ABSTRACT

In this paper an extension to the Dyakonov-Shur model is presented in which the electron fluid is considered to be moving on a suspended membrane subject to an uniform electric field, modeling the transverse oscillations of an graphene layer. The new dispersion relation is investigated and a new instability mechanism is proposed. Some numerical simulations were also performed showing the effective coupling of electron flow and membrane vibration.

Keywords: Graphene transistor, graphene hydrodynamics, Dyakonov-Shur, instability, Kirchhoff-Love plate theory, flexural modes.

INTRODUCTION

Dyakonov and Shur showed that the electron flow in a field effect transistor can be modeled by an hydrodynamic system of equations [Dyakonov and Shur, 1993, Dyakonov, 1997, Dyakonov, 2011] for the density $n$ and velocity $v$ fields of the form

\[
\begin{align*}
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) &= 0 \\
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} &= -\frac{e}{m} \frac{\partial \Phi}{\partial x}
\end{align*}
\]

where $e$ is the electron charge, $m$ its mass and $\Phi$ the electric potential applied, this set of equations is akin to the shallow water problem and originates the dispersion relation $\omega = k(v_0 \pm S)$, with $S$ being the sound speed of the fluid and $v_0$ the equilibrium drift velocity. Moreover for the asymmetric boundary conditions of constant charge density at drain and constant current at source,

\[
 n(0,t) = n_0 \quad \text{and} \quad n(L,t)v(L,t) = n_0 v_0
\]

respectively, a plasmonic instability develops itself [Crowne, 1997, Dmitriev et al., 1996].

The recent advances in graphene based transistors revived the interest in such approach and several hydrodynamic models have been recently studied for graphene systems [Chaves et al., 2017,Svintsov et al., 2013] and in such models two important issues must be addressed: the challenging definition of mass and the nature of scattering mechanisms. One important aspect is the coupling of the fluid dynamics with graphene phonons, in particular scattering with flexural ones [Castro et al., 2010].

The case of electron coupling with flexural phonons can be studied considering that the inviscid electron fluid moves on a suspended membrane clamped between the source and drain of transistor as outlined in Figure 1.
CHARGED VIBRATING MEMBRANE

In the case of a parallel plates approximation for the gate and membrane the system capacitance is given by the relation

$$C = \frac{enA}{\Phi} = \frac{\varepsilon A}{d},$$

where $A$ is the area of the plates, $d$ is the separation and $\varepsilon$ the medium permittivity. If the separation of the plates is written in the form

$$d = d_0 + \eta$$

the electric potential between the plates is

$$\Phi = \frac{en(d_0 + \eta)}{\varepsilon}.\]

In this system the Lagrangian density of the flexural phonons [Liu et al., 2016] is corrected by the electromagnetic Lagrangian density in the absence of magnetic fields

$$\mathcal{L} = \mathcal{L}_{\text{flex}} + \mathcal{L}_{\text{em}} = \frac{1}{2} \left[ \rho \ddot{\eta}^2 - \gamma (\nabla \eta)^2 - D (\nabla^2 \eta)^2 \right] + \frac{e^2 n^2 (d_0 + \eta)}{\varepsilon} + \frac{e^2 n^2}{2\varepsilon},$$

and therefore the Euler-Lagrange equation for the 1D+1 system is

$$\rho \frac{\partial^2 \eta}{\partial t^2} - \gamma \frac{\partial^2 \eta}{\partial x^2} + D \frac{\partial^4 \eta}{\partial x^4} = -\frac{e^2 n^2}{\varepsilon},$$

where $\rho$ is the surface mass density, $D$ the bending stiffness and $\gamma$ the strain force applied.

Combining the previous equation (4) with the hydrodynamic model (1) the coupled system obtained is given by

$$\begin{cases}
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = 0 \\
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{\partial}{\partial x} \left( \frac{e^2 d_0 n (1 + \eta/d_0)}{m \varepsilon} \right) = 0 \\
\frac{\partial^2 \eta}{\partial t^2} - \frac{\gamma}{\rho} \frac{\partial^2 \eta}{\partial x^2} + \frac{D}{\rho} \frac{\partial^4 \eta}{\partial x^4} = -\frac{e^2 n^2}{\rho \varepsilon}
\end{cases}$$

these equations can be nondimensionalized redefining the variables in terms of the channel length $L$, gate to membrane distance $d_0$ and characteristic velocity and density $v_0$ and $n_0$ writing

$$x^* \equiv x/L \quad t^* \equiv t v_0/L \quad v^* \equiv v/v_0 \quad n^* \equiv n/n_0 \quad \eta^* \equiv \eta/d_0$$

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so the coupled system becomes

\[
\begin{aligned}
\frac{\partial n^*}{\partial t^*} + \frac{\partial}{\partial x} (n^*v^*) &= 0 \\
\frac{\partial v^*}{\partial t^*} + v^* \frac{\partial v^*}{\partial x^*} + \frac{S^2}{v_0^2} \frac{\partial}{\partial x^*} [n^*(1 + \eta^*)] &= 0 \\
\frac{\partial^2 \eta^*}{\partial t^*^2} - \frac{\Gamma^2}{v_0^2} \frac{\partial^2 \eta^*}{\partial x^*^2} + \frac{\Delta^2}{v_0^2} \frac{\partial^4 \eta^*}{\partial x^*^4} &= -C^2 \frac{\partial^2 n^*}{\partial x^*^2}
\end{aligned}
\]  

(7)

where the following quantities were employed

\[
S^2 = \frac{e^2 d_0 n_0}{\rho \varepsilon}, \quad \Gamma^2 = \frac{\gamma}{\rho}, \quad \Delta^2 = \frac{D}{\rho L^2} \quad \text{and} \quad C^2 = \frac{e^2 n_0^2 L^2}{\rho \varepsilon d_0}.
\]  

(8)

**DISPERSION RELATION ANALYSIS**

Expanding the fields, around its equilibrium value, in the form \(a = a_0 + \tilde{a} \exp[-i\omega t + ikx]\) and imposing \(\eta_0 = 0\), the set of equations (5) can be linearized in the following system

\[
\begin{aligned}
(\omega - kv_0) \hat{n} - kn_0 \hat{v} &= 0 \\
-k \frac{e^2 d_0}{m \varepsilon} \hat{n} + (\omega - kv_0) \hat{v} - k \frac{e^2 n_0}{m \varepsilon} \hat{\eta} &= 0 \\
2e^2 \frac{\eta_0}{\rho \varepsilon} \hat{n} + \left(\frac{\gamma}{\rho} k^2 + \frac{D}{\rho} k^4 - \omega^2\right) \hat{\eta} &= -\frac{e^2 n_0^2}{\rho \varepsilon}
\end{aligned}
\]  

(9)

which leads to the sixth order secular equation

\[
[\Gamma^2 k^2 + \Delta^2 L^2 k^4 - \omega^2] [(\omega - kv_0)^2 - S^2 k^2] + \frac{2S^2 C^2}{L^2} k^2 = 0.
\]  

(10)

A typical plot of the dispersion relation is presented in Figure 2 where the mode mixing of the Dyakonov-Shur mode \(\omega = k(v_0 + S)\) and the flexural mode \(\omega^2 = \Gamma^2 k^2 + \Delta^2 L^2 k^4\) clearly visible. This two bare modes cross each other at \(k = \frac{\sqrt{(v_0 + S)^2 - \Gamma^2}}{\Delta L}\). At this value the frequency gap, \(\delta \omega\), is given in Figure 3.

With such dispersion relation the real part of the frequency vanishes for a critical value, \(k_c\), of the wave number (Figure 3) and the system exhibits an instability for \(0 < k < k_c\), such critical wave number is given by

\[
k_c^2 = \frac{\Gamma^2 (v_0^2 - S^2) \pm \sqrt{(S^2 - v_0^2) \left(\Gamma^4 (S^2 - v_0^2) + \frac{8S^2 C^2}{L^2}\right)}}{2(S^2 - v_0^2) L^2 \Delta^2}
\]  

(11)

which give a positive real number for \(S > v_0\) regardless the value of \(\Gamma\), which coincides with the threshold for the Dyakonov-Shur instability.
Fig. 2 - Dispersion relation from (10) with parameters $S = 5$, $\Gamma = 2$, $v_0 = 1$, $L = 1$, $C = 1$, $\Delta = 1$.

Fig. 3 - Variation of frequency gap at mode crossing with parameters $v_0 = 1$, $L = 1$, $C = 1$, $\Delta = 1$.

**NUMERICAL EXPERIMENTS**

The hydrodynamic equations of (7) were simulated using a second order Richtmyer two step Lax-Wendroff scheme [Kutler, 1969, LeVeque, 1992] and the Kirchhoff-Love part with a second order implicit time centered space method [Asakura et al., 2014], the spatial discretization was defined with $\Delta x = 1/150$ and $\Delta t = \Delta x/2S$ in order to satisfy the Courant-Friedrichs-Lewy condition. Both the Dyakonov-Shur boundary conditions (2) and periodic ones were tested.

Setting the initial condition $\eta(x,0)=0$ for the membrane and a small amplitude random density around $n0$ the excitation of the membrane is obtained in both boundary conditions. After a small transient time, Figure 4, the membrane starts to oscillate on the fundamental mode, either in the presence of the Dyakonov-Shur instability, where multiple frequencies are excited Figure 5, or self driven in the case of periodic boundaries, where the membrane develops a single frequency oscillation Figure 6.
Fig. 4 - Membrane displacement for initial simulation time up to 40Δt. Computed with parameters S=5, Γ =2, v₀=1, L=1, C= 1, Δ = 1.

Fig. 5 - Numerical simulation results for the electron density and membrane displacement at x=L/2 with the parameters S= 5, Γ = 2, v₀= 1, L= 1, C= 1, Δ = 1. Dyakonov-Shur instability boundary conditions.

Fig. 6 - Numerical simulation results for the electron density and membrane displacement at x=L/2 with the parameters S= 5, Γ = 2, v₀= 1, L= 1, C= 1, Δ = 1. Dyakonov-Shur instability boundary conditions.
CONCLUDING REMARKS

The coupling of the Dyakonov-Shur model with a Kirchhoff-Love vibrating membrane is able to self drive the membrane oscillations and it doesn't seem to mitigate the plasma oscillations when in its presence. Moreover, the obtained dispersion relation for the system exhibits an instability due to the vanishing real frequency at $k_c$ and the threshold for $k_c$ is consistent with the usual Dyakonov-Shur instability criterion $S>v_0$.

Such processes are expected to be usable in the drive and control of future terahertz oscillators in which the applied mechanical strain on the membrane could modify the plasmon behavior. However, a more in depth study of the herein presented model is required.

REFERENCES


