained
393 % STRF to predict the response
394 strfData(stimVal,respVal)
395 testingIdx = [1:globDat.nSample];
396 [strfTrained,predresp]=strfFwd(strfTrained,testingIdx);
397
398 % Let’s remove any NaN’s from the prediction and look at the
399 % correlation between the prediciton and actual response
400 nonnanidx = find(~isnan(predresp));
401 predresp=predresp(nonnanidx);
402 respVal=respVal(nonnanidx);
403 $Apply the predicted response to the NARX network
404 inputSeries=tonndata(predresp,false,false);
405 targetSeries=tonndata(respVal,false,false);
406
407 [xc,xic,aic,tc] = preparets(netc,inputSeries,{},targetSeries);
408 yc = netc(xc,xic,aic);
409 $closedLoopPerformance = perform(netc,tc,yc);
```
NARXpredresp=cell2mat(yc);
NARXpredresp=[zeros([1 12]) NARXpredresp]’; %Delay and transpose

j=0;
for k=0:0.05:max(predresp);
    j=j+1;
    % Passing it through a linear threshold
    threshold = k;
    predrespT=(predresp-k).*(predresp>threshold);
    predcorr1(j) = corr(predrespT,respVal);
    Ck(j)=k;
end

% Display the maximum correlation found and at which threshold
[maxPred, maximizingPred]=max(predcorr1);
disp(['Neuron : ' cellids{currentNeuron}]);
disp(['Maximum found correlation of: ' num2str(maxPred) ...'
    found at threshold : ' num2str(Ck(maximizingPred))]);

for k=0:0.05:max(predresp);
    j=j+1;
    % Passing it through a linear threshold
    threshold = k;
    NARXpredrespT=(NARXpredresp-k).*(NARXpredresp>threshold);
    predcorr1N(j) = corr(NARXpredrespT,respVal);
    CkN(j)=k;
end

% Note: corr values must be square rooted when comparing with literature,
% for corr returns R^2 instead of R. All indicated values in the written
% work are in R, not in R^2.

% Display the maximum correlation found and at which threshold
[maxPred, maximizingPred]=max(predcorr1N);
disp(['Maximum found correlation of: ' num2str(maxPred) ...'
    found at threshold : ' num2str(CkN(maximizingPred))]);
toc;

if (dispFigures~=0)
    figure;
    plot(Ck,predcorr1);
    tresp2=tresp(32:end);
    % Now let’s graph the predicion and response to compare:
    figure; plot(respVal, ‘r’); hold on; plot(predresp, ‘b’); plot(NARXpredresp,’g’
    ); plot(tresp2,’m’);
    figure; plot(tresp2.*NARXpredresp); hold on; plot(respVal,’r’);
    % Then a scatter plot of predicted response vs original response (it’s from
Matlab Code Listing

456  $ this graph that the correlation R is derived
457      figure; plot(conv(respVal,ones([30 1]),'same'),predresp,'b.);
458      hold on; plot(conv(respVal,ones([30 1])+0.3,'same'),NARXpredresp,'r.); 
459
460  end
461  $-----------------------------------------End Testing----------------------------------$

A.2 Speedup - Profiler
Figure A.1: The execution Profiler of our algorithm (the main function is "trial" which is much like like mainStandalon, only in the form of function, so as to be called by a parameter sweeper with different settings. The important functions here being linFwd and linGradTimeDomain.

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Calls</th>
<th>Total Time</th>
<th>Self Time*</th>
<th>Total Time Plot (dark band = self time)</th>
</tr>
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<tbody>
<tr>
<td>trial</td>
<td>1</td>
<td>42.044 s</td>
<td>0.681 s</td>
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<tr>
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<td>2</td>
<td>20.421 s</td>
<td>0.056 s</td>
<td></td>
</tr>
<tr>
<td>striOpt</td>
<td>1</td>
<td>20.420 s</td>
<td>0.000 s</td>
<td></td>
</tr>
<tr>
<td>linFwd</td>
<td>286</td>
<td>14.673 s</td>
<td>14.673 s</td>
<td></td>
</tr>
<tr>
<td>striErr</td>
<td>284</td>
<td>14.611 s</td>
<td>0.022 s</td>
<td></td>
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<tr>
<td>linErr</td>
<td>284</td>
<td>14.589 s</td>
<td>0.034 s</td>
<td></td>
</tr>
<tr>
<td>preprocWavelets3d</td>
<td>2</td>
<td>8.598 s</td>
<td>2.703 s</td>
<td></td>
</tr>
<tr>
<td>data</td>
<td>116</td>
<td>7.258 s</td>
<td>0.006 s</td>
<td></td>
</tr>
<tr>
<td>data-&gt;strict_format</td>
<td>96</td>
<td>5.824 s</td>
<td>5.824 s</td>
<td></td>
</tr>
<tr>
<td>striGrad</td>
<td>141</td>
<td>5.684 s</td>
<td>0.006 s</td>
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<tr>
<td>linGrad</td>
<td>141</td>
<td>5.679 s</td>
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<td>5.671 s</td>
<td>5.671 s</td>
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<td>4.569 s</td>
<td>4.523 s</td>
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<tr>
<td>prepares</td>
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<td>0.008 s</td>
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<td>0.036 s</td>
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<tr>
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<td>2.770 s</td>
<td>0.001 s</td>
<td></td>
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<tr>
<td>network_sim</td>
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<td>0.026 s</td>
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<tr>
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<td>1.428 s</td>
<td>0.000 s</td>
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<tr>
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<td>0.004 s</td>
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<tr>
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<td>1.422 s</td>
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<tr>
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<td>0.000 s</td>
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</tr>
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<td>0.001 s</td>
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</tr>
<tr>
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<td>1.343 s</td>
<td>0.001 s</td>
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<td>1.336 s</td>
<td>0.319 s</td>
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### Matlab Code Listing

#### Profile Summary

<table>
<thead>
<tr>
<th>Function Name</th>
<th>Calls</th>
<th>Total Time</th>
<th>Self Time*</th>
<th>Total Time Plot</th>
</tr>
</thead>
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<td>0.013 s</td>
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<td>0.057 s</td>
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<td>0.000 s</td>
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<tr>
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<td>0.024 s</td>
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<td>0.011 s</td>
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<td>0.003 s</td>
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<td>1.962 s</td>
<td>0.005 s</td>
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<tr>
<td>linGrad</td>
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<td>1.957 s</td>
<td>0.005 s</td>
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<tr>
<td>linGradTimeDomainGPU</td>
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<td>1.952 s</td>
<td>1.920 s</td>
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<td>1.845 s</td>
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<td>0.002 s</td>
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<td>network_sim&gt;simData</td>
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<td>1.420 s</td>
<td>0.001 s</td>
<td></td>
</tr>
<tr>
<td>network_sim&gt;simDataCellICMatrix</td>
<td>4</td>
<td>1.419 s</td>
<td>0.010 s</td>
<td></td>
</tr>
<tr>
<td>map</td>
<td>3</td>
<td>1.360 s</td>
<td>0.001 s</td>
<td></td>
</tr>
<tr>
<td>mapping.mtimes</td>
<td>3</td>
<td>1.360 s</td>
<td>0.000 s</td>
<td></td>
</tr>
<tr>
<td>km_map</td>
<td>1</td>
<td>1.346 s</td>
<td>0.323 s</td>
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</tr>
</tbody>
</table>

Figure A.2: Notice the improvement in the core functions, which in this very short case only represent a improvement from 42s to 27s. Yet the improvement in the core functions is from 15s to 3s. And those are the ones which are limiting, and will increase with data, and number of channels - smaller step etc. The rest is mostly constant overhead.
Matlab Code Listing

```
% Matlab Code Listing

0.00 1209 1280  if options.wavMethod == 0
0.09 1209 1281  agrad = sqspmv(mat(adgrad,[]), b);
0.01 1209 1282  elseif options.wavMethod == 1
0.18 1209 1283  agrad = sqser(mat(adgrad,[]));
0.19 1209 1284  dimag = length(strf.delay);
0.20 1209 1285  elseif options.wavMethod == 2
0.21 1209 1286  agrad = sum(sqser(mat(adgrad,[])),2);
0.22 1209 1287  dimag = 1;
0.23 1209 1288  end
1209 1289
0.23 1209 1290  agrad = agrad(1);
0.24 1209 1291
0.26 1209 1292  [ad,aidx] = max(agrid); %
0.46 1209 1293  stride = find(adgrad*(options.threshold.*ad));
1209 1294
0.50 1209 1295  [sg,sgdx] = indlux(mat(strf.gids, dimag), adidx);
0.51 1209 1296  aidx = unique(sg);
0.52 1209 1297  % changes to apply threshold to each group
0.53 1209 1298  for aidx = 1:aidx
0.54 1209 1299  ty = find(advec{1}(strf.gids, aidx));
1300 1209 1300
1301 1209 1302  stride_c = find(advec{2}.up, 1):length(advec{2}.up); %
14.06 1209 1303  stride_c = stride_c(find(advec{2}.up, 1):end)
14.07 1209 1304  stride = [stride stride_c];
14.08 1209 1305  end
14.14 1209 1306
0.01 1209 1307  if options.activeSet
0.30 1209 1308  actvec = unique([activeSet stride size(grad,2)]);
1209 1309 1209 1310  size = [size(1) size(2)];
1209 1311 1209 1312  end
1209 1313
1209 1314
14.10 1209 1315  grad = grad(:); % length(gids, 2);
0.16 1209 1316  grad = grad(idx); % length(gids, 2);
0.17 1209 1317  xnew = xold + options.stepSize * grad;
0.18 1209 1318  xold = x;
1317 1319 1317
1317 1320
1317 1321

Figure A.3: Before correction - in red, time spent in that segment of code - overall 2000s
Matlab Code Listing

```matlab
else if options.method == '2'
    grad = num2nan(num2grad(u) ./ 2);
    disp('2')
end
end

% after correction, same results, but the direct mapping performs the same in 200s
% (10 x less)

% changed to apply threshold to each group

% after group

% with_t = find(group == 1);

% stds_t = stds_t(find(abs(grad)<=threshold*mean(abs(grad))));
end

if options.activeSet
    activeSet = unique(activeSet \\ stds * grad); \\ stds * grad);
end

grad = grad ./ norm(grad);
flag = 1;

end

% calculate and record error

% calculate and record error

% options.diagnostics.x(i, iter) = err;
if options.earlyStop
    if err
end
```

Figure A.4: After correction, same results, but the direct mapping performs the same in 200s (10 x less)
References


REFERENCES


REFERENCES


